CHAPTER 3

CURRENT ELECTRICITY

3.1 CURRENT ELECTRICITY

1. *What is current electricity ?*

Current electricity. In chapters 1 and 2, we studied the phenomena associated with the electric charges at rest. The physics of charges at rest is called ***electrostatics*** or ***static electricity***. We shall now study the motion or *dynamics* of charges. As the term current implies some sort of motion, so the motion of electric charges constitutes an electric current.

The study of electric charges in motion is called ***current electricity***.

3.2 ELECTRIC CURRENT

2. *Define electric current.*

Electric current. If two bodies charged to different potentials are connected together by means of a conducting wire, charges begin to flow from one body to another. The charges continue to flow till the potentials of the two bodies become equal.

The flow of electric charges through a conductor constitutes an ***electric current***. Quantitatively, *electric current in a conductor across an area held perpendicular to the direction of flow of charge is defined as the amount of charge flowing across that area per unit time.*

If a charge ΔQ passes through an area in time t to $t + \Delta t$, then the current I at time t is given by

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

If the current is steady *i.e.*, the rate of flow of charge does not change with time, then

$$I = \frac{Q}{t}$$

or $$\text{Electric current} = \frac{\text{Electric charge}}{\text{Time}}$$

where Q is the charge that flows across the given area in time t.

Lightning, which is the flow of electric charge between two clouds or from a cloud to the earth, is an example of a transient current (a current of short duration). But the charges flow in a steady manner in devices like a torch, cell-driven clock, transistor radios, hearing aids, etc.

3. *Give the SI unit of current.*

SI unit of current is ampere. *If one coulomb of charge crosses an area in one second, then the current through that area is one ampere* (A).

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

or $$1\text{ A} = 1\text{ Cs}^{-1}.$$

Ampere is one basic SI unit. We shall formally define it in chapter 4 in terms of magnetic effect of current. Smaller currents are expressed in following units :

$$1 \text{ milliampere} = 1\text{ mA} = 10^{-3}\text{ A}$$

$$1 \text{ microampere} = 1\,\mu\text{A} = 10^{-6}\text{ A}$$

The orders of magnitude of some electric currents we come across in daily life are as follows :

Current in a domestic appliance $\simeq 1$ A

Current carried by a lightning $\simeq 10^4$ A

Current in our nerves $\simeq 10^{-6}$ A $= 1\,\mu$A.

4. *Distinguish between conventional and electronic currents.*

Conventional and electronic currents. By convention, the direction of motion of positive charges is taken as the direction of electric current. However, a negative charge moving in one direction is equivalent to an equal positive charge moving in the opposite direction, as shown in Fig. 3.1. As the electrons are negatively charged particles, so the direction of electronic current (*i.e.*, the current constituted by the flow of electrons) is opposite to that of the conventional current.

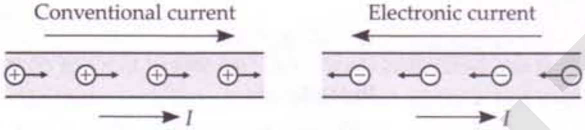


Fig. 3.1 Flow of negative charge is equivalent to the flow of positive charge in the opposite direction.

5. *Is electric current a scalar or vector quantity ?*

Electric current is a scalar quantity. Although electric current has both magnitude and direction, yet it is a scalar quantity. This is because the laws of ordinary algebra are used to add electric currents and the laws of vector addition are not applicable to the addition of electric currents. For example, in Fig. 3.2, two different currents of 3 A and 4 A flowing in two mutually perpendicular wires *AO* and *BO* meet at the junction *O* and then flow along wire *OC*. The current in wire *OC* is 7 A which is the scalar addition of 3 A and 4 A and not 5 A as required by vector addition.

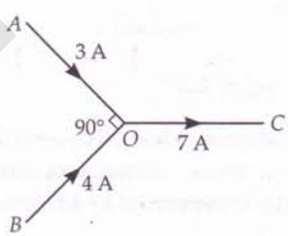


Fig. 3.2 Addition of electric currents is scalar.

Examples based on

Definitions of Electric Current

Formulae Used

1. Electric current $= \dfrac{\text{Charge}}{\text{Time}}$ or $I = \dfrac{q}{t}$
2. As $q = ne$, so $I = \dfrac{ne}{t}$
3. In case of an electron revolving in a circle of radius r with speed v, period of revolution of the electron is

$$T = \frac{2\pi r}{v}$$

Frequency of revolution, $\nu = \dfrac{1}{T} = \dfrac{v}{2\pi r}$

Current at any point of the orbit is

I = Charge flowing in 1 revolution × No. of revolutions per second

or $I = e\nu = \dfrac{ev}{2\pi r}$.

Units Used

Electric charge is in coulomb (C), time in second (s), and current in ampere (A)

Constant Used

Charge on an electron, $e = 1.6 \times 10^{-19}$C.

Example 1. *10^{20} electrons, each having a charge of 1.6×10^{-19} C, pass from a point A towards another point B in 0.1 s. What is the current in ampere ? What is its direction ?*

Solution. Here $n = 10^{20}$, $e = 1.6 \times 10^{-19}$ C, $t = 0.1$ s

Current,

$$I = \frac{q}{t} = \frac{ne}{t} = \frac{10^{20} \times 1.6 \times 10^{-19}\,\text{C}}{0.1\,\text{s}} = \mathbf{160\ A.}$$

The direction of current is from *B* to *A*.

Example 2. *Show that one ampere is equivalent to a flow of 6.25×10^{18} elementary charges per second.* [CBSE D 92C]

Solution. Here $I = 1$ A, $t = 1$ s, $e = 1.6 \times 10^{-19}$ C

As $I = \dfrac{q}{t} = \dfrac{ne}{t}$

∴ Number of electrons,

$$n = \frac{It}{e} = \frac{1 \times 1}{1.6 \times 10^{-19}} = \mathbf{6.25 \times 10^{18}}.$$

Example 3. *How many electrons pass through a lamp in one minute, if the current is 300 mA ?* [Himachal 95 ; Punjab 02]

Solution. $I = 300$ mA $= 300 \times 10^{-3}$ A,

$t = 1$ minute $= 60$ s, $e = 1.6 \times 10^{-19}$ C

As $I = \dfrac{q}{t} = \dfrac{ne}{t}$

∴ Number of electrons,

$$n = \frac{It}{e} = \frac{300 \times 10^{-3} \times 60}{1.6 \times 10^{-19}} = \mathbf{1.125 \times 10^{20}}.$$

Example 4. *How many electrons per second flow through a filament of a 120 V and 60 W electric bulb ? Given electric power is the product of voltage and current.*

Solution. Here $V = 120$ V, $P = 60$ W, $t = 1$ s

As $P = VI$, therefore, $I = \frac{P}{V} = \frac{60}{120} = 0.5$ A

Number of electrons,

$$n = \frac{It}{e} = \frac{0.5 \times 1}{1.6 \times 10^{-19}} = \mathbf{3.125 \times 10^{18}}.$$

Example 5. *In the Bohr model of hydrogen atom, the electron revolves around the nucleus in a circular path of radius 5.1×10^{-11} m at a frequency of 6.8×10^{15} revolutions per second. Calculate the equivalent current.*

Solution. Here $r = 5.1 \times 10^{-11}$ m,

$\nu = 6.8 \times 10^{15}$ rps, $e = 1.6 \times 10^{-19}$ C

Current,

$$I = e\nu = 1.6 \times 10^{-19} \times 6.8 \times 10^{15} = \mathbf{1.088 \times 10^{-3}\ A}.$$

Example 6. *In a hydrogen atom, an electron moves in an orbit of radius 5.0×10^{-11} m with a speed of 2.2×10^6 ms^{-1}. Find the equivalent current. (Electronic charge $= 1.6 \times 10^{-19}$ coulomb).* **[Roorkee 84]**

Solution. Here $r = 5.0 \times 10^{-11}$ m,

$v = 2.2 \times 10^6$ ms^{-1}, $e = 1.6 \times 10^{-19}$ C

Period of revolution of electron,

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 5.0 \times 10^{-11}}{2.2 \times 10^6}\ \text{s}$$

Frequency,
$$\nu = \frac{1}{T} = \frac{2.2 \times 10^6}{2\pi \times 5.0 \times 10^{-11}}$$

$$= \frac{2.2 \times 7 \times 10^{17}}{2 \times 22 \times 5} = 7 \times 10^{15}\ \text{s}^{-1}$$

Current,
$$I = e\nu = 1.6 \times 10^{-19} \times 7 \times 10^{15}$$
$$= \mathbf{1.12 \times 10^{-3}\ A}.$$

Example 7. *Figure 3.3 shows a plot of current I through the cross-section of a wire over a time interval of 10 s. Find the amount of charge that flows through the wire during this time period.*

[CBSE OD 15C]

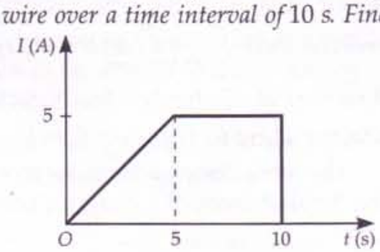


Fig. 3.3

Solution. Amount of charge that flows in 10 s

= Area under the I-t graph

$= \frac{1}{2} \times 5 \times 5 + (10 - 5)\,5 = \mathbf{37.5\ C}$.

Example 8. *The amount of charge passing through cross-section of a wire is* $q(t) = at^2 + bt + c$

(i) Write the dimensional formulae for a, b and c.

(ii) If the values of a, b and c in SI units are 5, 3 and 1 respectively, find the value of current at t = 5 second.

Solution. (*i*) Given $q(t) = at^2 + bt + c$

Dimension of $a = \left[\frac{q}{t^2}\right] = \frac{\text{AT}}{\text{T}^2} = \text{AT}^{-1}$

Dimension of $b = \left[\frac{q}{t}\right] = \frac{\text{AT}}{\text{T}} = \text{A}$

Dimension of $c = [q] = \text{AT}$

(*ii*) Current, $I = \frac{dq}{dt} = \frac{d}{dt}(at^2 + bt + c) = 2at + b$

At $t = 5$ s, $I = 2 \times 5 \times 5 + 3 = \mathbf{53\ A}$.

Problems For Practice

1. One billion electrons pass from a point P towards another point Q in 10^{-3} s . What is the current in ampere ? What is its direction ?
(Ans. 1.6×10^{-7} A, direction of current is from Q to P)
2. If 2.25×10^{20} electrons pass through a wire in one minute, find the magnitude of the current flowing through the wire. **[Punjab 02]** (Ans. 0.6 A)
3. A solution of sodium chloride discharges 6.1×10^{16} Na^+ ions and 4.6×10^{16} Cl^- ions in 2 s. Find the current passing through the solution.
(Ans. 8.56×10^{-3} A)
4. An electric current of 2.0 μA exists in a discharge tube. How much charge flows across a cross-section of the tube in 5 minutes ? (Ans. 6.0×10^{-4} C)
5. In a hydrogen atom, the electron makes about 0.6×10^{16} revolutions per second around the nucleus. Determine the average current at any point on the orbit of the electron. (Ans. 0.96 mA)
6. An electron moves in a circular orbit of radius 10 cm with a constant speed of 4.0×10^6 ms^{-1}. Determine the electric current at a point on the orbit.
(Ans. 1.02×10^{-12} A)
7. In a hydrogen discharge tube, the number of protons drifting across a cross-section per second is 1.1×10^{18}, while the number of electrons drifting in the opposite direction across another cross-section is 3.1×10^{18} per second. Find the current flowing in the tube. (Ans. 0.672 A)

HINTS

1. $I = \dfrac{ne}{t} = \dfrac{10^9 \times 1.6 \times 10^{-19}}{10^{-3}} = \mathbf{1.6 \times 10^{-7}\ A}$

2. $I = \dfrac{ne}{t} = \dfrac{2.25 \times 10^{20} \times 1.6 \times 10^{-19}}{60} = \mathbf{0.6\ A}$

3. $I = I_{cations} + I_{anions} = \dfrac{(n^+ + n^-)\,e}{t}$

   $= \dfrac{(6.1 \times 10^{16} + 4.6 \times 10^{16}) \times 1.6 \times 10^{-19}}{2} = \mathbf{8.56 \times 10^{-3}\ A}$

4. $q = It = 2.0 \times 10^{-6} \times 5 \times 60 = \mathbf{6.0 \times 10^{-4}\ C}$.

5. $I = \nu e = 0.6 \times 10^{16} \times 1.6 \times 10^{-19}$

   $= 0.96 \times 10^{-3}\ A = \mathbf{0.96\ mA}$

6. $T = \dfrac{2\pi r}{v} = \dfrac{2\pi \times 0.10}{4 \times 10^6}\ s \quad \therefore \quad \nu = \dfrac{1}{T} = \dfrac{4 \times 10^6}{2\pi \times 0.10}\ s^{-1}$

   $I = \nu e = \dfrac{4 \times 10^6 \times 1.6 \times 10^{-19}}{2\pi \times 0.10} = \mathbf{1.02 \times 10^{-12}\ A}$.

7. $I = I_p + I_n = \dfrac{(n_p + n_e)\,e}{t}$

   $= \dfrac{(1.1 \times 10^{18} + 3.1 \times 10^{18}) \times 1.6 \times 10^{-19}}{1} = \mathbf{0.672\ A}$

3.3 MAINTENANCE OF STEADY CURRENT IN A CIRCUIT

6. *With the help of a mechanical analogy, explain how the flow of electric current is maintained in an electric circuit.*

Maintenance of steady current in an electric circuit. The flow of electric current in a circuit is analogous to the flow of water in a pipe. As shown in Fig. 3.4, suppose we wish to maintain a steady flow of water in a horizontal pipe from A to B. As pressure at A is higher than that at B, so water flows spontaneously from the upper tank to the lower tank. To maintain a steady flow, a water pump must do work at a steady rate to pump water back from the lower tank to the upper tank. Obviously, the water pump makes water flow from lower to higher pressure. It helps to maintain the pressure difference between A and B.

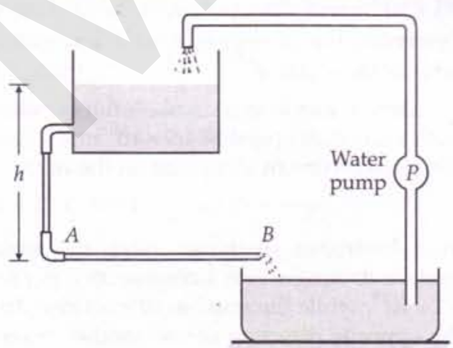


Fig. 3.4 A closed water flow circuit.

A steady flow of electric current in a conductor is maintained in a similar way. As shown in Fig. 3.5, positive charge flows spontaneously in a conductor from higher potential (A) to lower potential (B) *i.e.*, in the direction of the electric field. To maintain steady current through the conductor, some external device must do work at a steady rate to take positive charge from lower potential (B) to the higher potential (A). Such a device is the source of *electromotive force* (emf) which may be an electrochemical cell or an electric-generator. A source of emf transfers positive charge form lower potential to higher potential *i.e.*, in the opposite direction of the electric field. Clearly, a charge flow circuit is analogous to the water flow circuit.

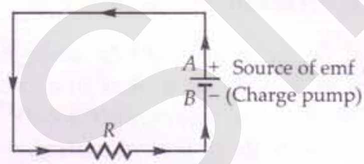


Fig. 3.5 A closed charge flow circuit.

3.4 ELECTROMOTIVE FORCE : EMF

7. *Define emf of a battery. Is it really a force ? When is the emf of a battery equal to the potential difference between its terminals ? Define emf of 1 volt.*

Electromotive force. A battery is a device which maintains a potential difference between its two terminals A and B.

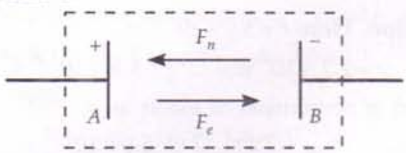


Fig. 3.6 A schematic diagram of a battery.

Figure 3.6 shows a schematic diagram of a battery. Due to certain chemical reactions, a force (of non-electrostatic origin) is exerted on the charges of the electrolyte. This force drives positive charges towards terminal A and negative charges towards terminal B. Suppose the force on a positive charge q is $\vec{F_n}$. As the charges build up on the two terminals A and B, a potential difference is set up between them. An electric field $\vec{E}$ is set up in the electrolyte from A to B. This field exerts a force $\vec{F_e} = q\vec{E}$ on the charge q, in the opposite direction of $\vec{F_n}$. In the steady state, the charges stop accumulating further and $F_n = F_e$.

The work done by the non-electrostatic force during the displacement of a charge q from B to A is

$$W = F_n\, d$$

where d is the distance between the terminals A and B.

The work done per unit charge is

$$\mathcal{E} = \frac{W}{q} = \frac{F_n\, d}{q}$$

The quantity $\mathcal{E} = W/q$ is called the *electromotive force or emf* of the battery or any other source.

*The **electromotive force** of a source may be defined as the work done by the source in taking a unit positive charge from lower to the higher potential.*

If the two terminals of the battery are not connected externally, then

$$F_n = F_e = qE$$

$$\therefore \quad F_n d = F_e\, d = q\, Ed = qV$$

where $V = Ed$ is the p.d. between the two terminals. Thus,

$$\mathcal{E} = \frac{F_n\, d}{q} = \frac{qV}{q} = V$$

Hence *the emf of a source is equal to the maximum potential difference between its terminals when it is in the open circuit i.e.,* when it is not sending any current in the circuit.

Basically, an electrochemical cell consists of two electrodes P and N immersed in an electrolyte, as shown in Fig. 3.7

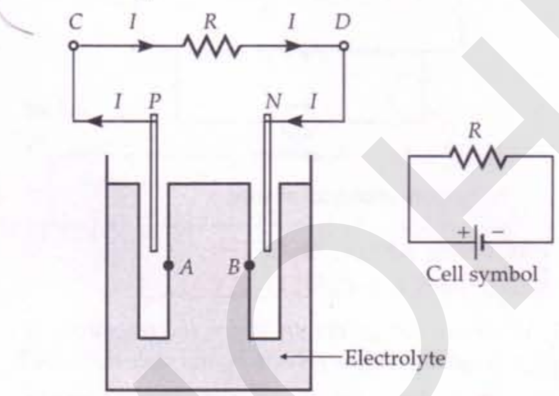


Fig. 3.7 An electrochemical cell connected to an external resistance and the symbolic representation. Here $V_P - V_A = V_+ > 0$ and $V_N - V_B = -V_- < 0$.

The two electrodes exchange charges with the electrolyte. Consequently, the positive electrode P develops a positive potential V_+ ($V_+ > 0$) with respect to its adjacent electrolyte marked A. The negative electrode N develops a negative potential $-V_-$ ($V_- > 0$) with respect to the adjacent electrolyte B. When no current flows through the cell, the electrolyte has the same potential throughout, so that the potential difference between the two electrodes P and N is

$$V_+ - (-V_-) = V_+ + V_- = \mathcal{E}, \text{ the emf.}$$

Obviously, $V_+ + V_- > 0$.

In case of a closed circuit, we can define emf in another way as follows :

The emf of a source may be defined as the energy supplied by the source in taking a unit positive charge once round the complete circuit. Again, we note that

$$\text{emf} = \frac{\text{Work done}}{\text{Charge}} \quad \text{or} \quad \mathcal{E} = \frac{W}{q}$$

Literally, emf means the force which causes the flow of charges in a circuit. However, the term emf is a misnomer. The emf is not a force at all. It is a special case of potential difference, so it has the nature of work done per unit charge.

SI unit of emf is volt. *If an electrochemical cell supplies an energy of 1 joule for the flow of 1 coulomb of charge through the whole circuit (including the cell), then its emf is said to be one volt.*

3.5 EMF VS. POTENTIAL DIFFERENCE

8. *Give important points of differences between electromotive force and potential difference.*

Differences between electromotive force and potential difference.

	Electromotive force	Potential difference
1.	It is the work done by a source in taking a unit charge once round the complete circuit.	It is the amount of work done in taking a unit charge from one point of a circuit to another.
2.	It is equal to the maximum potential difference between the two terminals of a source when it is in an open circuit.	Potential difference may exist between any two points of a closed circuit.
3.	It exists even when the circuit is not closed.	It exists only when the circuit is closed.
4.	It has non-electrostatic origin.	It originates from the electrostatic field set up by the charges accumulated on the two terminals of the source.
5.	It is a cause. When emf is applied in a circuit, potential difference is caused.	It is an effect.
6.	It is equal to the sum of potential differences across all the components of a circuit including the p.d. required to send current through the cell itself.	Every circuit component has its own potential difference across its ends.
7.	It is larger than the p.d. across any circuit element.	It is always less than the emf.
8.	It is independent of the external resistance in the circuit.	It is always less than the emf.

3.6 OHM'S LAW : RESISTANCE

9. *State Ohm's law. Define resistance and state its SI unit.*

Ohm's law. On the basis of his experimental observations, a German physicist *George Simon Ohm* derived a relationship between electric current and potential difference in 1828. This relationship is known as *Ohm's law* and can be stated as follows :

The current flowing through a conductor is directly proportional to the potential difference applied across its ends, provided the temperature and other physical conditions remain unchanged.

Thus, Potential difference ∝ Current

$$V \propto I$$

or $$V = RI$$

The proportionality constant R is called the *resistance* of the conductor. Its value is independent of V and I but depends on the nature of the conductor, its length and area of cross-section and physical conditions like temperature, etc. Ohm's law may also be expressed as

$$\frac{V}{I} = R$$

The graph between the potential difference V applied across a conductor to the current I flowing through it is a straight line, as shown in Fig. 3.8.

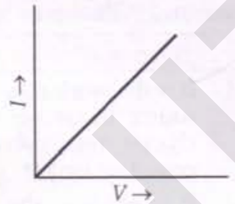


Fig. 3.8 *V-I* graph for an ohmic conductor.

Resistance. *The resistance of a conductor is the property by virtue of which it opposes the flow of charges through it.* The more the resistance, the less is the current I for a given potential difference. *It is equal to the ratio of the potential difference applied across the conductor to the current flowing through it.* Thus

$$R = \frac{V}{I}$$

SI unit of resistance is ohm (Ω). If the potential difference (V) is 1 volt and current (I) is 1 ampere, then the resistance (R) is 1 ohm.

$$\therefore \quad 1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}}$$

or $$1\,\Omega = 1\,\text{VA}^{-1}.$$

Thus, *the resistance of a conductor is said to be 1 ohm if a current of 1 ampere flows through it on applying a potential difference of 1 volt across its ends.*

Any material that has some resistance is called a resistor. Pictorial symbols for resistors and meters are given in Fig. 3.9.

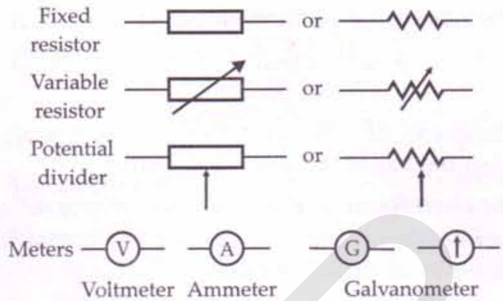


Fig. 3.9 Symbols for resistors and meters.

10. *Briefly explain how can we measure the resistance of a wire.*

Measurement of resistance. Fig. 3.10 shows a simple circuit for measuring the resistance of a wire. Here the battery and ammeter are connected in series with the wire and the voltmeter in parallel with it. The ratio of the voltmeter reading (V) and the ammeter reading (I) gives the resistance (R) of the wire.

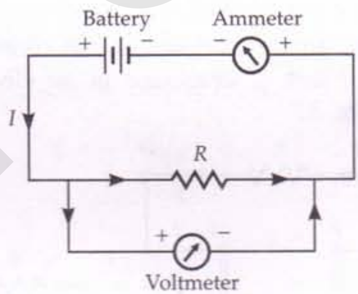


Fig. 3.10 To measure resistance of wire.

3.7 FACTORS AFFECTING THE RESISTANCE : RESISTIVITY

11. *What are the factors on which the resistance of a conductor depends ? Define resistivity and state its SI unit.*

Factors affecting the resistance. At a constant temperature, the resistance of a conductor depends on the following factors :

1. Length. *The resistance R of a conductor is directly proportional to its length i.e.,*

$$R \propto l$$

2. Area of cross-section. *The resistance R of a uniform conductor is inversely proportional to its area of cross-section A, i.e.,*

$$R \propto \frac{l}{A}$$

3. Nature of the material. The resistance of a conductor also depends on the nature of its material. For example, the resistance of a nichrome wire is 60 times that of a copper wire of equal length and area of cross-section.

Combining the above factors, we get

$$R \propto \frac{l}{A} \qquad \text{or} \qquad R = \rho \frac{l}{A}$$

where ρ is the constant of proportionality called *resistivity* or *specific resistance* of the material of the conductor. It depends on the nature of the material of the conductor and on the physical conditions like temperature and pressure but it is independent of its size or shape.

Resistivity or specific resistance. If in the above equation, we take

$$l = 1 \text{ unit and } A = 1 \text{ square unit}$$

then $R = \rho$

Thus, *the resistivity or specific resistance of a material may be defined as the resistance of a conductor of that material, having unit length and unit area of cross-section. Or, it is the resistance offered by the unit cube of the material of a conductor.*

SI unit of resistivity. We can write

$$\rho = \frac{R \times A}{l}$$

$$\therefore \quad \text{SI unit of } \rho = \frac{\text{ohm} \times \text{metre}^2}{\text{metre}}$$

$$= \text{ohm meter } (\Omega \text{ m})$$

Thus, the SI unit of resistivity is ***ohm metre*** (Ω m).

3.8 CURRENT DENSITY, CONDUCTANCE AND CONDUCTIVITY

12. *Define the terms current density, conductance and conductivity. Write their SI units. Express Ohm's law in vector form.*

Current density. *The current density at any point inside a conductor is defined as the amount of charge flowing per second through a unit area held normal to the direction of the flow of charge at that point.* It is a vector quantity having the same direction as that of the motion of the positive charge. It is a characteristic property of any point inside the conductor and is denoted by $\vec{j}$.

As shown in Fig. 3.11(*a*), if a current *I* is flowing uniformly and normally through an area of cross-section *A* of a conductor, then the magnitude of current density at any point of this cross-section will be

$$j = \frac{q/t}{A} = \frac{I}{A}$$

If the area *A* is not perpendicular to the direction of current and normal to this area makes angle θ with the direction of current as shown in Fig. 3.11(*b*), then the component of *A* normal to the direction of current flow will be

$$A_n = A \cos \theta$$

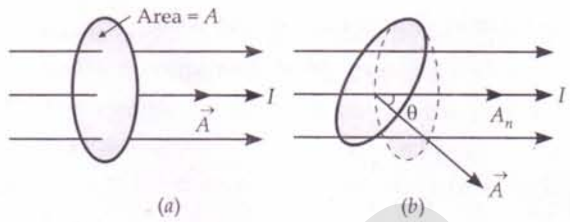


Fig. 3.11 Current density.

∴ Current density,

$$j = \frac{I}{A_n} = \frac{I}{A \cos \theta}$$

or

$$I = jA \cos \theta = \vec{j} \cdot \vec{A}$$

This equation again shows that electric current, being scalar product of two vectors, is a scalar quantity.

The SI unit of current density is ***ampere per square metre*** **(Am^{-2})** and its dimensions are $[AL^{-2}]$.

NOTE The current *I* through a particular surface *S* in a conductor is the flux of $\vec{j}$ through that surface and is given by the surface integral

$$I = \int_S \vec{j} \cdot \vec{dS}$$

where $\vec{dS}$ is a small element of the given surface area.

Conductance. *The conductance of a conductor is the ease with which electric charges flow through it. It is equal to the reciprocal of its resistance and is denoted by G.*

Thus,

$$\text{Conductance} = \frac{1}{\text{Resistance}}$$

or

$$G = \frac{1}{R}$$

The SI unit of conductance is ***ohm***$^{-1}$ or ***mho*** or ***siemens*** **(S)**

Conductivity. *The reciprocal of the resistivity of a material is called its conductivity* and is denoted by σ.

Thus,

$$\text{Conductivity} = \frac{1}{\text{Resistivity}}$$

or

$$\sigma = \frac{1}{\rho}$$

The SI unit of conductivity is **ohm^{-1} m^{-1}** or **mho m^{-1}** or **Sm^{-1}**.

Vector form of Ohm's Law. If *E* is the magnitude of electric field in a conductor of length *l*, then the potential difference across its ends is

$$V = El$$

Also from Ohm's law, we can write

$$V = IR = \frac{I\rho l}{A}$$

$$\therefore \quad El = \frac{I}{A}\rho l$$

or $$E = j\rho$$

As the direction of current density $\vec{j}$ is same as that of electric field $\vec{E}$, we can write the above equation as

$$\vec{E} = \rho\,\vec{j}$$

or $$\vec{j} = \sigma\,\vec{E}$$

The above equation is the ***vector form of Ohm's law***. It is equivalent to the scalar form $V = RI$.

3.9 CLASSIFICATION OF MATERIALS IN TERMS OF RESISTIVITY

13. *How can we classify solids on the basis of their resistivity values ?*

Classification of solids on the basis of their resistivity values. The electrical resistivity of substances varies over a very wide range, as shown in Table 3.1. Various substances can be classified into *three* categories :

1. Conductors. *The materials which conduct electric current fairly well are called conductors.* Metals are good conductors. They have low resistivities in the range of $10^{-8}\ \Omega$ m to $10^{-6}\ \Omega$ m. Copper and aluminium have the lowest resistivities of all the metals, so their wires are used for transporting electric current over large distances without the appreciable loss of energy. On the other hand nichrome has a resistivity of about 60 times that of copper. It is used in the elements of electric heater and electric iron.

2. Insulators. *The materials which do not conduct electric current are called insulators.* They have high resistivity, more than $10^4\ \Omega$ m. Insulators like glass, mica, bakelite and hard rubber have very high resistivities in the range $10^{14}\ \Omega$ m to $10^{16}\ \Omega$ m. So they are used for blocking electric current between two points.

3. Semiconductors. *These are the materials whose resistivities lie in between those of conductors and insulators i.e.,* between $10^{-6}\ \Omega$ m to $10^4\ \Omega$ m. Germanium and silicon are typical semiconductors. For moderately high resistances in the range of kΩ, resistors made of carbon (graphite) or some semiconducting material are used.

Table 3.1 Electrical resistivities of some substances

Material	Resistivity at 0°C ρ (Ωm)	Temperature coefficient of resistivity at 0°C, $\alpha = \frac{1}{\rho}\left(\frac{d\rho}{dT}\right)$ ($°C^{-1}$)	No. of valence electrons per unit cell
A. Conductors			
Silver	1.6×10^{-8}	0.0041	1
Copper	1.7×10^{-8}	0.0068	1
Aluminium	2.7×10^{-8}	0.0043	3
Tungsten	5.6×10^{-8}	0.0045	6
Iron	10×10^{-8}	0.0065	8
Platinum	11×10^{-8}	0.0039	10
Mercury	98×10^{-8}	0.0009	2
Nichrome (alloy of Ni, Fe, Cr)	100×10^{-8}	0.0004	
Manganin (alloy of Cu, Ni, Fe, Mn)	48×10^{-8}	0.002×10^{-3}	
B. Semiconductors			
Carbon (graphite)	3.5×10^{-5}	– 0.0005	4
Germanium	0.46	– 0.05	4
Silicon	2300	– 0.07	4
C. Insulators			
Pure water	2.5×10^5		–
Glass	$10^{10} - 10^{14}$		–
Hard Rubber	$10^{13} - 10^{16}$		–
NaCl	$\sim 10^{14}$		8
Fused quartz	$\sim 10^{16}$		–

14. *What are the two common varieties of commercial resistors ?*

Common commercial resistors. The commercial resistors are of two major types :

1. ***Wire-bound resistors.*** These are made by winding the wires of an alloy like manganin, constantan or nichrome on an insulating base. The advantage of using these alloys is that they are relatively insensitive to temperature. But inconveniently large length is required for making a high resistance.

2. ***Carbon resistors.*** They are made from mixture of carbon black, clay and resin binder which are pressed and then moulded into cylindrical rods by heating. The rods are enclosed in a ceramic or plastic jacket.

The carbon resistors are widely used in electronic circuits of radio receivers, amplifiers, etc. They have the following *advantages* :

(*i*) They can be made with resistance values ranging from few ohms to several million ohms.

(*ii*) They are quite cheap and compact.

(*iii*) They are good enough for many purposes.

3.10 COLOUR CODE FOR CARBON RESISTORS

15. *Describe the colour code used for carbon resistors.*

Colour code for resistors. *A colour code is used to indicate the resistance value of a carbon resistor and its percentage accuracy.* The colour code used throughout the world is shown in Table 3.2.

Table 3.2 Resistor colour code

Colour	*Letter as an aid to memory*	*Number*	*Multiplier*	*Colour*	*Tolerance*
Black	B	0	10^0	Gold	5%
Brown	B	1	10^1	Silver	10%
Red	R	2	10^2	No fourth band	20%
Orange	O	3	10^3		
Yellow	Y	4	10^4		
Green	G	5	10^5		
Blue	B	6	10^6		
Violet	V	7	10^7		
Grey	G	8	10^8		
White	W	9	10^9		

How to remember colour code :

B	B	R	O	Y of	Great	Britain had	Very	Good	Wife
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
0	1	2	3	4	5	6	7	8	9

There are *two* systems of marking the colour codes :

First system. A set of coloured co-axial rings or bands is printed on the resistor which reveals the following facts :

1. The first band indicates the *first significant figure.*
2. The second band indicates the second *significant figure.*
3. The third band indicates the *power of ten* with which the above two significant figures must be multiplied to get the resistance value in ohms.
4. The fourth band indicates the tolerance or possible variation in percent of the indicated value. If the fourth band is absent, it implies a tolerance of ± 20% .

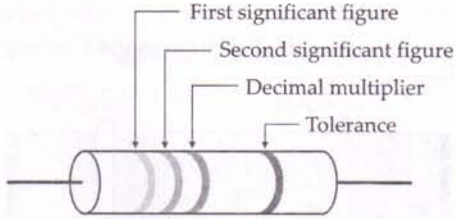


Fig. 3.12 Meanings of four bands.

Illustrations : **1.** In Fig. 3.13, the colours of the four bands are red, red, red and silver ; the resistance value is

Red	Red	Red	Silver
↓	↓	↓	↓
2	2	2	± 10%

$$R = 22 \times 10^2 \ \Omega \pm 10\%.$$

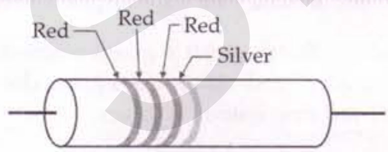


Fig. 3.13

2. In Fig. 3.14, the colours of the four bands are yellow, violet, brown and gold ; the resistance value is

Yellow	Violet	Brown	Gold
↓	↓	↓	↓
4	7	1	± 5%

$$R = 47 \times 10^1 \ \Omega \pm 5\%.$$

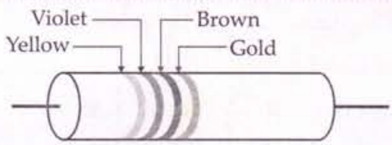


Fig. 3.14

3. When there are only three coloured bands printed on a resistor and there is no gold or silver band, the tolerance is 20%. In Fig. 3.15, there are only three bands of green, violet and red colours ; the resistance value is

Green	Violet	Red	No 4th band
↓	↓	↓	↓
5	7	2	± 20%

$$R = 57 \times 10^2 \ \Omega \pm 20\%.$$

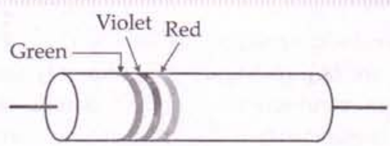


Fig. 3.15

Second System :

1. The colour of the body gives the first significant figure.

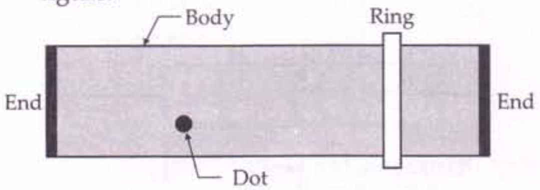


Fig. 3.16

2. The colour of the end gives the second significant figure.
3. The colour of the dot gives the number of zeroes to be placed after the second figure.
4. The colour of the ring gives the tolerance or percent accuracy of the indicated value.

Illustration. Suppose for a given resistor, the body colour is yellow, end colour is violet, dot colour is orange and the ring colour is silver.

Body Yellow	End Violet	Dot Orange	Ring Silver
↓	↓	↓	↓
4	7	3	± 10%

$\therefore R = 47 \times 10^3\ \Omega \pm 10\% = \mathbf{47\ k\Omega \pm 10\%}$.

Examples based on
Ohm's law, Resistance, Resistivity, Conductance, Conductivity, Current Density and Colour Code of Carbon Resistors

Formulae Used

1. Ohm's law, $R = \frac{V}{I}$ or $V = IR$
2. Resistance of a uniform conductor, $R = \rho \frac{l}{A}$
3. Resistivity or specific resistance, $\rho = \frac{RA}{l}$
4. Conductance $= \frac{1}{R}$
5. Conductivity $= \frac{1}{\text{Resistivity}}$ or $\sigma = \frac{1}{\rho} = \frac{l}{RA}$
6. Current density $= \frac{\text{Current}}{\text{Area}}$ or $j = \frac{I}{A}$
7. Colour code of carbon resistors. Refer to Table 3.2.

Units Used

Potential difference V is in volt (V), current I in ampere (A), resistance R in ohm (Ω), resistivity ρ in Ωm, conductance in ohm^{-1} or mho or siemens (S), conductivity in $\Omega^{-1} m^{-1}$ or Sm^{-1} and current density j in Am^{-2}.

Example 9. *In a discharge tube, the number of hydrogen ions (i.e., protons) drifting across a cross-section per second is 1.0×10^{18}, while the number of electrons drifting in the opposite direction across another cross-section is 2.7×10^{18} per second. If the supply voltage is 230 V, what is the effective resistance of the tube ?* [NCERT]

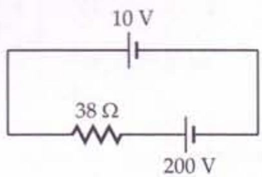
Solution. The current carried by a negatively charged electron is equivalent to the current carried by a proton in the opposite direction, therefore, total current in the direction of protons is

$$I = \text{Total charge flowing per second} = (n_e + n_p)e$$
$$= [2.7 \times 10^{18} + 1.0 \times 10^{18}] \times 1.6 \times 10^{-19}$$
$$= 3.7 \times 1.6 \times 10^{-1} = 0.592 \text{ A}$$

Effective resistance,

$$R = \frac{V}{I} = \frac{230}{0.592}\ \Omega = 388.5\ \Omega \simeq \mathbf{3.9 \times 10^2\ \Omega}.$$

Example 10. *A 10 V battery of negligible internal resistance is connected across a 200 V battery and a resistance of 38 Ω as shown in the figure. Find the value of the current in circuit.* [CBSE D13]



Solution. $I = \frac{V}{R} = \frac{200 - 10}{38} = \mathbf{5\ A}.$

Example 11. *A copper wire of radius 0.1 mm and resistance 1 k Ω is connected across a power supply of 20 V. (i) How many electrons are transferred per second between the supply and the wire at one end ? (ii) Write down the current density in the wire.*

Solution. Here $r = 0.1\text{ mm} = 0.1 \times 10^{-3}$ m,

$$R = 1\text{ k}\Omega = 10^3\ \Omega,\ V = 20\text{ V}$$

(*i*) Current, $I = \frac{V}{R} = \frac{20}{10^3} = 0.02$ A

No. of electrons,

$$n = \frac{q}{e} = \frac{It}{e}$$
$$= \frac{0.02 \times 1}{1.6 \times 10^{-19}} = \mathbf{1.25 \times 10^{17}}.$$

(*ii*) Current density,

$$j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{0.02}{3.14 \times (0.1 \times 10^{-3})^2}$$
$$= \mathbf{6.37 \times 10^5\ Am^{-2}}.$$

Example 12. *Current flows through a constricted conductor, as shown in Fig. 3.17. The diameter $D_1 = 2.0$ mm and the current density to the left of the constriction is $j = 1.27 \times 10^6$ Am^{-2}. (i) What current flows into the constriction ? (ii) If the current density is doubled as it emerges from the right side of the constriction, what is diameter D_2 ?*

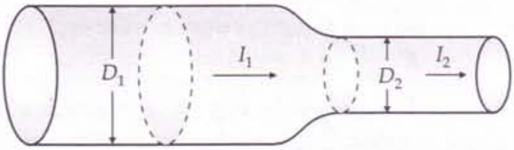


Fig. 3.17

Solution. Here $D_1 = 2.0$ mm, $j_1 = 1.27 \times 10^6 \text{ Am}^{-2}$, $j_2 = 2\, j_1$

(*i*) Current flowing into the constriction,

$$I_1 = j_1\, A = j_1 \times \pi \left(\frac{D_1}{2}\right)^2$$

$$= 1.27 \times 10^6 \times 3.14 \times (1 \times 10^{-3})^2 = \mathbf{3.987\ A.}$$

(*ii*) For a steady flow of current,

$$I_1 = I_2$$

or $$j_1 A_1 = j_2 \times A_2$$

or $$j_1 \times \pi \left(\frac{D_1}{2}\right)^2 = j_2 \times \pi \left(\frac{D_2}{2}\right)^2$$

or $$j_1 \times \pi \left(\frac{D_1}{2}\right)^2 = 2\, j_1 \times \pi \left(\frac{D_2}{2}\right)^2 \quad [\because\ j_2 = 2\, j_1]$$

or $$D_2 = \frac{1}{\sqrt{2}}\, D_1 = 0.707\, D_1$$

$$= 0.707 \times 2.0 \text{ mm} = \mathbf{1.414\ mm.}$$

Example 13. *A current of 2 mA is passed through a colour coded carbon resistor with first, second and third rings of yellow, green and orange colours. What is the voltage drop across the resistor ?*

Solution.

Yellow	Green	Orange
↓	↓	↓
4	5	3

$\therefore$ $R = 45 \times 10^3\ \Omega$

Given $I = 2 \text{ mA} = 2 \times 10^{-3}$ A

$\therefore$ $V = RI = 45 \times 10^3 \times 2 \times 10^{-3} \text{ V} = \mathbf{90\ V.}$

Example 14. *An arc lamp operates at 80 V, 10 A. Suggest a method to use it with a 240 V d.c. source. Calculate the value of the electric component required for this purpose.* [CBSE F 94]

Solution. Resistance of the arc lamp is

$$R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$$

In order to use arc lamp with a source of 240 V, a resistance R' should be connected in series with it so that current through the circuit does not exceed 10 A. Then

$$I\,(R + R') = V \quad \text{or} \quad 10(8 + R') = 240$$

or $$R' = 24 - 8 = \mathbf{16\ \Omega}.$$

Example 15. *Calculate the resistivity of a material of a wire 10 m long, 0.4 mm in diameter and having a resistance of 2.0 Ω.* [Haryana 02]

Solution. Here $l = 10$ m, $r = 0.2 \text{ mm} = 0.2 \times 10^{-3}$ m, $R = 2\ \Omega$

Resistivity,

$$\rho = \frac{RA}{l} = \frac{R \times \pi r^2}{l}$$

$$= \frac{2 \times 3.14 \times (0.2 \times 10^{-3})^2}{10} = \mathbf{2.513 \times 10^{-8}\ \Omega m.}$$

Example 16. *The external diameter of a 5 metre long hollow tube is 10 cm and the thickness of its wall is 5 mm. If the specific resistance of copper be 1.7×10^{-5} ohm-metre, then determine its resistance.*

Solution. The cross-sectional area of the tube is

$$A = \pi\,(r_2^2 - r_1^2)$$

$$= 3.14 \times [(5 \times 10^{-2})^2 - (4.5 \times 10^{-2})^2]$$

$$= 14.9 \times 10^{-4} \text{ m}^2$$

Also, $\rho = 1.7 \times 10^{-8}\ \Omega\text{m}, l = 5$ m

$\therefore$ Resistance,

$$R = \rho\, \frac{l}{A} = \frac{1.7 \times 10^{-8} \times 5}{14.9 \times 10^{-4}}$$

$$= \mathbf{5.7 \times 10^{-5}\ \Omega.}$$

Example 17. *Find the resistivity of a conductor in which a current density of 2.5 Am^{-2} is found to exist, when an electric field of 15 Vm^{-1} is applied on it.* [ISCE 98]

Solution. Here $j = 2.5 \text{ Am}^{-2}$, $E = 15 \text{ Vm}^{-1}$

Resistivity, $$\rho = \frac{RA}{l} = \frac{V}{I} \cdot \frac{A}{l}$$

$$= \frac{V / l}{I / A} = \frac{E}{j} = \frac{15}{2.5} = \mathbf{6\ \Omega\ m.}$$

Example 18. *Calculate the electrical conductivity of the material of a conductor of length 3 m, area of cross-section 0.02 mm^2 having a resistance of 2 Ω.*

Solution. Here $l = 3$ m, $R = 2\ \Omega$,

$$A = 0.02 \text{ mm}^2 = 0.02 \times 10^{-6} \text{ m}^2$$

$$\text{Electrical conductivity} = \frac{1}{\text{Resistivity}}$$

or $$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{3}{2 \times 0.02 \times 10^{-6}} \qquad \left[R = \rho \cdot \frac{l}{A}\right]$$

$$= \mathbf{75 \times 10^6\ \Omega^{-1} m^{-1}}.$$

Example 19. *A wire of resistance 4 Ω is used to wind a coil of radius 7 cm. The wire has a diameter of 1.4 mm and the specific resistance of its material is 2×10^{-7} Ωm. Find the number of turns in the coil.*

Solution. Let n be the number of turns in the coil.

Then total length of wire used

$$= 2\pi R \times n = 2\pi \times 7 \times 10^{-2} \times n \text{ metre}$$

Total resistance,

$$R = \rho \frac{l}{A} \quad \text{or} \quad 4 = \frac{2 \times 10^{-7} \times 2\pi \times 7 \times 10^{-2} \times n}{\pi (0.7 \times 10^{-3})^2}$$

$\therefore \quad n = \mathbf{70}.$

Example 20. *A wire of 10 ohm resistance is stretched to thrice its original length. What will be its (i) new resistivity, and (ii) new resistance ?* [CBSE D 98C]

Solution. (*i*) Resistivity ρ remains unchanged because it is the property of the material of the wire.

(*ii*) In both cases, volume of wire is same. So

$$V = A'l' = Al$$

or $$\frac{A'}{A} = \frac{l}{l'} = \frac{l}{3l} = \frac{1}{3} \qquad [\because\ l' = l + 2l = 3l]$$

$$\therefore \quad \frac{R'}{R} = \frac{\rho \frac{l'}{A'}}{\rho \frac{l}{A}} = \frac{l'}{l} \times \frac{A}{A'} = \frac{3}{1} \times \frac{3}{1} = 9$$

Hence $R' = 9R = 9 \times 10 = \mathbf{90\ \Omega}.$

Example 21. *A wire has a resistance of 16 Ω. It is melted and drawn into a wire of half its length. Calculate the resistance of the new wire. What is the percentage change in its resistance ?*

Solution. In both cases, volume of the wire is same.

$$\therefore \quad V = A'l' = Al$$

or $$\frac{A'}{A} = \frac{l}{l'} = \frac{l}{\frac{1}{2}l} = \frac{2}{1}$$

$$\therefore \quad \frac{R'}{R} = \frac{\rho \frac{l'}{A'}}{\rho \frac{l}{A}} = \frac{l'}{l} \times \frac{A}{A'} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

or $$R' = \frac{1}{4} R = \frac{1}{4} \times 16 = \mathbf{4\ \Omega}.$$

Change in resistance

$$= \frac{R - R'}{R} \times 100 = \frac{12}{16} \times 100 = \mathbf{75\%}.$$

Example 22. *The resistance of a wire is R ohm. What will be its new resistance if it is stretched to n times its original length ?*

Solution. In both cases, volume of the wire is same.

$$\therefore \quad V = Al = A'l'$$

or $$\frac{A}{A'} = \frac{l'}{l} = n \qquad [\because\ l' = nl]$$

$$\therefore \quad \frac{R'}{R} = \frac{\rho \frac{l'}{A'}}{\rho \frac{l}{A}} = \frac{l'}{l} \cdot \frac{A}{A'} = n \cdot n = n^2$$

or $$R' = \mathbf{n^2 R}.$$

Example 23. *A cylindrical wire is stretched to increase its length by 10%. Calculate the percentage increase in resistance.*

Solution. New length, $l' = l + 10\%$ of l

$$= l + 0.1\,l = 1.1\,l$$

or $$\frac{l'}{l} = 1.1$$

$$Al = A'l'$$

or $$\frac{A}{A'} = \frac{l'}{l}$$

$$\therefore \quad \frac{R'}{R} = \frac{l'}{l} \times \frac{A}{A'} = \left(\frac{l'}{l}\right)^2 = (1.1)^2 = 1.21$$

The percentage increase in resistance,

$$\frac{R' - R}{R} \times 100 = \left(\frac{R'}{R} - 1\right) \times 100 = (1.21 - 1) \times 100 = \mathbf{21\%}.$$

Example 24. *Two wires A and B of equal mass and of the same metal are taken. The diameter of the wire A is half the diameter of wire B. If the resistance of wire A is 24 Ω, calculate the resistance of wire B.*

Solution. Mass of wire = volume × density

= area of cross-section × length × density

$$\therefore \quad m = \pi r_A^2\, l_A\, d = \pi r_B^2\, l_B\, d$$

or $$\frac{l_B}{l_A} = \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{1/2}{1}\right)^2 = \frac{1}{4}$$

$$\therefore \quad \frac{R_B}{R_A} = \frac{\rho \frac{l_B}{\pi r_B^2}}{\rho \frac{l_A}{\pi r_A^2}} = \frac{l_B}{l_A} \times \left(\frac{r_A}{r_B}\right)^2 = \frac{1}{4} \times \left(\frac{1}{2}\right)^2 = \frac{1}{16}$$

or $$R_B = \frac{1}{16} R_A = \frac{1}{16} \times 24\ \Omega = \mathbf{1.5\ \Omega}.$$

Example 25. *A piece of silver has a resistance of 1 Ω. What will be the resistance of a constantan wire of one-third length and one-half diameter, if the specific resistance of constantan is 30 times that of silver ?*

Solution. For silver,

$$R = \frac{4\,\rho\, l}{\pi\, d^2} = 10\,\Omega$$

For constantan,

$$R' = \frac{4\rho' l'}{\pi d'^2} = \frac{4 \times 30\,\rho \times \frac{l}{3}}{\pi\left(\frac{d}{2}\right)^2}$$

$$= \frac{40 \times 4\rho\, l}{\pi d^2} = 40\ R = 40 \times 1 = \mathbf{40\,\Omega}.$$

Example 26. *On applying the same potential difference between the ends of wires of iron and copper of the same length, the same current flows in them. Compare their radii. Specific resistances of iron and copper are respectively* 1.0×10^{-7} *and* $1.6 \times 10^{-8}\ \Omega m$. *Can their current-densities be made equal by taking appropriate radii ?*

Solution. On applying same potential difference, same current flows in the two wires. Hence the resistances of the two wires should be equal.

But $\quad R = \rho\,\frac{l}{A} = \rho\,\frac{l}{\pi r^2}$

For the two wires of same length l, we have

$$R_1 = \rho_1\,\frac{l}{\pi\, r_1^2} \quad \text{and} \quad R_2 = \rho_2\,\frac{l}{\pi\, r_2^2}.$$

As $\quad R_1 = R_2$

$$\therefore \quad \frac{\rho_1}{r_1^2} = \frac{\rho_2}{r_2^2} \quad \text{or} \quad \frac{r_1}{r_2} = \sqrt{\frac{\rho_1}{\rho_2}}$$

$$\therefore \quad \frac{r_{iron}}{r_{copper}} = \sqrt{\frac{\rho_{iron}}{\rho_{copper}}} = \sqrt{\frac{1.0 \times 10^{-7}}{1.6 \times 10^{-8}}} = \mathbf{2.5.}$$

No, current densities cannot be equal because they depend on nature of the metals.

Problems For Practice

1. A voltage of 30 V is applied across a colour coded carbon resistor with first, second and third rings of blue, black and yellow colours. What is the current flowing through the resistor ? **[CBSE D 05]** (Ans. 0.5×10^{-4} A)

2. A potential difference of 10 V is applied across a conductor of resistance 1 kΩ. Find the number of electrons flowing through the conductor in 5 minutes. (Ans. 1.875×10^{19})

3. What length of a copper wire of cross-sectional area 0.01 mm^2 would be required to obtain a resistance of 1 kΩ ? Resistivity of copper = $1.7 \times 10^{-8}\,\Omega$m. (Ans. 588.2 m)

4. A metal wire of specific resistance $64 \times 10^{-8}\,\Omega$ m and length 1.98 m has a resistance of 7 Ω. Find its radius. (Ans. 2.4×10^{-4} m)

5. Calculate the resistance of a 2 m long nichrome wire of radius 0.321 mm. Resistivity of nichrome is $15 \times 10^{-6}\,\Omega$ m. If a potential difference of 10 V is applied across this wire, what will be the current in the wire ? (Ans. 9.26 Ω, 1.08 A)

6. An electron beam has an aperture of 1.0 mm^2. A total of 6×10^{16} electrons flow through any perpendicular cross-section per second. Calculate (*i*) the current and (*ii*) the current density in the electron beam. [Ans. (*i*) 9.6×10^{-3} A (*ii*) 9.6×10^3 Am^{-2}]

7. Calculate the electric field in a copper wire of cross-sectional area 2.0 mm^2 carrying a current of 1 A. The resistivity of copper = $1.7 \times 10^{-8}\ \Omega$m. (Ans. 0.85×10^{-2} Vm^{-1})

8. A given copper wire is stretched to reduce its diameter to half its previous value. What would be its new resistance ? **[CBSE D 92C]** (Ans. $R' = 16\ R$)

9. What will be the change in resistance of a constantan wire when its radius is made half and length reduced to one-fourth of its original length ? (Ans. No change)

10. A wire of resistance 5 Ω is uniformly stretched until its new length becomes 4 times the original length. Find its new resistance. (Ans. 80 Ω)

11. A metallic wire of length 1 m is stretched to double its length. Calculate the ratio of its initial and final resistances assuming that there is no change in its density on stretching. **[CBSE D 94]** (Ans. 1 : 4)

12. A wire of certain radius is stretched so that its radius decreases by a factor n. Calculate its new resistance. (Ans. $n^4\ R$)

13. A wire 1 m long and 0.13 mm in diameter has a resistance of 4.2 Ω. Calculate the resistance of another wire of the same material whose length is 1.5 m and diameter 0.155 mm. (Ans. 4.4 Ω)

14. A rheostat has 100 turns of a wire of radius 0.4 mm having resistivity $4.2 \times 10^{-7}\,\Omega$m. The diameter of each turn is 3 cm. What is the maximum value of resistance that it can introduce ? (Ans. 7.875 Ω)

15. Given that resistivity of copper is $1.68 \times 10^{-8}\ \Omega$m. Calculate the amount of copper required to draw a wire 10 km long having resistance of 10 Ω. The density of copper is 8.9×10^3 kgm^{-3}. (Ans. 1495.2 kg)

16. The size of a carbon block is 1.0 cm × 1.0 cm × 50 cm. Find its resistance (*i*) between the opposite square faces (*ii*) between the opposite rectangular faces of the block. The resistivity of carbon is $3.5\times10^{-5}\,\Omega$ cm. (Ans. $0.175\,\Omega,\ 7.0\times10^{-5}\,\Omega$)

17. Two wires *A* and *B* of the same material have their lengths in the ratio 1 : 5 and diameters in the ratio 3 : 2. If the resistance of the wire *B* is 180 Ω, find the resistance of the wire *A*. (Ans. 16 Ω)

18. A uniform wire is cut into four segments. Each segment is twice as long as the earlier segment. If the shortest segment has a resistance of 4 Ω, find the resistance of the original wire. (Ans. 60 Ω)

19. Calculate the conductance and conductivity of a wire of resistance 0.01 Ω, area of cross-section $10^{-4}\,\text{m}^2$ and length 0.1 m. **[Haryana 2000]**

(Ans. 100 S, $10^5\,\text{Sm}^{-1}$)

HINTS

1. $R = 60\times10^4\,\Omega,\ V = 30$ V

$$I=\frac{V}{R}=\frac{30}{60\times10^4}=\mathbf{0.5\times10^{-4}\ A.}$$

2. $I=\frac{V}{R}=\frac{10\text{ V}}{1\text{ k}\Omega}=\frac{10\text{ V}}{1000\,\Omega}=10^{-2}\text{ A}$

$$n=\frac{q}{e}=\frac{It}{e}=\frac{10^{-2}\times5\times60}{1.6\times10^{-19}}=\mathbf{1.875\times10^{19}}.$$

4. As $R=\frac{\rho l}{A}=\frac{\rho l}{\pi r^2}$

$$\therefore\ r^2=\frac{\rho l}{\pi R}=\frac{64\times10^{-8}\times1.98\times7}{22\times7}=5.76\times10^{-8}\text{m}^2$$

or $r=\mathbf{2.4\times10^{-4}\ m.}$

5. Use $R=\rho\frac{l}{\pi r^2}$ and $I=\frac{V}{R}$.

6. (*i*) $I=\frac{q}{t}=\frac{ne}{t}=\frac{6\times10^{16}\times1.6\times10^{-19}}{1}$

$=\mathbf{9.6\times10^{-3}\ A.}$

(*ii*) Current density,

$$j=\frac{I}{A}=\frac{9.6\times10^{-3}}{1.0\times10^{-6}}=\mathbf{9.6\times10^{3}\ Am^{-2}}.$$

7. $E=\frac{V}{l}=\frac{IR}{l}=\frac{I\rho l}{lA}=\frac{I\rho}{A}=\frac{1\times1.7\times10^{-8}}{2.0\times10^{-6}}$

$=\mathbf{0.85\times10^{-2}\ Vm^{-1}}.$

8. When the diameter of the wire is reduced to its half value, area of cross-section becomes one-fourth and the length increases to four times the original length.

$$\therefore\ R'=\rho\frac{l'}{A'}=\rho.\frac{4l}{\frac{1}{4}A}=16\rho\frac{l}{A}=\mathbf{16\,R.}$$

9. $R=\rho\frac{l}{A}=\rho\frac{l}{\pi r^2}$

$$R'=\rho\frac{l/4}{\pi(r/2)^2}=\rho\frac{l}{\pi r^2}=R.$$

10. $R=\rho\frac{l}{A}=5\,\Omega$

$$R'=\rho\frac{4l}{A/4}=16\rho\frac{l}{A}=16\,R=16\times5=\mathbf{80\,\Omega}.$$

11. $R=\rho\frac{l}{A}$

$$R'=\rho\frac{2l}{A/2}=4\rho\frac{l}{A}=4\,R$$

$\therefore\ R:R'=\mathbf{1:4}.$

12. $V=A'l'=Al$

or $V=\pi\left(\frac{r}{n}\right)^2 l'=\pi r^2 l$ or $l'=n^2 l$

$$\therefore\ R'=\rho\frac{l'}{\pi r'^2}=\rho\frac{n^2 l}{\pi(r/n)^2}=n^4\rho\frac{l}{\pi r^2}=\mathbf{n^4\,R}.$$

13. $R_2=R_1\left[\frac{l_2}{l_1}\right]\left[\frac{D_1}{D_2}\right]^2$

$$=4.2\left[\frac{1.5}{1}\right]\left[\frac{0.13\times10^{-3}}{0.155\times10^{-3}}\right]^2=\mathbf{4.4\,\Omega}.$$

14. Length of the wire used, $l=100\pi\,D$

$$R=\rho\frac{l}{\pi r^2}=\rho.\frac{100\pi D}{\pi r^2}=\frac{100\rho D}{r^2}$$

$$=\frac{100\times4.2\times10^{-7}\times3\times10^{-2}}{(0.4\times10^{-3})^2}=\mathbf{7.875\,\Omega}.$$

15. As $R=\rho\frac{l}{A}$

$$\therefore\ A=\frac{\rho l}{R}=\frac{1.68\times10^{-8}\times10\times10^3}{10}=1.68\times10^{-5}\text{m}^2$$

Mass of copper required,

m = Volume × density = Al × density

$=1.68\times10^{-5}\times10\times10^3\times8.9\times10^3$

$=\mathbf{1495.2\ kg}.$

16. (*i*) $R=\rho\frac{l}{A}=\frac{3.5\times10^{-5}\times50\times10^{-2}}{1.0\times10^{-2}\times1.0\times10^{-2}}=\mathbf{0.175\,\Omega}.$

(*ii*) $R=\rho\frac{l}{A}=\frac{3.5\times10^{-5}\times1.0\times10^{-2}}{1.0\times10^{-2}\times50\times10^{-2}}=\mathbf{7.0\times10^{-5}\,\Omega}.$

17. $$\frac{R_A}{R_B}=\frac{\rho\frac{l_A}{\pi d_A^2/4}}{\rho\frac{l_B}{\pi d_B^2/4}}=\frac{l_A}{l_B}\left(\frac{d_B}{d_A}\right)^2=\frac{1}{5}\times\left(\frac{2}{3}\right)^2=\frac{4}{45}$$

$$R_A=\frac{4}{45}R_B=\frac{4}{45}\times180=\mathbf{16\,\Omega}.$$

18. Let the lengths of the four segments be l, $2l$, $4l$ and $8l$. Then their corresponding resistances will be R, $2R$, $4R$ and $8R$.

Given $R = 4\,\Omega$

Resistance of the original wire

$$= R + 2R + 4R + 8R = 15R = 15 \times 4 = \mathbf{60\,\Omega}.$$

19. Conductance, $G = \frac{1}{R} = \frac{1}{0.01} = \mathbf{100\ S}$.

Conductivity,

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{0.1}{0.01 \times 10^{-4}} = \mathbf{10^5\ Sm^{-1}}.$$

3.11 CARRIERS OF CURRENT

16. *Mention different types of charge carriers in solids, liquids and gases.*

Carriers of current. *The charged particles which by flowing in a definite direction set up an electric current are called current carriers.* The different types of current carriers are as follows :

1. In solids. In metallic conductors, electrons are the charge carriers. The electric current is due to the drift of electrons from low to high potential regions. In n-type semi-conductors, electrons are the majority charge carriers while in p-type semiconductors, holes are the majority charge carriers. A *hole* is a vacant state from which an electron has been removed and it acts as a positive charge carrier.

2. In liquids. In electrolytic liquids, the charge carriers are positively and negatively charged ions. For example, $CuSO_4$ solution has Cu^{2+} and SO_4^{2-} ions, which act as the charge carriers.

3. In gases. In ionised gases, positive and negative ions and electrons are the charge carriers.

4. In vacuum tubes. In vacuum tubes like radio valves, cathode ray oscilloscope, picture tube etc ; free electrons emitted by the heated cathode act as charge carriers.

17. *Why is it that electrons carry current in metals ?*

Metallic conduction. In metals, the atoms are closely packed. The valence electrons of one atom are close to the neighbouring atoms and experience electrical forces due to them. So they do not remain attached to a particular atom, but can hop from one atom to another and are free to move throughout the lattice. These free electrons are responsible for conduction in metals.

The fact, that the negatively charged electrons carry current in metals, was first experimentally confirmed by the American physicists *Tolman and Stewart in* 1917. They measured the angular momentum of the charges flowing steadily in a circular loop. Their observations indicated that

1. The sign of the charges is negative.
2. The ratio e/m of the charges is equal to that measured for the electrons in other experiments.

It was thus established directly that current in metals is carried by negatively charged electrons.

3.12 MECHANISM OF CURRENT FLOW IN A CONDUCTOR : DRIFT VELOCITY AND RELAXATION TIME

18. *Explain the mechanism of the flow of current in a metallic conductor. Hence define the terms drift velocity and relaxation time. Deduce a relation between them.*

Mechanism of the flow of electric charges in a metallic conductor : Concepts of drift velocity and relaxation time. Metals have a large number of free electrons, nearly 10^{28} per cubic metre. In the absence of any electric field, these electrons are in a state of continuous random motion due to thermal energy. At room temperature, they move with velocities of the order of $10^5\,ms^{-1}$. However, these velocities are distributed randomly in all directions. There is no preferred direction of motion. On the average, the number of electrons travelling in any direction will be equal to number of electrons travelling in the opposite direction. If $\vec{u}_1, \vec{u}_2, ..., \vec{u}_N$ are the random velocities of N free electrons, then *average velocity* of electrons will be

$$\vec{u} = \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N}{N} = 0$$

Thus, there is no net flow of charge in any direction.

In the presence of an external field $\vec{E}$, each electron experiences a force $-e\vec{E}$ in the opposite direction of $\vec{E}$ (since an electron has negative charge) and undergoes an acceleration $\vec{a}$ given by

$$\vec{a} = \frac{\text{Force}}{\text{Mass}} = -\frac{e\vec{E}}{m}$$

where m is the mass of an electron. As the electrons accelerate, they frequently collide with the positive metal ions or other electrons of the metal. Between two successive collisions, an electron gains a velocity component (in addition to its random velocity) in a direction opposite to $\vec{E}$. However, the gain in velocity lasts for a short time and is lost in the next collision. At each collision, the electron starts afresh with a random thermal velocity.

If an electron having random thermal velocity $\vec{u}_1$ accelerates for time τ_1 (before it suffers next collision), then it will attain a velocity,

$$\vec{v}_1 = \vec{u}_1 + \vec{a}\,\tau_1$$

Similarly, the velocities of the other electrons will be

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\,\tau_2,$$

$$\vec{v}_3 = \vec{u}_3 + \vec{a}\,\tau_3, \ldots,$$

$$\vec{v}_N = \vec{u}_N + \vec{a}\,\tau_N$$

The average velocity $\vec{v}_d$ of all the N electrons will be

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots\ldots + \vec{v}_N}{N}$$

$$= \frac{(\vec{u}_1 + \vec{a}\,\tau_1) + (\vec{u}_2 + \vec{a}\,\tau_2) + \ldots + (\vec{u}_N + \vec{a}\,\tau_N)}{N}$$

$$= \frac{\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_N}{N} + \vec{a}\,\frac{\tau_1 + \tau_2 + \ldots + \tau_N}{N}$$

$$= 0 + \vec{a}\,\tau$$

where $\tau = (\tau_1 + \tau_2 + \ldots\ldots + \tau_N)/N$ is the average time between two successive collisions. *The average time that elapses between two successive collisions of an electron is called* ***relaxation time***. For most conductors, it is of the order of 10^{-14} s. The velocity gained by an electron during this time is

$$\vec{v}_d = \vec{a}\,\tau = -\frac{e\,\vec{E}\,\tau}{m}.$$

The parameter $\vec{v}_d$ is called ***drift velocity*** of electrons. *It may be defined as the average velocity gained by the free electrons of a conductor in the opposite direction of the externally applied electric field.*

It may be noted that although the electric field accelerates an electron between two collisions, yet it does not produce any net acceleration. This is because the electron keeps colliding with the positive metal ions. The velocity gained by it due to the electric field is lost in next collision. As a result, it acquires a constant average velocity $\vec{v}_d$ in the opposite direction of $\vec{E}$. The motion of the electron is similar to that of a small spherical metal ball rolling down a long flight of stairs. As the ball falls from one stair to the next, it acquires acceleration due to the force of gravity. The moment it collides with the stair, it gets decelerated. The net effect is that after falling through a number of steps, the ball begins to roll down the stairs with zero average acceleration *i.e.*, at constant average speed. Moreover, as the average time τ between two successive collisions is small, an electron slowly and steadily drifts in the opposite direction of $\vec{E}$, as shown in Fig. 3.18.

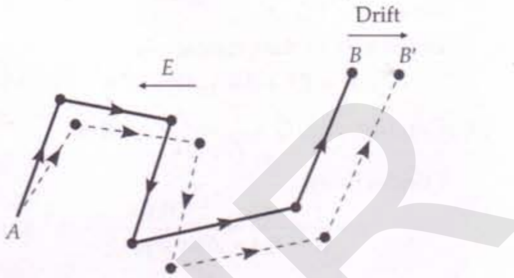


Fig. 3.18 Slow and steady drift of an electron in the opposite direction of $\vec{E}$. The solid lines represent the path in the absence of $\vec{E}$ and dashed lines in the presence of $\vec{E}$.

3.13 RELATION BETWEEN ELECTRIC CURRENT AND DRIFT VELOCITY : DERIVATION OF OHM'S LAW

19. *Derive relation between electric current and drift velocity. Hence deduce Ohm's law. Also write the expression for resistivity in terms of number density of free electrons and relaxation time.*

Relation between electric current and drift velocity. Suppose a potential difference V is applied across a conductor of length l and of uniform cross-section A. The electric field E set up inside the conductor is given by

$$E = \frac{V}{l}$$

Under the influence of field $\vec{E}$, the free electrons begin to drift in the opposite direction $\vec{E}$ with an average drift velocity v_d.

Let the number of electrons per unit volume or electron density $= n$

Charge on an electron $= e$

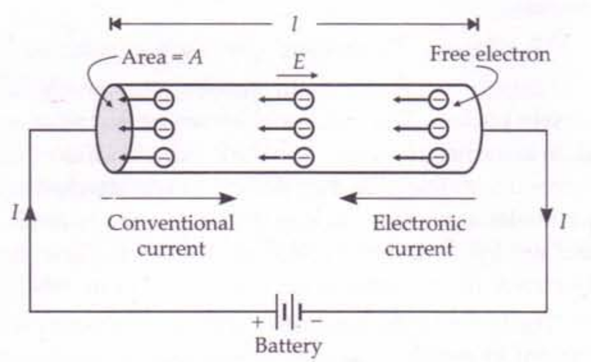


Fig. 3.19 Drift of electrons and electric field inside a conductor.

Number of electrons in length l of the conductor

$= n \times$ volume of the conductor $= n\ Al$

Total charge contained in length l of the conductor is

$$q = en\ Al$$

All the electrons which enter the conductor at the right end will pass through the conductor at the left end in time,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{l}{v_d}$$

$\therefore$ Current, $I = \frac{q}{t} = \frac{enAl}{l / v_d}$ or $I = enAv_d$

This equation relates the current I with the drift velocity v_d.

The current density 'j' is given by

$$j = \frac{I}{A} = env_d$$

In vector form $\vec{j} = en\,\vec{v}_d$

The above equation is valid for both positive and negative values of q.

Deduction of Ohm's law. When a potential difference V is applied across a conductor of length l, the drift velocity in terms of V is given by

$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$$

If the area of cross-section of the conductor is A and the number of electrons per unit volume or the electron density of the conductor is n, then the current through the conductor will be

$$I = enAv_d = enA \cdot \frac{eV\tau}{ml}$$

or
$$\frac{V}{I} = \frac{ml}{ne^2\,\tau A}.$$

At a fixed temperature, the quantities m, l, n, e, τ and A, all have constant values for a given conductor. Therefore,

$$\frac{V}{I} = \text{a constant, } R$$

This proves Ohm's law for a conductor and here

$$R = \frac{ml}{ne^2\,\tau A}$$

is the resistance of the conductor.

Resistivity in terms of electron density and relaxation time. The resistance R of a conductor of length l, area of cross-section A and resistivity ρ is given by

$$R = \rho\,\frac{l}{A}$$

But
$$R = \frac{ml}{ne^2\,\tau A}$$

where τ is the relaxation time. Comparing the above two equations, we get

$$\rho = \frac{m}{ne^2\,\tau}$$

Obviously, ρ is independent of the dimensions of the conductor but depends on its two parameters :

1. Number of free electrons per unit volume or electron density of the conductor.
2. The relaxation time τ, the average time between two successive collisions of an electron.

20. *Write relation between quantities $\vec{j}$, σ and $\vec{E}$.*

Relation between $\vec{j}$, σ and $\vec{E}$. For an electron,

$$q = -e$$

and
$$\vec{v}_d = -\frac{e\vec{E}\tau}{m}$$

$$\therefore \quad \vec{j} = nq\vec{v}_d = n(-e)\left(-\frac{e\vec{E}\tau}{m}\right) = \frac{ne^2\tau}{m}\vec{E}$$

But $\frac{ne^2\tau}{m} = \frac{1}{\rho} = \sigma$, conductivity of the conductor

$$\therefore \quad \vec{j} = \sigma\vec{E} \quad \text{or} \quad \vec{E} = \rho\,\vec{j}$$

This is Ohm's law in terms of vector quantities like current density $\vec{j}$ and electric field $\vec{E}$.

21. *What causes resistance in a conductor ?*

Cause of resistance. Collisions are the basic cause of resistance. When a potential difference is applied across a conductor, its free electrons get accelerated. On their way, they frequently collide with the positive metal ions *i.e.,* their motion is opposed and this opposition to the flow of electrons is called resistance. Larger the number of collisions per second, smaller is the relaxation time τ, and larger will be the resistivity ($\rho = m / ne^2\,\tau$).

The number of collisions that the electrons make with the atoms/ions depends on the arrangement of atoms or ions in a conductor. So the *resistance depends on the nature of the material* (copper, silver, etc.) of the conductor.

The resistance of a conductor depends on its length. A long wire offers more resistance than short wire because there will be more collisions in the longer wire.

The resistance of conductor depends on its area of cross-section. A thick wire offers less resistance than a thin wire because in a thick wire, more area of cross-section is available for the flow of electrons.

22. *Alloys of metals have greater resistivity than their constituent metals. Why ?*

High resistivity of nichrome. In an alloy, *e.g.*, nichrome (Ni – Cr alloy), Ni^{2+} and Cr^{3+} ions have different charge and size. They occupy random locations relative to each other, though their ionic *sites* form a regular crystalline lattice. An electron, therefore, passes through a very random medium and is very frequently deflected. So there is a small relaxation time and hence large resistivity. In general, *alloys have more resistivity than that of their constituent metals.*

23. *Explain the cause of instantaneous current in an electric circuit.*

Cause of instantaneous current. Although the drift speed of electrons is very small, typically 1 mm/s, yet an electric bulb lights up as soon as we turn the switch on. This is because electrons are present everywhere in an electric circuit. When a potential difference is applied to the circuit, an electric field is set up throughout the circuit, almost with the speed of light. Electrons in every part of the circuit begin to drift under the influence of this electric field and a current begins to flow in the circuit almost immediately.

The above situation is analogous to the flow of water in a long pipe. As soon as the pressure is applied at one end of the water filled pipe, a pressure wave is transmitted along the pipe with a speed of about 1400 ms^{-1}. When this wave reaches the other end, water starts flowing out. But water inside pipe moves forward with a much smaller speed.

Examples based on Drift Velocity

Formulae Used

1. Current in terms of drift velocity (v_d) is $I = en\,A\,v_d$
2. Current density, $j = env_d$
3. No. of atoms in one gram atomic mass of an element, N = Avogadro's number = 6.023×10^{23}.
4. In terms of relaxation time τ,

$$R = \frac{ml}{ne^2\,\tau\,A} \quad \text{and} \quad \rho = \frac{m}{ne^2\,\tau}$$

5. Relation between current density and electric field,

$$j = \sigma E \quad \text{or} \quad E = \rho j$$

Units Used

Drift velocity v_d is in ms^{-1}, free-electron density in m^{-3}, cross-sectional area A in m^2, current density j in Am^{-2}, all resistances in Ω.

Constants Used

$$e = 1.6 \times 10^{-19}\text{ C and } N_A = 6.023 \times 10^{23}\text{mol}^{-1}.$$

Example 27. *Assuming that there is one free electron per atom in copper, determine the number of free electrons in 1 metre^3 volume of copper. Density of copper is 8.9×10^3 kgm^{-3} and atomic weight 63.5. (Avogadro's number, $N = 6.02 \times 10^{26}$ per kg-atom).*

Solution. If the atomic weight of a material is M kg and the density is d kgm^{-3}, then the volume of its 1 kg-atom will be (M/d) m^3.

According to Avogadro's hypothesis, there are 6.02×10^{26} atoms in 1 kg-atom of the material. This number is called Avogadro's number (N). Thus

Number of atoms in (M/d) m^3 volume of a material $= N$

∴ Number of atoms in 1 m^3 volume

$$= \frac{N}{M/d} = \frac{d \times N}{M}$$

Assuming 1 free electron per atom in copper, the number of free electrons in 1 m^3 volume of copper will be

$$n = \frac{d \times N}{M}$$

Now $d = 8.9 \times 10^3$ kg m^{-3}, $N = 6.02 \times 10^{26}$, $M = 63.5$ kg

$$\therefore \quad n = \frac{8.9 \times 10^3 \times 6.02 \times 10^{26}}{63.5} = \mathbf{8.4 \times 10^{28}\ m^{-3}}.$$

Example 28. *A copper wire has a resistance of 10 Ω and an area of cross-section 1 mm^2. A potential difference of 10 V exists across the wire. Calculate the drift speed of electrons if the number of electrons per cubic metre in copper is 8×10^{28} electrons.* [CBSE D 96]

Solution. Here $R = 10\,\Omega$, $A = 1\text{ mm}^2 = 10^{-6}\text{m}^2$, $V = 10$ V, $n = 8 \times 10^{28}$ electrons / m^3

Now $I = en\,A\,v_d$

$$\therefore \quad \frac{V}{R} = en\,A\,v_d$$

or

$$v_d = \frac{V}{enAR} = \frac{10}{1.6 \times 10^{-19} \times 8 \times 10^{28} \times 10^{-6} \times 10}$$

$$= 0.078 \times 10^{-3}\text{ms}^{-1} = \mathbf{0.078\ mm\ s^{-1}}.$$

Example 29. *(a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m^2, carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg m^{-3}, and its atomic mass is 63.5 u. Take Avogadro's number $= 6.0 \times 10^{23}$ mol^{-1}.*

(b) Compare the drift speed obtained above with (i) thermal speeds of copper atoms at ordinary temperatures, (ii) speeds of electrons carrying the current and (iii) speed of propagation of electric field along the conductor which causes the drift motion. [NCERT]

Solution. Mass of 1 m^3 of Cu

$$= 9.0 \times 10^3 \text{ kg} = 9 \times 10^6 \text{ g}$$

Since Avogadro's number is 6.0×10^{23} and atomic mass of Cu is 63.5 u, therefore, 63.5 g of Cu contains 6.0×10^{23} atoms.

So 9×10^6 g of Cu contains

$$\frac{6.0 \times 10^{23}}{63.5} \times 9 \times 10^6 \text{ atoms} = 8.5 \times 10^{28} \text{ atoms}$$

Number of conduction electrons,

$$n = \text{number of Cu atoms} = 8.5 \times 10^{28}$$

Now $I = 1.5$ A, $A = 10^{-7}\ m^2$, $e = 1.6 \times 10^{-19}$C

$$\therefore\ v_d = \frac{I}{enA} = \frac{1.5}{1.6 \times 10^{-19} \times 8.5 \times 10^{28} \times 10^{-7}}$$

$$= \frac{15}{16 \times 85 \times 10} = \mathbf{1.1 \times 10^{-3} ms^{-1}}.$$

(*b*) (*i*) At any temperature *T*, the thermal speed of a copper atom of mass *M* is given by

$$v_{rms} = \sqrt{\frac{3k_B T}{M}}$$

But ordinary temperature, $T \simeq 300$ K,

Boltzmann constant, $k_B = 1.38 \times 10^{-23} JK^{-1}$,

Mass of a copper atom,

$$M = \frac{63.5}{6.0 \times 10^{23}} \text{ g} = \frac{63.5 \times 10^{-3}}{6.0 \times 10^{23}} \text{kg}$$

$$\therefore\ v_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300 \times 6.0 \times 10^{23}}{63.5 \times 10^{-3}}}$$

$$= \sqrt{117354.33} = 342.57 \text{ ms}^{-1}$$

From part (*a*), drift speed of electrons,

$$v_d = 1.1 \times 10^{-3} ms^{-1}$$

$$\therefore\ \frac{v_d \text{ (electrons)}}{v_{rms} \text{ (Cu atoms)}} = \frac{1.1 \times 10^{-3}}{342.57} = \mathbf{3.21 \times 10^{-6}}.$$

(*ii*) The maximum kinetic energy $\frac{1}{2} mv_F^2$ of electron in copper corresponds to a temperature,

$$T_0 = 10^5 \text{ K}$$

$$\therefore\ \frac{1}{2} mv_F^2 = k_B T$$

or

$$v_F = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 10^5}{9.1 \times 10^{-31}}}$$

$$= \mathbf{1.74 \times 10^6 \ ms^{-1}}.$$

$$\frac{v_d \text{ (electron)}}{v_F \text{ (electron)}} = \frac{1.1 \times 10^{-3}}{1.74 \times 10^6} \simeq \mathbf{10^{-9}}.$$

(*iii*) An electric field propagates along a conductor with the speed of an electromagnetic wave *i.e.*, $3 \times 10^8\ ms^{-1}$.

$$\frac{v_d \text{ (electron)}}{\text{speed of propagation of electric field}} = \frac{1.1 \times 10^{-3}}{3 \times 10^8}$$

$$\simeq \mathbf{10^{-11}}.$$

Example 30. *Calculate the electric field in a copper wire of cross-sectional area 2.0 mm^2 carrying a current of 1 A. The conductivity of copper $= 6.25 \times 10^7\ Sm^{-1}$.*

Solution. Here $A = 2.0 \text{ mm}^2 = 2.0 \times 10^{-6} \text{m}^2$, $I = 1$ A, $\sigma = 6.25 \times 10^7 Sm^{-1}$

As $$j = \frac{I}{A} = \sigma E$$

$$\therefore\ E = \frac{I}{A\sigma} = \frac{1}{2.0 \times 10^{-6} \times 6.25 \times 10^7}$$

$$= \mathbf{8 \times 10^{-3}\ Vm^{-1}}.$$

Example 31. *A potential difference of 100 V is applied to the ends of a copper wire one metre long. Calculate the average drift velocity of the electrons. Compare it with the thermal velocity at 27°C. Given conductivity of copper, $\sigma = 5.81 \times 10^7\ \Omega^{-1}\ m^{-1}$ and number density of conduction electrons, $n = 8.5 \times 10^{28} m^{-3}$.* [NCERT]

Solution. Electric field,

$$E = \frac{V}{l} = \frac{100 \text{ V}}{1 \text{ m}} = 100 \text{ Vm}^{-1}$$

As $$j = \sigma E = env_d$$

$\therefore$ Drift speed,

$$v_d = \frac{\sigma E}{e\,n} = \frac{5.81 \times 10^7 \times 100}{1.6 \times 10^{-19} \times 8.5 \times 10^{28}}$$

$$= \mathbf{0.43\ ms^{-1}}.$$

Now, $k_B = 1.38 \times 10^{-23} JK^{-1}$,

$T = 27 + 273 = 300$ K

$m_e = 9.1 \times 10^{-31}$ kg

Thermal velocity of electron at 27°C,

$$v_{rms} = \sqrt{\frac{3k_B T}{m_e}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}}$$

$$= \mathbf{1.17 \times 10^5 ms^{-1}}.$$

$$\frac{v_d}{v_{rms}} = \frac{0.43}{1.17 \times 10^5} = \mathbf{3.67 \times 10^{-6}}.$$

Example 32. *Find the time of relaxation between collision and free path of electrons in copper at room temperature. Given resistivity of copper $= 1.7 \times 10^{-8}\ \Omega m$, number density of electrons in copper $= 8.5 \times 10^{28} m^{-3}$, charge on electron $= 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg and drift velocity of free electrons $= 1.6 \times 10^{-4}\ ms^{-1}$.*

Solution. Here $\rho = 1.7 \times 10^{-8}\ \Omega\text{m}, n = 8.5 \times 10^{28}\ \text{m}^{-3}$, $e = 1.6 \times 10^{-19}\text{C}, m_e = 9.1 \times 10^{-31}\text{kg}, v_d = 1.6 \times 10^{-4}\ \text{ms}^{-1}$.

As resistivity, $\rho = \dfrac{m_e}{ne^2\tau}$

$\therefore$ Relaxation time,

$$\tau = \frac{m_e}{e^2 n\rho} = \frac{9.1 \times 10^{-31}}{(1.6 \times 10^{-19})^2 \times 8.5 \times 10^{28} \times 1.7 \times 10^{-8}}$$

$$= \mathbf{2.5 \times 10^{-14}\ s}$$

Mean free path of electron

$$= v_d\tau = 1.6 \times 10^{-4} \times 2.5 \times 10^{-14}$$

$$= \mathbf{4.0 \times 10^{-18}\ m.}$$

Example 33. *An aluminium wire of diameter 0.24 cm is connected in series to a copper wire of diameter 0.16 cm. The wires carry an electric current of 10 ampere. Find (i) current-density in the aluminium wire (ii) drift velocity of electrons in the copper wire. Given : Number of electrons per cubic metre volume of copper* $= 8.4 \times 10^{28}$.

Solution. (*i*) Radius of Al wire,

$$r = \frac{0.24}{2} = 0.12\ \text{cm} = 0.12 \times 10^{-2}\text{m}$$

Area of cross-section,

$$A = \pi r^2 = 3.14 \times (0.12 \times 10^{-2})^2 = 4.5 \times 10^{-6}\text{m}^2$$

$\therefore$ Current density,

$$j = \frac{I}{A} = \frac{10}{4.5 \times 10^{-6}} = \mathbf{2.2 \times 10^6\ Am^{-2}}.$$

(*ii*) Area of cross-section of Cu wire is

$$A = \pi \times (0.08 \times 10^{-2})^2 = 2.0 \times 10^{-6}\ \text{m}^2$$

Also,

$$n = 8.4 \times 10^{28}\text{m}^{-3}, e = 1.6 \times 10^{-19}\ \text{C}, I = 10\ \text{A}$$

$$\therefore\ v_d = \frac{I}{en\,A} = \frac{10}{1.6 \times 10^{-19} \times 8.4 \times 10^{28} \times 2.0 \times 10^{-6}}$$

$$= \mathbf{3.7 \times 10^{-4}\ ms^{-1}}.$$

Example 34. *A current of 1.0 ampere is flowing through a copper wire of length 0.1 metre and cross-section* $1.0 \times 10^{-6}\ m^2$. *(i) If the specific resistance of copper be* $1.7 \times 10^{-8}\ \Omega m$, *calculate the potential difference across the ends of the wire. (ii) Determine current density in the wire. (iii) If there be one free electron per atom in copper, then determine the drift velocity of electrons. Given : density of copper* $= 8.9 \times 10^3 kg\ m^{-3}$, *atomic weight = 63.5,* $N = 6.02 \times 10^{26}$ *per kg-atom.*

Solution. Here $I = 1.0\,\text{A}, l = 0.1\ \text{m}, A = 1.0 \times 10^{-6}\text{m}^2$, $\rho = 1.7 \times 10^{-8}\Omega\text{m}, d = 8.9 \times 10^3\ \text{kg m}^{-3}$

(*i*) Resistance of wire is

$$R = \frac{\rho l}{A} = \frac{1.7 \times 10^{-8} \times 0.1}{1.0 \times 10^{-6}} = 1.7 \times 10^{-3}\Omega$$

$\therefore$ Potential difference,

$$V = IR = 1.0 \times 1.7 \times 10^{-3} = \mathbf{1.7 \times 10^{-3}\ V.}$$

(*ii*) Current density,

$$j = \frac{I}{A} = \frac{1.0}{1.0 \times 10^{-6}} = \mathbf{1.0 \times 10^6\ Am^{-2}}.$$

(*iii*) Free-electron density,

$$n = \frac{d \times N}{M} = \frac{8.9 \times 10^3 \times 6.02 \times 10^{26}}{63.5}$$

$$= 8.4 \times 10^{28}\text{m}^{-3}$$

$\therefore$ Drift velocity,

$$v_d = \frac{j}{en} = \frac{1.0 \times 10^6}{1.6 \times 10^{-19} \times 8.4 \times 10^{28}}$$

$$= \mathbf{7.4 \times 10^{-5}\ ms^{-1}}.$$

Problems For Practice

1. The free electrons of a copper wire of cross-sectional area 10^{-6}m^2 acquire a drift velocity of 10^{-4}m/s when a certain potential difference is applied across the wire. Find the current flowing in the wire if the density of free electrons in copper is 8.5×10^{28} electrons/m^3. (Ans. 1.36 A)
2. Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $2.5 \times 10^{-7}\text{m}^2$ carrying a current of 2.7 A. Assume the density of conduction electrons to be $9 \times 10^{28}\text{m}^{-3}$. **[CBSE OD 14]** (Ans. 0.75 mms^{-1})
3. A current of 1.8 A flows through a wire of cross-sectional area 0.5 mm^2. Find the current density in the wire. If the number density of conduction electrons in the wire is $8.8 \times 10^{28}\text{m}^{-3}$, find the drift speed of electrons. (Ans. $3.6 \times 10^6\text{Am}^{-2}, 2.56 \times 10^{-4}\text{ms}^{-1}$)
4. The resistivity of copper at room temperature is $1.7 \times 10^{-8}\Omega\text{m}$. If the free electron density of copper is $8.4 \times 10^{28}\ \text{m}^{-3}$, find the relaxation time for the free electrons of copper. Given $m_e = 9.11 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C. (Ans. 2.49×10^{-14}s)
5. A copper wire of diameter 1.0 mm carries a current of 0.2 A. Copper has 8.4×10^{28} atoms per cubic metre. Find the drift velocity of electrons, assuming that one charge carrier of 1.6×10^{-19}C is associated with each atom of the metal. **[ISCE 97]** (Ans. $1.895 \times 10^{-5}\ \text{ms}^{-1}$)
6. A current of 2 A is flowing through a wire of length 4 m and cross-sectional area 1 mm^2. If each cubic metre of the wire contains 10^{29} free electrons, find the average time taken by an electron to cross the length of the wire. (Ans. 3.2×10^4s)

7. A 10 C of charge flows through a wire in 5 minutes. The radius of the wire is 1 mm. It contains 5×10^{22} electrons per centimetre^3. Calculate the current and drift velocity.
(Ans. 3.33×10^{-2} A, 1.326×10^{-6} ms^{-1})

8. A copper wire of diameter 0.16 cm is connected in series to an aluminium wire of diameter 0.25 cm. A current of 10 A is passed through them. Find (*i*) current density in the copper wire (*ii*) drift velocity of free electrons in the aluminium wire. The number of free electrons per unit volume of aluminium wire is 10^{29} m^{-3}.
(Ans. 4.976×10^6 Am^{-2}, 1.28×10^{-4} ms^{-1})

9. A current of 30 ampere is flowing through a wire of cross-sectional area 2 mm^2. Calculate the drift velocity of electrons. Assuming the temperature of the wire to be 27°C, also calculate the *rms* velocity at this temperature. Which velocity is larger ? Given that Boltzman's constant $= 1.38 \times 10^{-23}$ J K^{-1}, density of copper 8.9 g cm^{-3}, atomic mass of copper = 63. (Ans. 1.1×10^{-3} ms^{-1}, 1.17×10^5 ms^{-1})

10. What is the drift velocity of electrons in silver wire of length 1 m, having cross-sectional area 3.14×10^{-6} m^2 and carrying a current of 10 A ? Given atomic mass of silver = 108, density of silver $= 10.5 \times 10^3$ kg m^{-3}, charge on electron $= 1.6 \times 10^{-19}$ C and Avogadro's number $= 6.023 \times 10^{26}$ per kg-atom.
(Ans. 3.399×10^{-4} ms^{-1})

11. When a potential difference of 1.5 V is applied across a wire of length 0.2 m and area of cross-section 0.3 mm^2, a current of 2.4 A flows through the wire. If the number density of free electrons in the wire is 8.4×10^{28} m^{-3}, calculate the average relaxation time. Given that mass of electron $= 9.1 \times 10^{-31}$ kg and charge on electron $= 1.6 \times 10^{-19}$ C. (Ans. 4.51×10^{-16} s)

HINTS

1. $I = enAv_d = 1.6 \times 10^{-19} \times 8.5 \times 10^{28} \times 10^{-6} \times 10^{-4}$
$= \mathbf{1.36\ A.}$

2. $v_d = \dfrac{I}{enA} = \dfrac{2.7}{1.6 \times 10^{-19} \times 9 \times 10^{28} \times 2.5 \times 10^{-7}}$ ms^{-1}
$= 0.75 \times 10^{-3}$ ms^{-1} = **0.75 mms^{-1}.**

3. Current density, $j = \dfrac{I}{A} = \dfrac{1.8\text{ A}}{0.5 \times 10^{-6}\text{ m}^2}$
$= \mathbf{3.6 \times 10^6\ Am^{-2}}$.
Drift speed,
$v_d = \dfrac{j}{en} = \dfrac{3.6 \times 10^6}{1.6 \times 10^{-19} \times 8.8 \times 10^{28}}$
$= \mathbf{2.56 \times 10^{-4}\ ms^{-1}}$.

4. Relaxation time, $\tau = \dfrac{m_e}{e^2 n\rho}$
$= \dfrac{9.11 \times 10^{-31}}{(1.6 \times 10^{-19})^2 \times 8.4 \times 10^{28} \times 1.7 \times 10^{-8}}$
$= \mathbf{2.49 \times 10^{-14}\ s.}$

5. Diameter of wire, $D = 1.0\text{ mm} = 10^{-3}$ m
Area of cross-section,
$A = \dfrac{\pi D^2}{4} = \dfrac{\pi \times (10^{-3})^2}{4} = 7.854 \times 10^{-7}\text{ m}^2$
$v_d = \dfrac{I}{enA} = \dfrac{0.2}{1.6 \times 10^{-19} \times 8.4 \times 10^{28} \times 7.854 \times 10^{-7}}$
$= \mathbf{1.895 \times 10^{-5}\ ms^{-1}}$.

6. Drift velocity, $v_d = \dfrac{I}{enA}$
$= \dfrac{2}{1.6 \times 10^{-19} \times 10^{29} \times 1 \times 10^{-6}}$
$= 1.25 \times 10^{-4}$ ms^{-1}.
Required time, $t = \dfrac{l}{v_d} = \dfrac{4}{1.25 \times 10^{-4}} = \mathbf{3.2 \times 10^4\ s.}$

7. $I = \dfrac{q}{t} = \dfrac{10\text{ C}}{5 \times 60\text{ s}} = \mathbf{3.33 \times 10^{-2}\ A.}$
$v_d = \dfrac{I}{enA} = \dfrac{I}{en(\pi r^2)}$
$= \dfrac{3.33 \times 10^{-2}}{1.6 \times 10^{-19} \times 5 \times 10^{22} \times 10^6 \times 3.14 \times (10^{-3})^2}$
$= \mathbf{1.326 \times 10^{-6}\ ms^{-1}}$.

8. As the two wires are connected in series, so current through each wire, $I = 10$ A.
(*i*) Current density in copper wire,
$j = \dfrac{I}{\pi D^2/4} = \dfrac{10 \times 4}{3.14 \times (0.16 \times 10^{-2})^2}$
$= \mathbf{4.976 \times 10^6\ Am^{-2}}$.
(*ii*) Area of cross-section of aluminium wire,
$A = \dfrac{\pi D^2}{4} = \dfrac{3.14 \times (0.25 \times 10^{-2})^2}{4}$
$= 4.9 \times 10^{-6}\text{ m}^2$
$v_d = \dfrac{I}{enA} = \dfrac{10}{1.6 \times 10^{-19} \times 10^{29} \times 4.9 \times 10^{-6}}$
$= \mathbf{1.28 \times 10^{-4}\ ms^{-1}}$.

9. No. of atoms in 63 gram of copper $= 6.023 \times 10^{23}$
No. of atoms in 8.9 gram or 1 cm³ of copper
$= \dfrac{6.023 \times 10^{23} \times 8.9}{63}$
No. of atoms per m^3 of copper
$= \dfrac{6.023 \times 10^{23} \times 8.9 \times 10^6}{63}$

Electron density,

$$n = \frac{6.023 \times 10^{23} \times 8.9 \times 10^6}{63} = 8.48 \times 10^{28}\,\text{m}^{-3}$$

Also $I = 30$ A, $A = 2\,\text{mm}^2 = 2 \times 10^{-6}\,\text{m}^2$,
$e = 1.6 \times 10^{-19}$C

∴ Drift velocity,

$$v_d = \frac{I}{enA} = \frac{30}{1.6 \times 10^{-19} \times 8.48 \times 10^{28} \times 2 \times 10^{-6}}$$
$$= \mathbf{1.1 \times 10^{-3}\ ms^{-1}}.$$

The *rms* velocity of electrons at 27°C (= 300 K) is given by

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23}}{9 \times 10^{-31}}}$$
$$= \mathbf{1.17 \times 10^5\ ms^{-1}}$$

The rms velocity is about 10^8 times the drift velocity.

10. Mass of silver wire,

$$m = Al\rho = 3.14 \times 10^{-6} \times 1 \times 10.5 \times 10^3$$

No. of electrons per unit volume of silver,

$$n = \frac{6.023 \times 10^{23}}{108} \times \frac{3.14 \times 10.5 \times 10^{-3}}{3.14 \times 10^{-6} \times 1}$$
$$= 5.8557 \times 10^{28}$$

$$\therefore \quad v_d = \frac{I}{enA}$$
$$= \frac{10}{1.6 \times 10^{-19} \times 5.8557 \times 10^{28} \times 3.14 \times 10^{-6}}$$
$$= \mathbf{3.399 \times 10^{-4}\ ms^{-1}}$$

11. $E = \frac{V}{l} = \frac{1.5\text{ V}}{0.2\text{ m}} = 7.5\ \text{Vm}^{-1}$.

Current density,

$$j = \frac{I}{A} = \frac{2.4}{0.3 \times 10^{-6}} = 8 \times 10^6\ \text{Am}^{-2}.$$

As $j = \sigma E = \frac{ne^2 \tau}{m} E$

$$\therefore \quad \tau = \frac{m \cdot j}{ne^2 E} = \frac{9.1 \times 10^{-31} \times 8 \times 10^6}{8.4 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 7.5}$$
$$= \mathbf{4.51 \times 10^{-16}\ s.}$$

3.14 MOBILITY OF CHARGE CARRIERS

24. *Define mobility of charge carrier. Write relations between electric current and mobility for (i) a conductor and (ii) a semiconductor. Hence write an expression for the conductivity of a semiconductor.*

Mobility. The conductivity of any material is due to its mobile charge carriers. These may be electrons in metals, positive and negative ions in electrolytes ; and electrons and holes in semiconductors.

The mobility of a charge carrier is the drift velocity acquired by it in a unit electric field. It is given by

$$\mu = \frac{v_d}{E}$$

As drift velocity, $v_d = \frac{qE\tau}{m}$

$$\therefore \quad \mu = \frac{v_d}{E} = q\frac{\tau}{m}$$

For an electron, $\mu_e = \frac{e\tau_e}{m_e}$

For a hole, $\mu_h = \frac{e\tau_h}{m_h}$

The mobilities of both electrons and holes are positive ; although their drift velocities are opposite to each other.

SI unit of mobility = $\text{m}^2\text{V}^{-1}\text{s}^{-1}$

Practical unit of mobility = $\text{cm}^2\ \text{V}^{-1}\text{s}^{-1}$.

$$1\ \text{m}^2\text{V}^{-1}\text{s}^{-1} = 10^4\ \text{cm}^2\ \text{V}^{-1}\text{s}^{-1}$$

Relation between electric current and mobility for a conductor

In a metallic conductor, the electric current is due to its free electrons and is given by

$$I = enAv_d$$

But $v_d = \mu_e E \quad \therefore \quad I = enA\mu_e E$

This is the relation between electric current and electron mobility.

Relation between electric current and mobility for a semiconductor

The conductivity of a semiconductor is both due to electrons and holes. So electric current in a semiconductor is given by

$$I = I_e + I_h = enAv_e + epAv_h$$
$$= enA\mu_e E + epA\mu_h E$$
$$= eAE(n\mu_e + p\mu_h) \qquad ...(i)$$

where n and p are the electron and hole densities of the semiconductor.

Conductivity of a semiconductor. According to Ohm's law,

$$I = \frac{V}{R} = \frac{El}{\rho l / A} = \frac{EA}{\rho} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$\frac{EA}{\rho} = eAE(n\mu_e + p\mu_h)$$

or

$$\frac{1}{\rho} = e(n\mu_e + p\mu_h)$$

But $1/\rho$ is the electrical conductivity σ. Therefore,

$$\sigma = e(n\mu_e + p\mu_h)$$

Table 3.3 Mobilities in some materials at room temperature, in $cm^2V^{-1}s^{-1}$

Materials	Electrons	Holes
Diamond	1800	1200
Silicon	1350	480
Germanium	3600	1800
InSb	800	450
GaAs	8000	300

Examples based on Mobility of Charge Carriers

Formulae Used

1. Mobility, $\mu = \dfrac{v_d}{E} = \dfrac{q\tau}{m}$
2. Electric current, $I = enAv_d = enA\,\mu E$
3. Conductivity of metallic conductor, $\sigma = ne\mu_e$
4. Conductivity of a semiconductor, $\sigma = ne\mu_e + pe\mu_h$

Units Used

Conductivity σ is in Sm^{-1} and mobility μ in $m^2V^{-1}s^{-1}$.

Example 35. *A potential difference of 6 V is applied across a conductor of length 0.12 m. Calculate the drift velocity of electrons, if the electron mobility is $5.6 \times 10^{-6}\ m^2V^{-1}s^{-1}$.*

Solution. Here $V = 6$ V, $l = 0.12$ m,

$$\mu = 5.6 \times 10^{-6}\ m^2V^{-1}s^{-1}$$

Drift velocity,

$$v_d = \mu E = \mu \cdot \frac{V}{l} = \frac{5.6 \times 10^{-6} \times 6}{0.12}\ \text{ms}^{-1}$$

$$= \mathbf{2.8 \times 10^{-4}\ ms^{-1}}.$$

Example 36. *The number density of electrons in copper is $8.5 \times 10^{28} m^{-3}$. Determine the current flowing through a copper wire of length 0.2 m, area of cross-section 1 mm^2, when connected to a battery of 3 V. Given the electron mobility $= 4.5 \times 10^{-6}\ m^2V^{-1}s^{-1}$ and charge on electron $= 1.6 \times 10^{-19}$ C.*

Solution. Here $n = 8.5 \times 10^{28}\ \text{m}^{-3}$, $l = 0.2$ m,

$A = 1\ \text{mm}^2 = 10^{-6}\text{m}^2$, $V = 3$ V,

$\mu = 4.5 \times 10^{-6}\ \text{m}^2\ \text{V}^{-1}\text{s}^{-1}$, $e = 1.6 \times 10^{-19}$C.

Electric field set up in the copper wire,

$$E = \frac{V}{l} = \frac{3}{0.2} = 15\ \text{Vm}^{-1}$$

Current,

$$I = enA\mu E$$

$$= 1.6 \times 10^{-19} \times 8.5 \times 10^{28} \times 10^{-6} \times 4.5 \times 10^{-6} \times 15$$

$$= \mathbf{0.918\ A}.$$

Example 37. *A semiconductor has the electron concentration $0.45 \times 10^{12}\ m^{-3}$ and hole concentration $5 \times 10^{20} m^{-3}$. Find its conductivity. Given : electron mobility $= 0.135\ m^2V^{-1}s^{-1}$ and hole mobility $= 0.048\ m^2V^{-1}s^{-1}$; $e = 1.6 \times 10^{-19}$ coulomb.*

Solution. Here $n = 0.45 \times 10^{12}\text{m}^{-3}$, $p = 5 \times 10^{20}\text{m}^{-3}$, $\mu_e = 0.135\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$, $\mu_h = 0.048\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$

Conductivity of the semiconductor is

$$\sigma = e(n\mu_e + p\mu_h)$$

$$= 1.6 \times 10^{-19}(0.45 \times 10^{12} \times 0.135 + 5 \times 10^{20} \times 0.048)\ \text{Sm}^{-1}$$

$$= 1.6 \times 10^{-7}(0.06075 + 0.24 \times 10^{8})\ \text{Sm}^{-1}$$

$$= 1.6 \times 10^{-7} \times 0.24 \times 10^{8}\text{Sm}^{-1} = \mathbf{3.84\ Sm^{-1}}.$$

Problems For Practice

1. A potential difference of 4.5 V is applied across a conductor of length 0.1 m. If the drift velocity of electrons is $1.5 \times 10^{-4}\text{ms}^{-1}$, find the electron mobility. (Ans. $3.33 \times 10^{-6}\text{m}^2\text{V}^{-1}\text{s}^{-1}$)
2. The number density of electrons in copper is $8.5 \times 10^{28}\text{m}^{-3}$. A current of 1 A flows through a copper wire of length 0.24 m and area of cross-section 1.2 mm^2, when connected to a battery of 3 V. Find the electron mobility. (Ans. $4.9 \times 10^{-6}\text{m}^2\text{V}^{-1}\text{s}^{-1}$)
3. Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are $0.54\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ and $0.18\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ respectively. If the electron and hole densities are equal to $3.6 \times 10^{19}\text{m}^{-3}$, calculate the germanium conductivity. **[BIT Ranchi 1997]** (Ans. $4.147\ \text{Sm}^{-1}$)

HINTS

1. $E = \dfrac{V}{l} = \dfrac{4.5\ \text{V}}{0.1\ \text{m}} = 45\ \text{Vm}^{-1}$.

$$\mu = \frac{v_d}{E} = \frac{1.5 \times 10^{-4}\ \text{ms}^{-1}}{45\ \text{Vm}^{-1}} = \mathbf{3.33 \times 10^{-6}\ m^2V^{-1}s^{-1}}.$$

2. $I = enA\mu E = enA\mu \cdot \dfrac{V}{l}$

$$\therefore \mu = \frac{Il}{enAV} = \frac{1 \times 0.24}{1.6 \times 10^{-19} \times 8.5 \times 10^{28} \times 1.2 \times 10^{-6} \times 3}$$

$$= \mathbf{4.9 \times 10^{-6}\ m^2V^{-1}s^{-1}}.$$

3. Here $\mu_e = 0.54\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$, $\mu_h = 0.18\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$, $n = p = 3.6 \times 10^{19}\text{m}^{-3}$

Conductivity,

$$\sigma = e(n\mu_e + p\mu_h) = en(\mu_e + \mu_h)$$

$$= 1.6 \times 10^{-19} \times 3.6 \times 10^{19}(0.54 + 0.18)$$

$$= \mathbf{4.147\ Sm^{-1}}.$$

3.15 TEMPERATURE DEPENDENCE OF RESISTIVITY

25. *Explain the variation of resistivity of metals, semiconductors, insulators and electrolytes with the change in temperature. Define temperature coefficient of resistivity.*

Temperature dependence of resistivity. The resistivity of any material depends on the number density n of free electrons and the mean collision time τ.

$$\rho = \frac{m}{ne^2 \tau}$$

1. Metals. For metals, the number density n of free electrons is almost independent of temperature. As temperature increases, the thermal speed of free electrons increases and also the amplitude of vibration of the metal ions increases. Consequently, the free electrons collide more frequently with the metal ions. The mean collision time τ decreases. Hence *the resistivity of a metal* ($\rho \propto 1/\tau$) *increases and the conductivity decreases with the increase in temperature.*

For most of the metals, resistivity increases linearly with the increase in temperature, around and above the room temperature. In such cases, resistivity ρ at any temperature T is given by

$$\rho = \rho_0 [1 + \alpha (T - T_0)] \qquad ...(1)$$

where ρ_0 is the resistivity at a lower reference temperature T_0 (usually 20° C) and α is the coefficient of resistivity. Obviously,

$$\alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)} = \frac{1}{\rho_0} \cdot \frac{d\rho}{dT}$$

Thus, the ***temperature coefficient of resistivity*** α *may be defined as the increase in resistivity per unit resistivity per degree rise in temperature.*

The unit of α is $°C^{-1}$. For metals α is positive. For many metallic elements, α is nearly $4 \times 10^{-3} °C^{-1}$. For such conductors, the temperature dependence of ρ at low temperatures is non-linear. At low temperatures, the resistivity of a pure metal increases as a higher power of temperature, as shown for copper in Fig. 3.20(a).

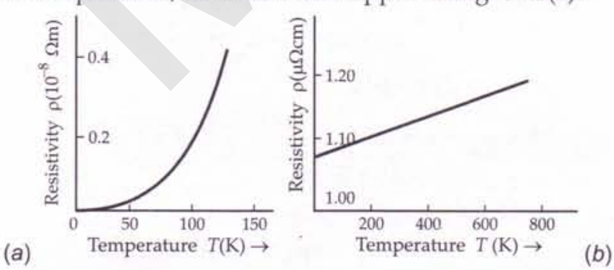


Fig. 3.20 (a) Variation of resistivity ρ of copper with temperature. (b) Variation of resistivity ρ of nichrome with temperature.

Alloys have high resistivity. The resistivity of nichrome has weak temperature dependence [Fig. 3.20(b)] while that of manganin is almost independent of temperature. At absolute zero, a pure metal has negligibly small resistivity while an alloy (like nichrome) has some residual resistivity. This fact can be used to distinguish a pure metal from an alloy.

As $\quad R = \rho \frac{l}{A} \quad$ *i.e.,* $\quad R \propto \rho$

Thus equation (1) can be written in terms of resistances as

$$R_t = R_0 (1 + \alpha t)$$

where R_t = the resistance at t°C

R_0 = the resistance at 0°C, and

t = the rise in temperature.

2. Semiconductors and insulators. In case of insulators and semiconductors, the relaxation time τ does not change with temperature but the number density of free electrons increases exponentially with the increase in temperature. Consequently, the conductivity increases or resistivity decreases exponentially with the increase in temperature.

The number density of electrons at temperature T is given by

$$n(T) = n_0 \, e^{-E_g / k_B T}$$

where k_B is the Boltzmann constant and E_g is the energy gap (positive energy) between conduction and valence bands of the substance.

As $\rho \propto \frac{1}{n}$, so we can write

$$\frac{1}{\rho(T)} = \frac{1}{\rho_0} e^{-E_g / k_B T}$$

or $$\rho(T) = \rho_0 \, e^{E_g / k_B T}$$

This equation implies that the resistivity of semiconductors and insulators rapidly increases with the decrease in temperature, becoming infinitely large as $T \to 0$.

At room temperature, $k_B T = 0.03$ eV. Whether the non-conducting substance is an insulator or a semiconductor, depends on the size of the energy gap, E_g:

(*i*) If $E_g \le 1$ eV, the resistivity at room temperature is not very high and the substance is a ***semiconductor***.

(*ii*) If $E_g > 1$ eV, the resistivity at room temperature is very high ($\sim 10^3 \, \Omega$ m) and the substance is an ***insulator***.

The coefficient of resistivity (α) is negative for carbon and semiconductors *i.e.*, their resistivity decreases with temperature, as shown in Fig. 3.21.

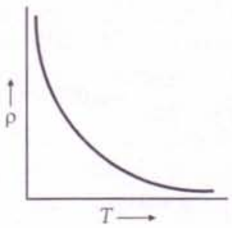


Fig. 3.21 Resistivity of a semiconductor decreases rapidly with temperature.

3. Electrolytes. As the temperature increases, the interionic attractions (solute-solute, solvent- solute and solvent-solvent types) decrease and also the viscous forces decrease, the ions move more freely. Hence conductivity increases or the resistivity decreases as the temperature of an electrolytic solution increases.

26. *Why alloys like constantan or manganin are used for making standard resistors ?*

Use of alloys in making standard resistors. Alloys like constantan or manganin are used for making standard resistance coils because of the following reasons :

(*i*) These alloys have high value of resistivity.

(*ii*) They have very small temperature coefficient. So their resistance does not change appreciably even for several degrees rise of temperature.

(*iii*) They are least affected by atmospheric conditions like air, moisture, etc.

(*iv*) Their contact potential with copper is small.

Examples based on Temperature Variation of Resistance

Formulae Used

Temperature coefficient of resistance

$$\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$$

If $t_1 = 0°C$ and $t_2 = t°C$, then

$$\alpha = \frac{R_t - R_0}{R_0 \times t} \quad \text{or} \quad R_t = R_0 (1 + \alpha t)$$

Units Used

Resistances are in Ω, temperatures in °C or K.

Example 38. *(i) At what temperature would the resistance of a copper conductor be double its resistance at 0°C ? (ii) Does this temperature hold for all copper conductors regardless of shape and size ?*

Given α for Cu = 3.9×10^{-3} °C^{-1}.

Solution. (*i*) $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{2\ R_0 - R_0}{R_0(t - 0)} = \frac{1}{t}$

$$\therefore \quad t = \frac{1}{\alpha} = \frac{1}{3.9 \times 10^{-3}} = 256°C$$

Thus the resistance of copper conductor becomes double at 256°C.

(*ii*) Since α does not depend on size and shape of the conductor, so the above result holds for all copper conductors.

Example 39. *The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.39 Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.975 Ω. Calculate the temperature of the bath.* [NCERT]

Solution. Here $R_0 = 5\ \Omega$, $R_{100} = 5.23\ \Omega$, $R_t = 5.795\ \Omega$

As $\quad R_t = R_0 (1 + \alpha t)$

$$\therefore \quad R_t - R_0 = R_0\, \alpha t \qquad ...(i)$$

and $$R_{100} - R_0 = R_0\, \alpha \times 100 \qquad ...(ii)$$

On dividing (*i*) by (*ii*), we get

$$\frac{R_t - R_0}{R_{100} - R_0} = \frac{t}{100}$$

or $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

$$= \frac{5.795 - 5}{5.23 - 5} \times 100 = \frac{0.795}{0.23} \times 100 = \mathbf{345.65°C}$$

Example 40. *A nichrome heating element connected to a 220 V supply draws an initial current of 2.2 A which settles down after a few seconds to a steady value of 2.0 A. Find the steady temperature of the heating element. The room temperature is 30°C and the average temperature coefficient of resistance of nichrome is 1.7×10^{-4} per°C.*

Solution. Here $V = 220$ V, $I_1 = 2.2$ A, $I_2 = 2.0$ A, $\alpha = 1.7 \times 10^{-4}$ °C^{-1}

Resistance at room temperature of 30°C,

$$R_1 = \frac{V}{I_1} = \frac{220}{2.2} = 100\ \Omega$$

Resistance at steady temperature,

$$R_2 = \frac{V}{I_2} = \frac{220}{2.0} = 110\ \Omega$$

As $$\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$$

$$\therefore \quad t_2 - t_1 = \frac{R_2 - R_1}{R_1 \alpha} = \frac{110 - 100}{100 \times 1.7 \times 10^{-4}} = 588°C$$

Steady temperature,

$$t_2 = 588 + t_1 = 588 + 30 = \mathbf{618°C.}$$

Example 41. *An electric toaster uses nichrome (an alloy of nickel and chromium) for its heating element. When a negligibly small current passes through it, its resistance at* *room temperature (27.0°C) is found to be* 75.3 Ω. *When the toaster is connected to a 230 V supply, the current settles after a few seconds to a steady value of 2.68 A. What is the steady temperature of the nichrome element ? The temperature coefficient of resistance of nichrome averaged over the temperature range involved is* $1.70 \times 10^{-4}\,°C^{-1}$. [NCERT]

Solution. Here $R_1 = 75.3\,\Omega,\ t_1 = 27°C$

$$R_2 = \frac{230}{2.68} = 85.8\,\Omega, \quad t_2 = ?$$

$$t_2 - t_1 = \frac{R_2 - R_1}{R_1\,\alpha} = \frac{85.8 - 75.3}{75.3 \times 1.70 \times 10^{-4}} = 820°C$$

Steady temperature,

$$t_2 = 820 + t_1 = 820 + 27 = \mathbf{847°C.}$$

At the steady temperature, the heating effect due to the current equals heat loss to the surroundings.

Example 42. *The resistance of a tungsten filament at 150°C is 133 ohm. What will be its resistance at 500°C ? The temperature coefficient of resistance of tungsten is 0.0045 per°C.*

Solution. Here $R_{150} = 133\,\Omega,\ \alpha = 0.0045°C,\ R_{500} = ?$

Now $R_t = R_0\,(1 + \alpha\, t)$

$\therefore\quad R_{150} = R_0\,(1 + \alpha \times 150)$

or $\quad 133 = R_0\,(1 + 0.0045 \times 150) \qquad ...(1)$

and $\quad R_{500} = R_0\,(1 + \alpha \times 500)$

or $\quad R_{500} = R_0\,(1 + 0.0045 \times 500) \qquad ...(2)$

Dividing (2) by (1), we get

$$\frac{R_{500}}{133} = \frac{1 + 0.0045 \times 500}{1 + 0.0045 \times 150} = \frac{3.25}{1.675}$$

or $\quad R_{500} = \dfrac{3.25}{1.675} \times 133 = \mathbf{258\,\Omega.}$

Example 43. *The resistance of a conductor at 20°C is 3.15 Ω and at 100°C is 3.75 Ω. Determine the temperature coefficient of resistance of the conductor. What will be the resistance of the conductor at 0°C ?*

Solution. $R_1 = R_0\,(1 + \alpha\, t_1)$ and $R_2 = R_0\,(1 + \alpha\, t_2)$

On dividing,

$$\frac{R_1}{R_2} = \frac{1 + \alpha\, t_1}{1 + \alpha\, t_2}$$

or $\quad R_1\,(1 + \alpha\, t_2) = R_2\,(1 + \alpha\, t_1)$

or $\quad \alpha = \dfrac{R_2 - R_1}{R_1 t_2 - R_2 t_1}.$

Here $t_1 = 20°C,\quad R_1 = 3.15\,\Omega,$
$t_2 = 100°C,\quad R_2 = 3.75\,\Omega$

$$\therefore\quad \alpha = \frac{3.75 - 3.15}{(3.15 \times 100) - (3.75 \times 20)}$$

$$= \frac{0.60}{315 - 75} = \frac{0.60}{240} = \mathbf{0.0025\,°C^{-1}.}$$

$$R_0 = \frac{R_t}{1 + \alpha\, t_1} = \frac{3.15}{1 + 0.0025 \times 20} = \mathbf{3.0\,\Omega.}$$

Example 44. *A standard coil marked 2 Ω is found to have a resistance of 2.118 Ω at 30°C. Calculate the temperature at which the marking is correct. The temperature coefficient of the resistance of the material of the coil is* $0.0042\,°C^{-1}$.

Solution. $R_1 = R_0\,(1 + \alpha\, t_1)$ and $R_2 = R_0\,(1 + \alpha\, t_2)$

$$\therefore\quad \frac{R_1}{R_2} = \frac{1 + \alpha\, t_1}{1 + \alpha\, t_2}$$

Here, $R_1 = 2\,\Omega,\ R_2 = 2.118\,\Omega,\ t_2 = 30°C,\ t_1 = ?$

$$\therefore\quad \frac{2}{2.118} = \frac{1 + 0.0042 \times t_1}{1 + 0.0042 \times 30} = \frac{1 + 0.0042 \times t_1}{1.126}$$

or $\quad 1 + 0.0042\, t_1 = \dfrac{2 \times 1.126}{2.118} = \dfrac{2.252}{2.118}$

$$\therefore\quad t_1 = \frac{1}{0.0042}\left[\frac{2.252}{2.118} - 1\right] = \frac{0.104}{0.0042 \times 2.118} \simeq \mathbf{15°C}$$

i.e., the marking will be correct at 15°C.

Example 45. *A potential difference of 200 V is applied to a coil at a temperature of 15°C and the current is 10 A. What will be the mean temperature of the coil when the current has fallen to 5 A, the applied voltage being same as before ? Given* $\alpha = \dfrac{1}{234}\,°C^{-1}$ *at 0°C.*

Solution. In the second case, the current decreases due to the increase in resistance on heating.

Now $\quad R_{15} = \dfrac{V}{I} = \dfrac{200}{10} = 20\,\Omega$

Let t be the temperature at which current falls to 5 A. Then

$$R_t = \frac{200}{5} = 40\,\Omega$$

As $\quad R_t = R_0\,(1 + \alpha\, t)$

$$R_{15} = R_0\left(1 + \frac{15}{234}\right) \quad \text{or} \quad 20 = \frac{R_0 \times 249}{234} \qquad ...(1)$$

$$R_t = R_0\left(1 + \frac{t}{234}\right) \quad \text{or} \quad 40 = R_0\left(\frac{234 + t}{234}\right) \qquad ...(2)$$

Dividing (2) by (1),

$$2 = \frac{234 + t}{249}$$

or $\quad t = 498 - 234 = \mathbf{264°C.}$

Example 46. *The resistances of iron and copper wires at 20°C are 3.9 Ω and 4.1 Ω respectively. At what temperature will the resistances be equal ? Temperature coefficient of resistivity for iron is* $5.0\times10^{-3}\,K^{-1}$ *and for copper it is* $4.0\times10^{-3}\,K^{-1}$. *Neglect any thermal expansion.*

Solution. Let resistance of iron wire at t°C

= Resistance of copper wire at t°C

$$\therefore\quad R_{20}\,[1+\alpha\,(t-20)] = R'_{20}\,[1+\alpha'\,(t-20)]$$

$$3.9[1+5.0\times10^{-3}(t-20)] = 4.1\,[1+4.0\times10^{-3}\,(t-20)]$$

$$[3.9\times5-4.1\times4]\times10^{-3}\times(t-20) = 4.1-3.9$$

$$t-20 = \frac{0.2}{3.1\times10^{-3}} = 64.5$$

$$t = 64.5+20 = \mathbf{84.5°C.}$$

Example 47. *A metal wire of diameter 2 mm and length 100 m has a resistance of 0.5475 Ω at 20°C and 0.805 Ω at 150°C. Find (i) the temperature coefficient of resistance (ii) resistance at 0°C (iii) resistivities at 0° and 20°C.*

Solution. Here $r = 1\text{ mm} = 10^{-3}\text{m}$, $l = 100$ m, $t_1 = 20°C$, $R_1 = 0.5475\,\Omega$, $t_2 = 150°C$, $R_2 = 0.805\,\Omega$

(*i*) Temperature coefficient of resistance is

$$\alpha = \frac{R_2-R_1}{R_1\,(t_2-t_1)} = \frac{0.805-0.5475}{0.5475\,(150-20)}$$

$$= \mathbf{3.6\times10^{-3}\,°C^{-1}}.$$

(*ii*) Resistance at 0°C is

$$R_0 = \frac{R_1}{1+\alpha\,t_1} = \frac{0.5475}{1+3.6\times10^{-3}\times20} = \frac{0.5475}{1.072}$$

$$= \mathbf{0.5107\,\Omega.}$$

(*iii*) Resistivity at 0°C,

$$\rho_0 = \frac{R_0\,A}{l} = \frac{R_0\times\pi r^2}{l} = \frac{0.5107\times3.14\times(10^{-3})^2}{100}$$

$$= \mathbf{1.60\times10^{-8}\,\Omega m.}$$

Resistivity at 20°C is

$$\rho_{20} = \rho_0\,(1+\alpha\,t)$$

$$= 1.60\times10^{-8}\,(1+3.6\times10^{-3}\times20)$$

$$= 1.60\times10^{-8}\times1.072 = \mathbf{1.72\times10^{-8}\,\Omega m.}$$

Problems For Practice

1. A platinum wire has a resistance of 10 Ω at 0°C and of 20 Ω at 273°C. Find its temperature coefficient of resistance. (Ans. $\frac{1}{273}$ °C^{-1})

2. A standard coil marked 3 Ω is found to have a true resistance of 3.115 Ω at 300 K. Calculate the temperature at which marking is correct. Temperature coefficient of resistance of the material of the coil is 4.2×10^{-3} °C^{-1}. (Ans. 290.2 K)

3. The resistance of a silver wire at 0°C is 1.25 Ω. Upto what temperature it must be heated so that its resistance is doubled ? The temperature coefficient of resistance of silver is 0.00375 °C^{-1}. Will the temperature be same for all silver conductors of all shapes ? (Ans. 267° C, Yes)

4. The resistance of a coil used in a platinum-resistance thermometer at 0°C is 3.00 Ω and at 100°C is 3.75 Ω. Its resistance at an unknown temperature is measured as 3.15 Ω. Calculate the unknown temperature. (Ans. 20°C)

5. The temperature coefficient of a resistance wire is 0.0012.5°C^{-1}. At 300 K, its resistance is 1 Ω. At what temperature the resistance of the wire will be 2 Ω ? [IIT 80] (Ans. 1127 K)

6. The temperature coefficient of resistivity of copper is 0.004°C^{-1}. Find the resistance of a 5 m long copper wire of diameter 0.2 mm at 100°C, if the resistivity of copper at 0°C is 1.7×10^{-8} Ωm. (Ans. 3.8 Ω)

HINTS

1. $\alpha = \frac{R_t-R_0}{R_0\times t} = \frac{20-10}{10\times273} = \frac{\mathbf{1}}{\mathbf{273}}\,°C^{-1}$.

2. Here $t = 300-273 = 27°C$

$$R_{27} = R_0\,(1+\alpha\times27)$$

$$3.115 = R_0(1+4.2\times10^{-3}\times27) \quad ...(1)$$

and $$3 = R_0(1+4.2\times10^{-3}\times t) \quad ...(2)$$

Dividing (2) by (1), we get

$$\frac{3}{3.115} = \frac{1+4.2\times10^{-3}\times t}{1+4.2\times10^{-3}\times27}$$

This gives,

$$t = 17.2°C = 17.2+273 = \mathbf{290.2\text{ K.}}$$

3. Proceed as in Example 38, page 3.25.

4. $$R_t = R_0\,(1+\alpha t)$$

$$R_{100} = R_0\,(1+\alpha\times100)$$

$$3.75 = 3.00\,(1+\alpha\times100)$$

$$\frac{3.75}{3}-1 = 100\alpha$$

$$\therefore\quad \alpha = \frac{0.75}{3\times100} = 0.0025°C^{-1}$$

$$t = \frac{R_t-R_0}{R_0\times\alpha} = \frac{3.15-3.00}{3.00\times0.0025} = \mathbf{20°C.}$$

5. $300\text{ K} = 300-273 = 27°C$

$$\therefore\quad R_{27} = R_0\,(1+\alpha\times27) = 1\,\Omega$$

and $$R_t = R_0\,(1+\alpha\times t) = 2\,\Omega$$

$$\therefore \frac{1+\alpha t}{1+27\alpha} = \frac{2}{1}$$

or $1+\alpha t = 2+54\alpha$

or $$t = \frac{1+54\,\alpha}{\alpha} = \frac{1+54\times 0.00125}{0.00125} = 854°C.$$

$$= 854 + 273 = \mathbf{1127\ K.}$$

6. $\rho_{100} = \rho_0\,(1+\alpha t) = 1.7\times 10^{-8}(1+0.004\times 100)$

$$= 2.38\times 10^{-8}\,\Omega m$$

$$R = \rho\frac{l}{\pi r^2} = \frac{2.38\times 10^{-8}\times 5}{3.14\times(0.1\times 10^{-3})^2} \simeq \mathbf{3.8\,\Omega.}$$

3.16 LIMITATIONS OF OHM'S LAW : OHMIC AND NON-OHMIC CONDUCTORS

27. *State the conditions under which Ohm's law is not obeyed in a conductor. What are ohmic and non-ohmic conductors ? Give examples of each type.*

Limitations of Ohm's law. Ohm's law is obeyed by many substances under certain conditions but it is not a fundamental law of nature.

Ohmic conductors. *The conductors which obey Ohm's law are called Ohmic conductors.* For these conductors, the linear relationship between voltage and current ($V \propto I$) holds good. The resistance ($R = V/I$) is independent of the current I through the conductor. In these conductors, the current I gets reversed in direction when the p.d. V is reversed, but the magnitude of current changes linearly with voltage. Thus *the V-I graph for ohmic conductors is a straight line passing through the origin.* A metallic conductor for small currents and the electrolyte like copper sulphate solution with copper electrodes are ohmic conductors, as shown in Figs. 3.22(*a*) and (*b*) respectively.

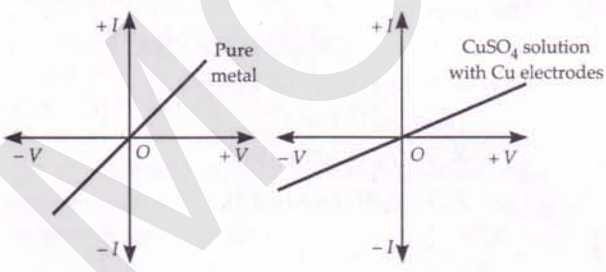


Fig. 3.22 Ohmic conductors.

Non-ohmic conductors. *The conductors which do not obey Ohm's law are called non-ohmic conductors.* The resistance of such conductors is not constant even at a given temperature, rather it is current dependent. Non-ohmic situations may be of the following types :

(*i*) The straight line *V-I* graph does not pass through the origin.

(*ii*) *V-I* relationship is non-linear.

(*iii*) *V-I* relationship depends on the sign of *V* for the same absolute value of *V*, and

(*iv*) *V-I* relationship is non-unique.

Examples of non-ohmic conductors. (*i*) ***Metallic conductor.*** For small currents, a metallic conductor obeys Ohm's law and its *V-I* graph is a straight line. But when large currents are passed through the same conductor, it gets heated up and its resistance increases. *V-I* graph no longer remains linear, *i.e.,* conductor becomes non-ohmic at higher currents, as shown in Fig. 3.23.

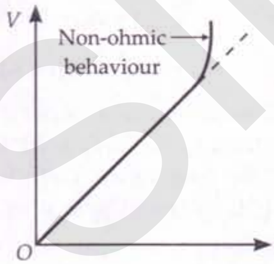


Fig. 3.23 *V-I* graph for a metallic conductor.

(*ii*) ***Water voltameter.*** Here a back e.m.f. is set up due to the liberation of hydrogen at the cathode and oxygen at the anode. No current flows through the voltameter until the applied p.d. exceeds the back e.m.f. V_0 (1.67 V for water voltameter). So *V-I* graph is a straight line but not passing through the origin, as shown in Fig. 3.24. Hence the electrolyte (water acidified with dil. H_2SO_4) is a non-ohmic conductor.

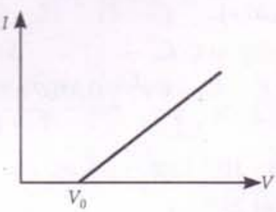


Fig. 3.24 *V-I* graph for a water voltameter.

(*iii*) ***p-n junction diode.*** It consists of a junction of *p*-type and *n*-type semiconductors (For details, refer to

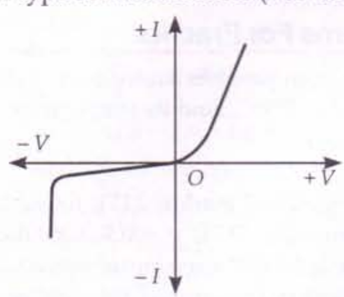


Fig. 3.25 *V-I* graph for a junction diode.

chapter 14 Vol. II). A voltage V is applied across the junction. The resulting current I is shown in Fig. 3.25. Obviously, I is not proportional to V. Further, very little current flows for fairly high negative voltage (called negative bias) and a current begins to flow for much smaller positive (forward) bias. Thus the junction diode allows current to flow only in one direction *i.e.*, it acts as a rectifier (converts a.c. into d.c.).

(*iv*) ***Thyristor.*** It consists of four alternate layers of p and n-type semiconductors. Its V-I relationship is as shown in Fig. 3.26. It can be easily seen that (*a*) the V-I relation is non-linear, (*b*) V-I relationship is different for positive and negative values of V, and (*c*) in certain portions, there are two or more values of current for the same value of voltage, *i.e.*, the V-I relationship is not unique. The region AB is interesting because the current carried by the device increases as the voltage decreases, *i.e.*, α is *negative* in this region.

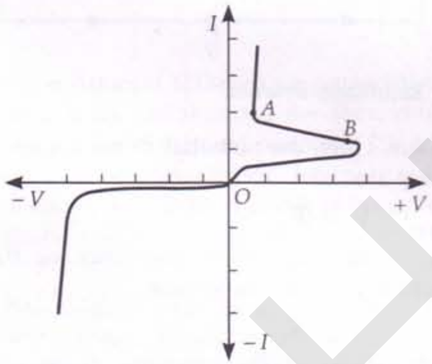


Fig. 3.26 V-I curve for a thyristor.

(*v*) ***Gallium arsenide.*** Fig. 3.27 shows the V-I graph for the semiconductor GaAs. It exhibits non-linear behaviour. Moreover, after a certain voltage, the current decreases as the voltage increases. That is, if ΔV is positive then ΔI is negative and hence the effective resistance ($= \Delta V / \Delta I$) is negative.

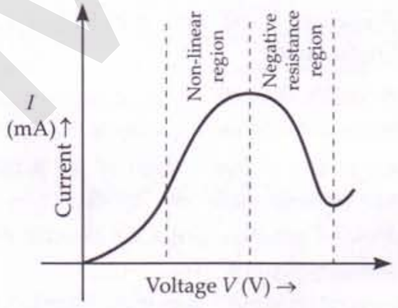


Fig. 3.27 V-I graph for GaAs.

3.17 SUPERCONDUCTIVITY

28. *What is superconductivity ? What is its cause ?*

Superconductivity. In 1911, Prof *Kamerlingh Onnes* at the University of Leiden (Holland), observed that the resistivity of mercury suddenly drops to zero at a temperature of about 4.2 K and it becomes a superconductor.

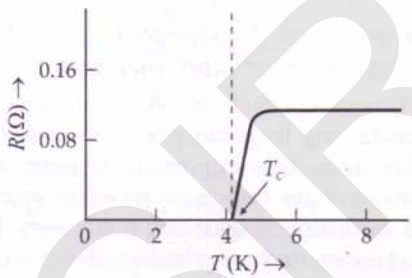


Fig. 3.28 Mercury loses complete resistance at 4.2 K.

The phenomenon of complete loss of resistivity by certain metals and alloys when they are cooled below a certain temperature is called ***superconductivity****. The temperature at which a substance undergoes a transition from normal conductor to superconductor in a zero magnetic field is called* ***transition*** *or* ***critical temperature*** (T_c).

A current once set up in a superconductor persists for a very long time without any apparent change in its magnitude.

Cause of superconductivity. It is believed that near the transition temperature, a weak attractive force acts on the electrons which brings them closer to form coupled pairs. Such coupled pairs are not deflected by ionic vibrations and so move without collisions.

29. *What is Meissener effect in superconductors ?*

Meissener effect. In 1933, *Meissner* and *Ochsenfeld* observed that if a conductor is cooled in a magnetic field to a temperature below the transition temperature, then at this temperature, the lines of magnetic induction B are pushed out of the specimen, as shown in Fig. 3.29. Thus B becomes zero inside a superconducting specimen.

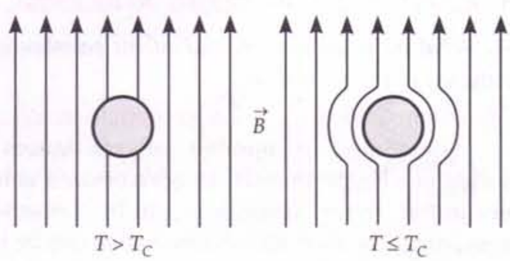


Fig. 3.29 Meissner effect in a superconductor.

The expulsion of the magnetic flux from a superconducting material when it is cooled to a temperature below the critical temperature in a magnetic field is called Meissner effect.

Meissner effect indicates that as the superconductivity appears in a material, it becomes perfectly *diamagnetic*.

30. *What is high T_C superconductivity ? Mention important applications of superconductors.*

High T_C superconductivity. A current once set up in a superconducting loop can persist for years without any applied emf. This important property of superconductors can have important practical applications. A serious difficulty in their use is the very low temperature at which they must be kept. Scientists all over the world are busy to construct alloys which would be superconducting at room temperature. Superconductivity at around 125 K has already been achieved and efforts are being made to improve upon this temperature.

Table 3.4 Critical temperatures of some superconducting materials

Material	T_C (K)
Hg	4.2
Pb_2Au	7.0
$YBa_2Cu_3O_7$	90
$Tl_2Ca_2Ba_2Cu_3O_{10}$	120

Applications of superconductors. The possible applications of superconductors are

1. For producing high magnetic fields required for research work in high energy physics.
2. For storage of memory in high speed computers.
3. In the construction of very sensitive galvanometers.
4. In levitation transportation (trains which move without rails).
5. In long distance power transmission without any wastage of power.

3.18 RESISTANCES IN SERIES AND PARALLEL

31. *What do you mean by equivalent resistance of a combination of resistances ?*

Equivalent resistance of a combination of resistances. Sometimes, a number of resistances are connected in a circuit in order to get a desired value of current in the circuit. Resistances can be connected *in series, in parallel* or their *mixed combination* can be used. *If a combination of two or more resistances in any electric circuit can be replaced by a single resistance such that there is no change in the current in the circuit and in the potential difference between the terminals of the combination, then the single resistance is called the* ***equivalent resistance*** *of the combination.*

32. *When are the resistances said to be connected in series ? Find an expression for the equivalent resistance of a number of resistances connected in series.*

Resistances in series. *If a number of resistances are connected end to end so that the same current flows through each one of them in succession, then they are said to be connected in series.* Fig. 3.30 shows three resistances R_1, R_2 and R_3 connected in series. When a potential difference V is applied across the combination, the same current I flows through each resistance.

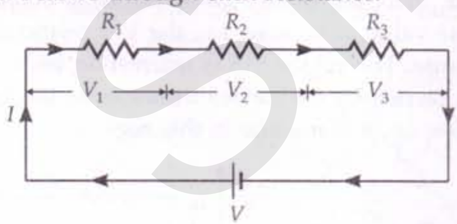


Fig. 3.30 Resistances in series.

By Ohm's law, the potential drops across the three resistances are

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$

If R_s is the equivalent resistance of the series combination, then we must have

$$V = IR_s$$

But V = Sum of the potential drops across the individual resistances

or $$V = V_1 + V_2 + V_3$$

or $$IR_s = IR_1 + IR_2 + IR_3$$

or $$R_s = R_1 + R_2 + R_3$$

The equivalent resistance of n resistances connected in series will be

$$R_s = R_1 + R_2 + R_3 + ... + R_n$$

Thus when a number of resistances are connected in series, their equivalent resistance is equal to the sum of the individual resistances.

Laws of resistances in series

(i) *Current through each resistance is same.*

(ii) *Total potential drop = Sum of the potential drops across the individual resistances.*

(iii) *Individual potential drops are directly proportional to individual resistances.*

(iv) *Equivalent resistance = Sum of the individual resistances.*

(v) *Equivalent resistance is larger than the largest individual resistance.*

33. *When are the resistances said to be connected in parallel ? Find the equivalent resistance of a number of resistances connected in parallel.*

Resistances in parallel. *If a number of resistances are connected in between two common points so that each of them provides a separate path for current, then they are said to be connected in parallel.* Fig. 3.31 shows three resistances R_1, R_2 and R_3 connected in parallel between points A and B. Let V be the potential difference applied across the combination.

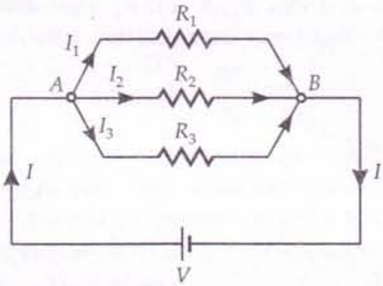


Fig. 3.31 Resistance in parallel.

Let I_1, I_2 and I_3 be the currents through the resistances R_1, R_2 and R_3 respectively. Then the current in the main circuit must be $I = I_1 + I_2 + I_3$

Since all the resistances have been connected between the same two points A and B, therefore, potential drop V is same across each of them. By Ohm's law, the currents through the individual resistances will be

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

If R_p is the equivalent resistance of the parallel combination, then we must have

$$I = \frac{V}{R_p}$$

But $\quad I = I_1 + I_2 + I_3$

or $$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The equivalent resistance R_p of n resistances connected in parallel is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}.$$

Thus *when a number of resistances are connected in parallel, the reciprocal of the equivalent resistance of the parallel combination is equal to the sum of the reciprocals of the individual resistances.*

Laws of resistances in parallel

(*i*) *Potential drop across each resistance is same.*

(*ii*) *Total current = Sum of the currents through individual resistances.*

(*iii*) *Individual currents are inversely proportional to the individual resistances.*

(*iv*) *Reciprocal of equivalent resistance = Sum of the reciprocals of the individual resistances.*

(*v*) *Equivalent resistance is less than the smallest individual resistance.*

Examples based on
Combination of Resistances in Series and Parallel

Formulae Used

1. The equivalent resistance R_s of a number of resistances connected in series is given by
$$R_s = R_1 + R_2 + R_3 + \ldots$$
2. The equivalent resistance R_P of a number of resistances connected in parallel is given by
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$
3. For two resistances in parallel,
Currents through the two resistors will be
$$I_1 = \frac{R_2\, I}{R_1 + R_2} \quad \text{and} \quad I_2 = \frac{R_1\, I}{R_1 + R_2}$$

Units Used

All resistances are in ohm (Ω).

Example 48. *A wire of resistance 4 R is bent in the form of a circle (Fig. 3.32). What is the effective resistance between the ends of the diameter ?* [CBSE D 10]

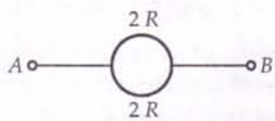


Fig. 3.32

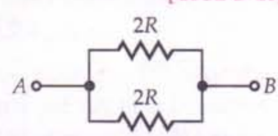


Fig. 3.33

Solution. As shown in Fig. 3.33, the two resistances of value 2 R each are in parallel with each other. So the resistance between the ends A and B of a diameter is

$$R' = \frac{2\,R \times 2\,R}{2\,R + 2\,R}$$

$$= \mathbf{R}.$$

Example 49. *Find the value of current I in the circuit shown in Fig. 3.34.* [CBSE F 03, IIT 83]

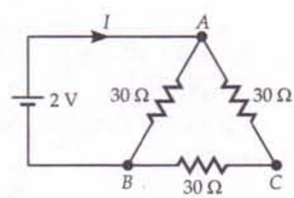


Fig. 3.34

Solution. In the given circuit, the resistance of arm $ACB (30 + 30 = 60\ \Omega)$ is in parallel with the resistance of arm $AB (= 30\ \Omega)$.

Hence the effective resistance of the circuit is

$$R = \frac{30 \times 60}{30 + 60} = 20\ \Omega$$

Current, $I = \frac{V}{R} = \frac{2}{20} = \mathbf{0.1\ A}.$

Example 50. *Determine the voltage drop across the resistor R_1 in the circuit given below with $\mathcal{E} = 60\ V$, $R_1 = 18\ \Omega$, $R_2 = 10\ \Omega$.*

Solution. As the resistances R_3 and R_4 are in series, their equivalent resistance

$$= 5 + 10 = 15\ \Omega.$$

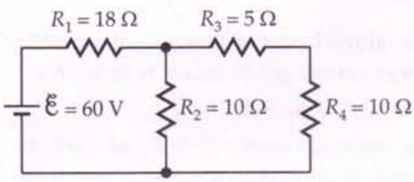


Fig. 3.35

The series combination of R_3 and R_4 is in parallel with R_2. Their equivalent resistance is

$$R' = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6\ \Omega$$

The combination R' is in series with R_1.

$\therefore$ Total resistance of the circuit,

$$R = 6 + 18 = 24\ \Omega$$

Current, $I = \frac{\mathcal{E}}{R} = \frac{60}{24} = 2.5$ A

$\therefore$ Voltage drop across R_1

$$= IR_1 = 2.5 \times 18\ \text{V} = \mathbf{45\ V}.$$

Example 51. *A letter A consists of a uniform wire of resistance 1 ohm per cm. The sides of the letter are each 20 cm long and the cross-piece in the middle is 10 cm long while the apex angle is 60°. Find the resistance of the letter between the two ends of the legs.*

Solution. Clearly,

$$AB = BC = CD = DE = BD = 10\ \text{cm}$$

$$\therefore \quad R_1 = R_2 = R_3 = R_4 = R_5 = 10\ \Omega$$

As R_2 and R_3 are in series, their combined resistance $= 10 + 10 = 20\ \Omega$. This combination is in parallel with $R_5 (= 10\ \Omega)$.

Hence resistance between points B and D is given by

$$\frac{1}{R} = \frac{1}{20} + \frac{1}{10} = \frac{3}{20} \quad \text{or} \quad R = \frac{20}{3}\ \Omega$$

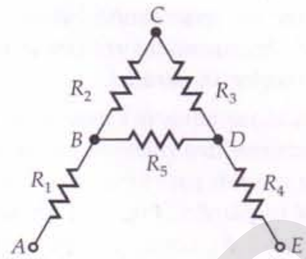


Fig. 3.36

Now resistances R_1, R and R_4 form a series combination. So resistance between the ends A and E is

$$R' = 10 + \frac{20}{3} + 10 = \mathbf{26.67\ \Omega}.$$

Example 52. *A set of n identical resistors, each of resistance $R\ \Omega$, when connected in series have an effective resistance $X\ \Omega$ and when the resistors are connected in parallel, their effective resistance is $Y\ \Omega$. Find the relation between R, X and Y.*

Solution. The effective resistance of the n resistors connected in series is

$$X = R + R + R + \dots n \text{ terms} = nR$$

The effective resistance Y of the n resistors connected in parallel is given by

$$\frac{1}{Y} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \dots n \text{ terms} = \frac{n}{R}$$

or $$Y = \frac{R}{n}$$

$$\therefore \quad XY = nR \cdot \frac{R}{n} = R^2.$$

Example 53. *A parallel combination of three resistors takes a current of 7.5 A from a 30 V supply. If the two resistors are $10\ \Omega$ and $12\ \Omega$, find the third one.*

[Punjab 91 ; Haryana 94]

Solution. Here $R_p = \frac{V}{I} = \frac{30}{7.5} = 4\ \Omega$

But $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or $$\frac{1}{4} = \frac{1}{10} + \frac{1}{12} + \frac{1}{R_3}$$

or $$\frac{1}{R_3} = \frac{1}{4} - \frac{11}{60} = \frac{1}{15} \quad \therefore \quad R_3 = \mathbf{15\ \Omega}.$$

Example 54. *When a current of 0.5 A is passed through two resistances in series, the potential difference between the ends of the series arrangement is 12.5 V. On connecting them in parallel and passing a current of 1.5 A, the potential difference between their ends is 6 V. Calculate the two resistances.*

Solution. *For series combination,* $V = 12.5$ V, $I = 0.5$ A

$$\therefore \quad R_1 + R_2 = \frac{12.5}{0.5} = 25.0\ \Omega \qquad ...(1)$$

For parallel combination, $V = 6.0$ V, $I = 1.5$ A

$$\therefore \quad R_p = \frac{V}{I} \quad \text{or} \quad \frac{R_1 R_2}{R_1 + R_2} = \frac{6.0}{1.5} = 4.0$$

or $\quad R_1 R_2 = 4(R_1 + R_2) = 4 \times 25 = 100$

$$(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4 R_1 R_2$$
$$= (25)^2 - 4 \times 100 = 225$$
$$R_1 - R_2 = 15 \qquad ...(2)$$

Solving (1) and (2), $R_1 = \mathbf{20\ \Omega}$, $R_2 = \mathbf{5\ \Omega}$.

Example 55. *Two square metal plates A and B are of same thickness and material. The side of B is twice that of A. These are connected in series, as shown in Fig. 3.37. Find the ratio* R_A / R_B *of the resistance of the two plates.*

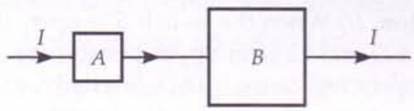


Fig. 3.37

Solution. Let l be the side of the square plate A and $2l$ that of square plate B. Let d be the thickness of each plate.

$$R_A = \frac{\rho l}{A} = \frac{\rho l}{l \times d} = \frac{\rho}{d}, \quad R_B = \frac{\rho \times 2l}{2l \times d} = \frac{\rho}{d}$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{\rho / d}{\rho / d} = \mathbf{1:1}.$$

Example 56. *Three conductors of conductances* G_1, G_2 *and* G_3 *are connected in series. Find their equivalent conductance.*

Solution. As conductance is reciprocal of resistance, therefore

$$R_1 = \frac{1}{G_1},\ R_2 = \frac{1}{G_2},\ R_3 = \frac{1}{G_3}$$

For the series combination, $R = R_1 + R_2 + R_3$

$$\therefore \quad \frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} = \frac{G_2 G_3 + G_1 G_3 + G_1 G_2}{G_1 G_2 G_3}$$

or equivalent conductance,

$$G = \frac{G_1 G_2 G_3}{G_2 G_3 + G_1 G_3 + G_1 G_2}.$$

Example 57. *A copper rod of length 20 cm and cross-sectional area 2 mm² is joined with a similar aluminium rod as shown in Fig. 3.38. Find the resistance of the combination between the ends. Resistivity of copper* $= 1.7 \times 10^{-8}\ \Omega m$ *and resistivity of aluminium* $= 2.6 \times 10^{-8}\ \Omega m$.

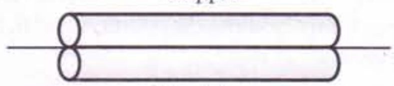


Fig. 3.38

Solution. *For copper rod,* $\rho = 1.7 \times 10^{-8}\ \Omega$m, $l = 20$ cm $= 20 \times 10^{-2}$ m, $A = 2$ mm$^2 = 2 \times 10^{-6}$ m^2

$\therefore$ Resistance,

$$R_1 = \frac{\rho l}{A} = \frac{1.7 \times 10^{-8} \times 20 \times 10^{-2}}{2 \times 10^{-6}} = 1.7 \times 10^{-3}\ \Omega$$

For aluminium rod,

$\rho = 2.6 \times 10^{-8}\ \Omega$m, $l = 20 \times 10^{-2}$ m, $A = 2 \times 10^{-6}$ m^2

$\therefore$ Resistance,

$$R_2 = \frac{2.6 \times 10^{-8} \times 20 \times 10^{-2}}{2 \times 10^{-6}} = 2.6 \times 10^{-3}\ \Omega$$

As the two rods are joined in parallel, their equivalent resistance is

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{1.7 \times 10^{-3} \times 2.6 \times 10^{-3}}{1.7 \times 10^{-3} + 2.6 \times 10^{-3}}$$
$$= \frac{1.7 \times 2.6 \times 10^{-3}}{4.3}$$
$$= 1.028 \times 10^{-3}\ \Omega = \mathbf{1.028\ m\ \Omega}.$$

Example 58. *A wire of uniform cross-section and length l has a resistance of* $16\ \Omega$. *It is cut into four equal parts. Each part is stretched uniformly to length l and all the four stretched parts are connected in parallel. Calculate the total resistance of the combination so formed. Assume that stretching of wire does not cause any change in the density of its material.*

Solution. Resistance of each of the four parts of length $l/4 = 4\ \Omega$. When each part is stretched to length l, its volume remains same.

$$V = A' l' = A l$$

or $\quad \frac{A'}{A} = \frac{l}{l'} = \frac{l/4}{l} = \frac{1}{4}$

$$\therefore \quad \frac{R}{R'} = \frac{l}{l'} \times \frac{A'}{A} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

or $\quad R' = 16 \times R = 16 \times 4 = 64\ \Omega$

i.e., resistance of each stretched part is $64\ \Omega$. When these four parts are connected in parallel, the total resistance R of the combination is given by

$$\frac{1}{R} = \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64} = \frac{4}{64} = \frac{1}{16}$$

or $\quad R = \mathbf{16\ \Omega}$.

Example 59. *Find, in the given network of resistors, the equivalent resistance between the points A and B, between A and D, and between A and C.* [IIT]

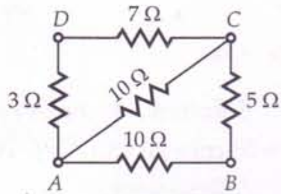


Fig. 3.39

Solution. The resistors $AD(=3\,\Omega)$ and $DC(=7\,\Omega)$ are in series to give a total resistance $R' = 10\,\Omega$. The resistance $R'(=10\,\Omega)$ and the resistor $AC(=10\,\Omega)$ are in parallel. Their equivalent resistance is

$$R'' = \frac{10 \times 10}{10 + 10} = 5\,\Omega$$

Now $R''(=5\,\Omega)$ and $CB(=5\,\Omega)$ are in series, their total resistance $R''' = 10\,\Omega$. Finally, $R'''(=10\,\Omega)$ and $AB(=10\,\Omega)$ are in parallel between A and B. Hence the equivalent resistance between points A and B is

$$R_{AB} = \frac{10 \times 10}{10 + 10} = \mathbf{5\,\Omega}.$$

Similarly, $\quad R_{AD} = \frac{\mathbf{39}}{\mathbf{16}}\,\Omega$ and $R_{AC} = \frac{\mathbf{15}}{\mathbf{4}}\,\Omega.$

Example 60. *Find the effective resistance between points A and B for the network shown in Fig. 3.40.*

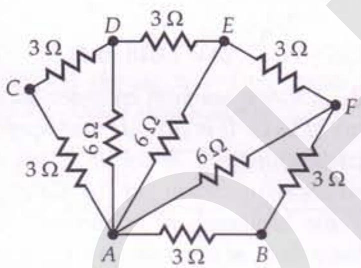


Fig. 3.40

Solution. At points A and D, a series combination of $3\,\Omega$, $3\,\Omega$ resistances (along AC and CD) is in parallel with $6\,\Omega$ resistance (along AD), therefore, resistance between A and D

$$= \frac{1}{\frac{1}{3+3} + \frac{1}{6}}\,\Omega = 3\,\Omega$$

Similarly, resistance between A and E

$$= \frac{1}{\frac{1}{3+3} + \frac{1}{6}} = 3\,\Omega$$

Resistance between A and F

$$= \frac{1}{\frac{1}{3+3} + \frac{1}{6}} = 3\,\Omega$$

Finally, resistance between A and B

$$= \frac{1}{\frac{1}{3+3} + \frac{1}{3}} = \mathbf{2\,\Omega}$$

Thus the effective resistance between A and B is $2\,\Omega$.

Example 61. *Find the effective resistance of the network shown in Fig. 3.41 between the points A and B when (i) the switch S is open (ii) switch S is closed.*

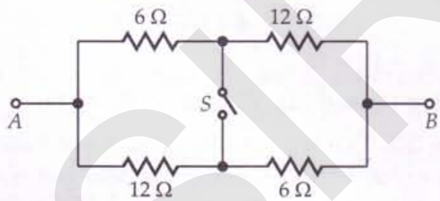


Fig. 3.41

Solution. (*i*) When the switch S is open, the resistances of $6\,\Omega$ and $12\,\Omega$ in upper portion are in series, the equivalent resistance is $18\,\Omega$. Similarly, resistances in the lower portion have equivalent resistance of $18\,\Omega$. Now the two resistances of $18\,\Omega$ are in parallel between points A and B.

$\therefore$ Effective resistance between points A and B

$$= \frac{18 \times 18}{18 + 18} = \mathbf{9\,\Omega}.$$

(*ii*) When the switch S is closed, the resistances of $6\,\Omega$ and $12\,\Omega$ on the left are in parallel. Their equivalent resistance is

$$\frac{6 \times 12}{6 + 12} = 4\,\Omega$$

Similarly, the resistances on the right have equivalent resistance of $4\,\Omega$. Now the two resistances of $4\,\Omega$ are in series.

$\therefore$ Effective resistance between points A and B

$$= 4 + 4 = \mathbf{8\,\Omega}.$$

Example 62. *Calculate the current shown by the ammeter A in the circuit shown in Fig. 3.42.* [CBSE OD 2000]

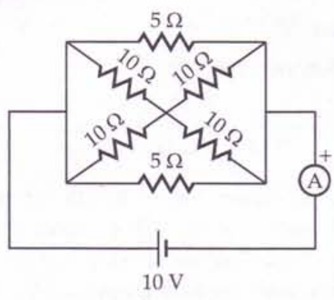


Fig. 3.42

Solution. The equivalent circuit is shown in Fig. 3.43.

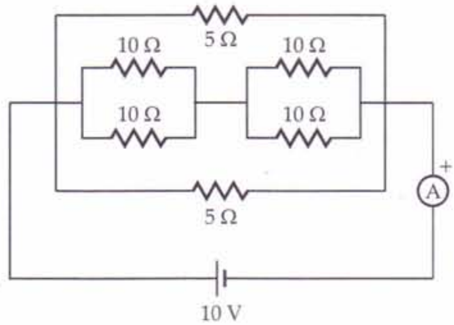


Fig. 3.43

For the two 10 Ω resistances connected in parallel,

$$\text{equivalent resistance} = \frac{10 \times 10}{10 + 10} = 5\,\Omega$$

For two such combinations connected in series,

equivalent resistance $= 5 + 5 = 10\,\Omega$

Now we have resistances of $5\,\Omega, 10\,\Omega$ and $5\,\Omega$ connected in parallel, so

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{10} + \frac{1}{5} = \frac{1}{2}$$

or $\quad R = 2\,\Omega$

Also $\quad V = 10$ V

$\therefore$ Current, $I = \frac{V}{R} = \frac{10}{2} = \mathbf{5\ A.}$

Example 63. *Calculate the value of the resistance R in the circuit shown in Fig. 3.44 so that the current in the circuit is 0.2 A. What would be the potential difference between points A and B ?* [CBSE OD 12]

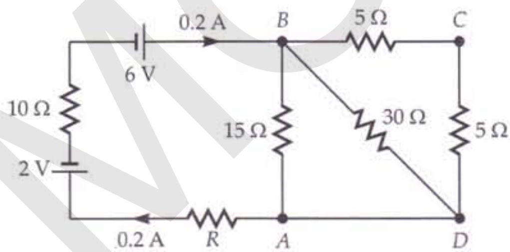


Fig. 3.44

Solution. $\frac{1}{R_{BA}} = \frac{1}{15} + \frac{1}{30} + \frac{1}{5+5}$

$$= \frac{6}{30} = \frac{1}{5}$$

$\therefore \quad R_{BA} = 5\,\Omega$

By Ohm's law,

$$0.2\text{ A} = \frac{6-2}{R+10+5} = \frac{4}{R+15}\text{A}$$

or $\quad R + 15 = \frac{4}{0.2} = 20 \quad$ or $\quad R = 5\Omega$

$$V_{AB} = 0.2 \times R_{AB} = 0.2 \times 5 = \mathbf{1.0\ V.}$$

Example 64. *In the circuit shown in Fig. 3.45, find the potential difference across the capacitor.*

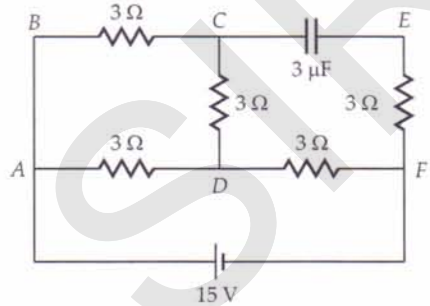


Fig. 3.45

Solution. In the steady state (when the capacitor is fully charged), no current flows through the branch *CEF*. The given circuit then reduces to the equivalent circuit shown in Fig. 3.46.

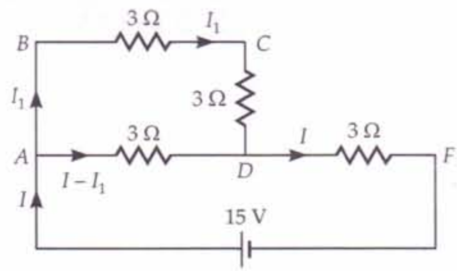


Fig. 3.46

The equivalent resistance of the circuit is

$$R = \frac{6 \times 3}{6+3} + 3 = 5\,\Omega$$

Current drawn from the battery,

$$I = \frac{15\text{ V}}{5\,\Omega} = 3\text{ A}$$

Current through the branch *BCD*,

$$I_1 = \frac{3}{6+3} \times I = \frac{3}{9} \times 3 = 1\text{ A}$$

Current through the arm $DF = I = 3$ A

P.D. across the capacitor

= P.D. between points *C* and *F*

= P.D. across *CD* + P.D. across *DF*

$= 3 \times 1 + 3 \times 3 = \mathbf{12\ V.}$

Example 65. *A battery of emf 10 V is connected to resistances as shown in Fig. 3.47. Find the potential difference between the points A and B.*

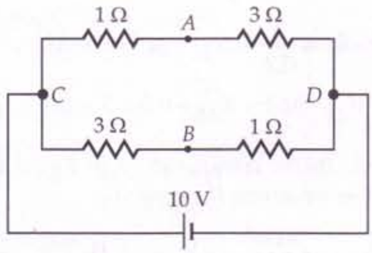


Fig. 3.47

Solution. Total resistance, $R = \frac{4 \times 4}{4+4} = 2\ \Omega$

Current, $I = \frac{V}{R} = \frac{10\ \text{V}}{2\ \Omega} = 5\ \text{A}$

As each of the two parallel branches has same resistance $(4\ \Omega)$, so the current of 5 A is divided equally through them.

Current through each branch $= 5/2 = 2.5$ A

Now $\quad V_C - V_A = 2.5 \times 1 = 2.5$ V

and $\quad V_C - V_B = 2.5 \times 3 = 7.5$ V

$\therefore \quad V_A - V_B = (V_C - V_B) - (V_C - V_A)$

$= 7.5 - 2.5 = \mathbf{5.0\ V}.$

Example 66. *What is the equivalent resistance between points A and B of the circuit shown in Fig. 3.48 ?* [IIT 97]

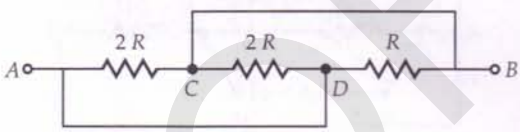


Fig. 3.48

Solution. Obviously, the points A and D are equipotential points. Also, the points B and C are equal potential points. So the given network of resistances reduces to the equivalent circuit shown in Fig. 3.49.

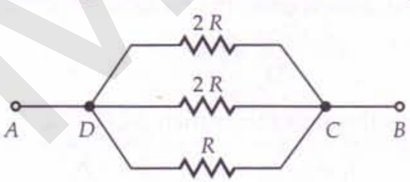


Fig. 3.49

The three resistances form a parallel combination. Their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} = \frac{1+1+2}{2R} = \frac{2}{R} \quad \text{or} \quad R_{eq} = \mathbf{R/2}.$$

Example 67. *In the circuit shown in Fig. 3.50, $R_1 = 4\ \Omega$, $R_2 = R_3 = 15\ \Omega$, $R_4 = 30\ \Omega$ and $\mathcal{E} = 10$ V. Work out the equivalent resistance of the circuit and the current in each resistor.* [CBSE D 2011]

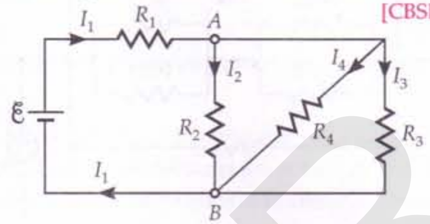


Fig. 3.50

Solution. The resistances R_2, R_3 and R_4 are in parallel. Their equivalent resistance R' is given by

$$\frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{15} + \frac{1}{15} + \frac{1}{30} = \frac{5}{30} = \frac{1}{6}$$

or $\quad R' = 6\ \Omega$

The resistance R_1 is in series with R'. Hence total resistance of the circuit is

$$R = R_1 + R' = 4 + 6 = 10\ \Omega$$

The current I_1 is the current sent by the cell $\mathcal{E}$ in the whole circuit.

$$\therefore \quad I_1 = \frac{\mathcal{E}}{R} = \frac{10}{10} = \mathbf{1\ A}$$

Potential drop between A and B,

$$V = I_1 R' = 1 \times 6 = 6\ \text{V}$$

This is the potential drop across each of the resistances R_2, R_3 and R_4 in parallel. Therefore, currents through these resistances are

$$I_2 = \frac{V}{R_2} = \frac{6}{15} = \mathbf{0.4\ A}; \quad I_3 = \frac{V}{R_3} = \frac{6}{15} = \mathbf{0.4\ A}$$

and $\quad I_4 = \frac{V}{R_4} = \frac{6}{30} = \mathbf{0.2\ A}.$

Example 68. *Find the equivalent resistance between the points A and B of the network of resistors shown in Fig. 3.51.*

Solution. The resistors R_1 and R_2 are in series. Their equivalent resistance

$$= 3 + 3 = 6\ \Omega$$

The $6\ \Omega$ resistance is in parallel with R_3, so that their equivalent resistance

$$= \frac{6 \times 3}{6+3} = 2\ \Omega$$

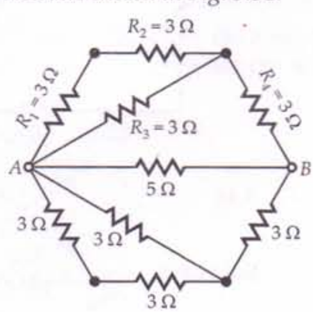


Fig. 3.51

Now the $2\ \Omega$ resistance is in series with R_4. So the total resistance of the upper portion $= 2 + 3 = 5\ \Omega$.

Similarly, total resistance of the lower portion

$$= 5\,\Omega$$

Now we have three $5\,\Omega$ resistors connected in parallel between the points A and B. Hence the equivalent resistance R of the entire network is given by

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5} \quad \text{or} \quad R = \frac{5}{3}\,\Omega.$$

Example 69. *Find the effective resistance between points A and B of the network of resistors shown in Fig. 3.52.*

Solution. By symmetry, the potential drops across GC and GD are equal, so no current flows in the arm CD. Similarly, no current flows in the arm DE. Hence the resistances in the arms CD and DE are ineffective. The given circuit reduces to the equivalent circuit shown in Fig. 3.53.

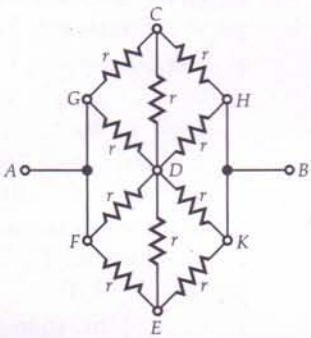


Fig. 3.52

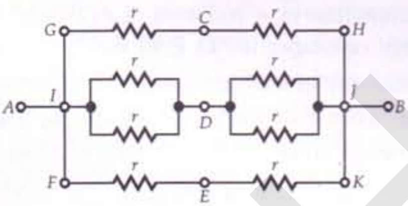


Fig. 3.53

Resistance of arm $GH = r + r = 2r$

Resistance of arm $IJ = \frac{r \times r}{r + r} + \frac{r \times r}{r + r} = r$

Resistance of arm $FK = r + r = 2r$

The above three resistances are in parallel between points A and B and their equivalent resistance R is given by

$$\frac{1}{R} = \frac{1}{2r} + \frac{1}{r} + \frac{1}{2r} = \frac{2}{r} \quad \therefore \quad R = \mathbf{0.5\ r}.$$

Example 70. *A regular hexagon with diagonals is made of identical wires, each having same resistance r, as shown in Fig. 3.54. Find the equivalent resistance between the points A and B.*

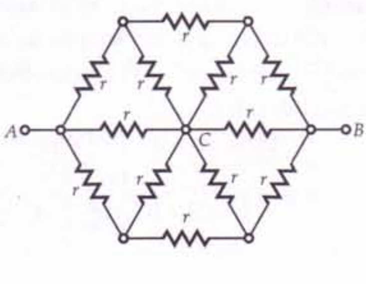


Fig. 3.54

Solution. As shown in Fig. 3.55, the given hexagon has a line of symmetry $C_1\,C\,C_2$. So all points on this line have the same potential *i.e.*, potential at C_1 = potential at C = potential at C_2. Hence the points C_1, C and C_2 can be made to coincide with each other.

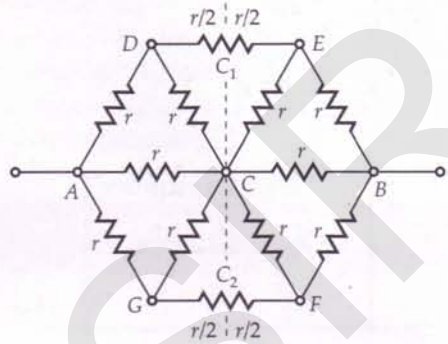
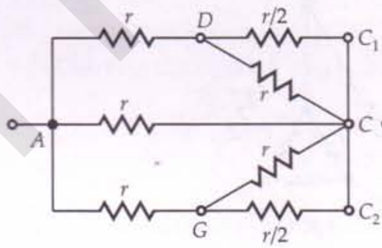


Fig. 3.55

After this is done, the circuit splits into identical parts, joined in series between the points A and B. One such part between A and C is shown in [Fig. 3.56] which, in turn, is equivalent to the circuit shown in Fig. 3.57.



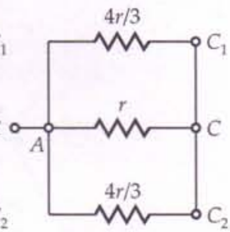


Fig. 3.56 Fig. 3.57

From Fig. 3.57, the equivalent resistance R' between the points A and C is given by

$$\frac{1}{R'} = \frac{3}{4r} + \frac{1}{r} + \frac{3}{4r} = \frac{10}{4r} \quad \text{or} \quad R' = \frac{4r}{10} = 0.4\,r$$

As two identical parts AC and CB are joined in series, hence the equivalent resistance of the entire circuit between points A and B is

$$R = R' + R' = 0.4\,r + 0.4\,r = \mathbf{0.8\ r}.$$

Example 71. *Find the equivalent resistance of the circuit shown in Fig. 3.58 between the points A and B. Each resistor has a resistance r.*

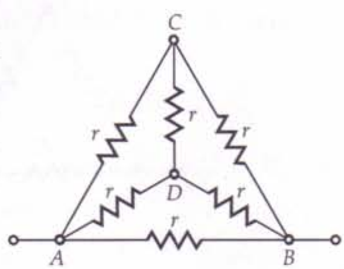


Fig. 3.58

Solution. By symmetry, potential drops across AC and AD are equal. So resistance in arm CD is ineffective. The given circuit reduces to the equivalent circuit shown in Fig. 3.59. Clearly the equivalent resistance R between points A and B is given by

$$\frac{1}{R}=\frac{1}{2r}+\frac{1}{2r}+\frac{1}{r}=\frac{4}{2r}=\frac{2}{r}$$

or $$R=\frac{r}{2}=\mathbf{0.5\ r.}$$

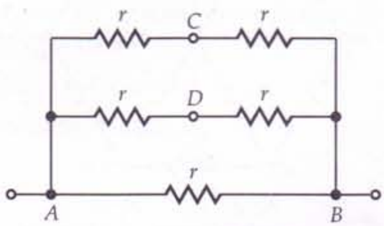


Fig. 3.59

Example 72. *Find the equivalent resistance of the circuit shown in Fig. 3.60 between the points P and Q. Each resistor has a resistance r.*

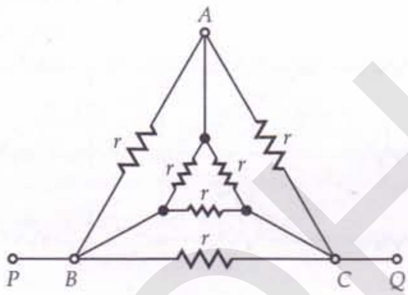


Fig. 3.60

Solution. Two resistances along each side of triangle are in parallel.

The equivalent resistance of each side

$$=\frac{r\times r}{r+r}=\frac{r}{2}$$

The given network of resistances reduces to the equivalent circuit shown in Fig. 3.61.

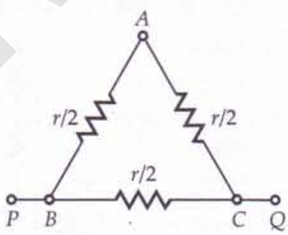


Fig. 3.61

The resistances in arms BA and AC are in series.

Their equivalent resistance $=r/2+r/2=r$. This resistance is in parallel with the resistance $r/2$ along BC.

∴ Effective resistance between points P and Q

$$=\frac{r\times(r/2)}{r+(r/2)}=\frac{\mathbf{r}}{\mathbf{3}}.$$

Problems For Practice

1. Given the resistances of $1\,\Omega$, $2\,\Omega$ and $3\,\Omega$. How will you combine them to get an equivalent resistance of (*i*) $\frac{11}{3}\,\Omega$ and (*ii*) $\frac{11}{5}\,\Omega$? **[CBSE F 2015]**

   [Ans. (*i*) parallel combination of $1\,\Omega$ and $2\,\Omega$ in series with $3\,\Omega$ (*ii*) parallel combination of $2\,\Omega$ and $3\,\Omega$ in series with $1\,\Omega$

2. Given three resistances of $30\,\Omega$ each. How can they be connected to give a total resistance of (*i*) $90\,\Omega$ (*ii*) $10\,\Omega$ (*iii*) $45\,\Omega$?

   [Ans. (*i*) in series (*ii*) in parallel (*iii*) two resistances in parallel and one in series]

3. A $5\,\Omega$ resistor is connected in series with a parallel combination of n resistors of $6\,\Omega$ each. The equivalent resistance is $7\,\Omega$. Find n. (Ans. 3)

4. A uniform wire of resistance $2.20\,\Omega$ has a length of 2 m. Find the length of the similar wire which connected in parallel with the 2 m long wire, will give a resistance of $2.0\,\Omega$. (Ans. 20 m)

5. A wire of $15\,\Omega$ resistance is gradually stretched to double its original length. It is then cut into two equal parts. These parts are then connected in parallel across a 3.0 volt battery. Find the current drawn from the battery. **[CBSE OD 09]**

   (Ans. 0.2 A)

6. The total resistance of two resistors when connected in series is $9\,\Omega$ and when connected in parallel, their total resistance becomes $2\,\Omega$. Calculate the value of each resistance. **[Punjab 2000]**

   (Ans. $6\,\Omega$, $3\,\Omega$)

7. Two wires a and b, each of length 40 m and area of cross-section $10^{-7}\ m^2$; are connected in series and a potential difference of 60 V is applied between the ends of this combined wire. Their resistances are respectively $40\,\Omega$ and $20\,\Omega$. Determine for each wire (*i*) specific resistance, (*ii*) electric-field, and (*iii*) current-density.

   [Ans. (*i*) $1.0\times10^{-7}\ \Omega m$, $5.0\times10^{-8}\ \Omega m$
   (*ii*) $1.0\ Vm^{-1}$, $0.5\ Vm^{-1}$
   (*iii*) $1.0\times10^{7}\ Am^{-2}$, $1.0\times10^{7}\ Am^{-2}$]

8. Three resistances, each of 4 Ω, are connected in the form of an equilateral triangle. Find the effective resistance between its corners. (Ans. 2.67 Ω)

9. Two resistors are in the ratio 1 : 4. If these are connected in parallel, their total resistance becomes 20 Ω. Find the value of each resistance.
[Punjab 2000]
(Ans. 25 Ω, 100 Ω)

10. Five resistors are connected as shown in Fig. 3.62. Find the equivalent resistance between the points *B* and *C*. **[Punjab 01]**
(Ans. 70 / 19 Ω)

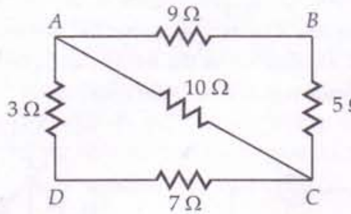


Fig. 3.62

11. Four resistors of 12 Ω each are connected in parallel. Three such combinations are then connected in series. What is the total resistance ? If a battery of 9 V emf and negligible internal resistance is connected across the network of resistors, find the current flowing through each resistor. **[Haryana 02]**
(Ans. 9 Ω, 0.25 A)

12. If the reading of the ammeter A_1 in Fig. 3.63 is 2.4 A, what will the ammeters A_2 and A_3 read ? Neglect the resistances of the ammeters. (Ans. 1.6 A, 4.0 A)

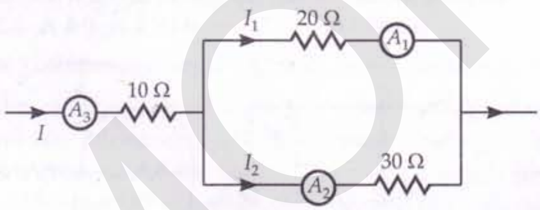


Fig. 3.63

13. The resistance of the rheostat shown in Fig. 3.64 is 30 Ω. Neglecting the meter resistance, find the minimum and maximum current through the ammeter as the resistance of the rheostat is varied.
(Ans. 0.18 A, 1.5 A)

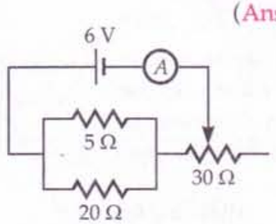


Fig. 3.64

14. Find the current through the 5 Ω resistor in the circuit shown in Fig. 3.65, when the switch *S* is (*i*) open and (*ii*) closed. [Ans. (*i*) 0.2 A, (*ii*) 0.6 A]

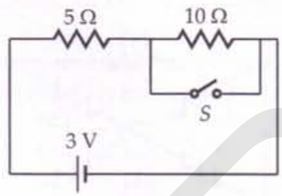


Fig. 3.65

15. The letter *A* consists of a uniform wire of resistance 1 Ω cm^{-1}. The sides of the letter are 40 cm long and the crosspiece 10 cm long divides the sides in the ratio 1 : 3 from the apex. Find the resistance of the letter between the two ends of the legs.
[Punjab 98C]
(Ans. 66.67 Ω)

16. Calculate the equivalent resistance between points *A* and *B* in each of the following networks of resistors :
[Ans. (*a*) 12 Ω (*b*) 40 / 3 Ω (*c*) 2 Ω (*d*) 10 / 3 Ω (*e*) 16 Ω (*f*) 5 Ω]

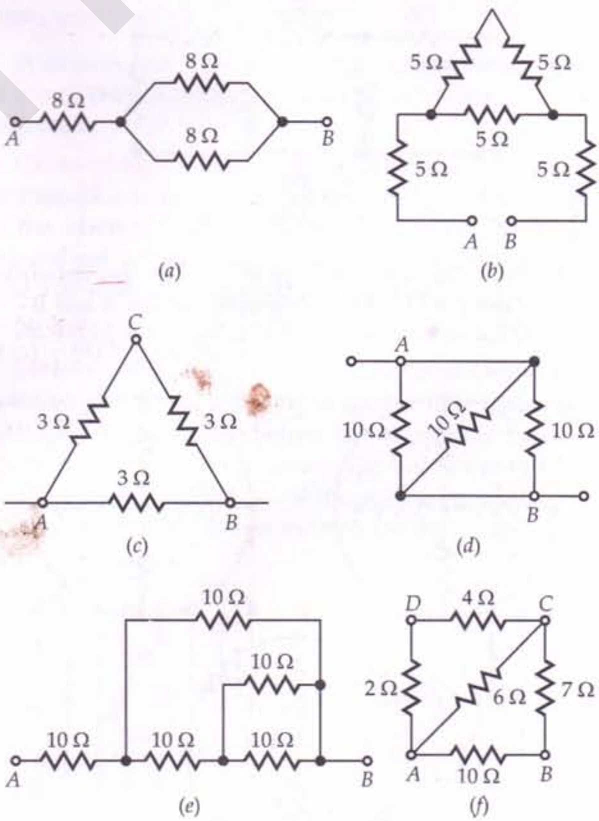


Fig. 3.66

17. Calculate the resistance between points A and B for the following networks :

[Ans. (a) $\frac{2}{3}\,\Omega$ (b) $\frac{4}{3}\,\Omega$ (c) $\frac{R}{3}\,\Omega$ (d) $6\,\Omega$]

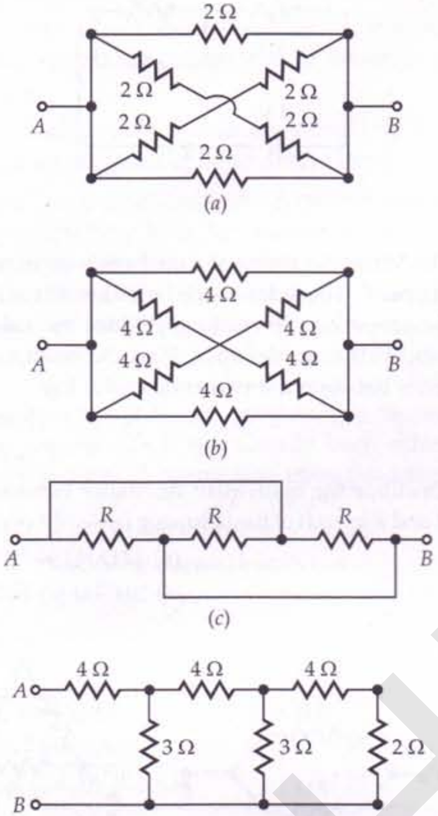


Fig. 3.67

18. Find the equivalent resistance of the networks shown in Fig. 3.68 between the points A and B.

[Ans. (a) $\frac{4}{3}r$ (b) $\frac{r}{4}$ (c) r]

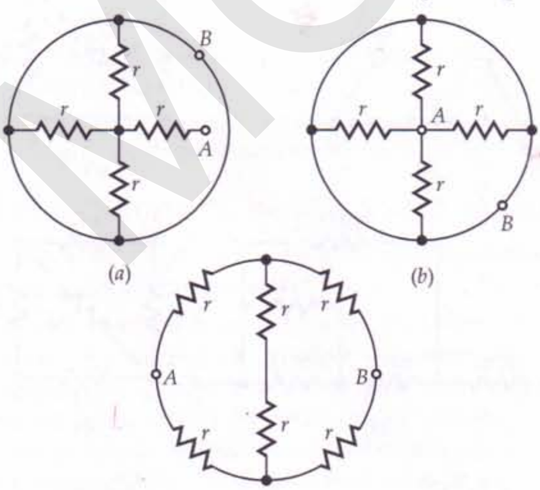


Fig. 3.68

19. Find the potential difference between the points A and B for the network shown in Fig. 3.69.

(Ans. 8.0 V)

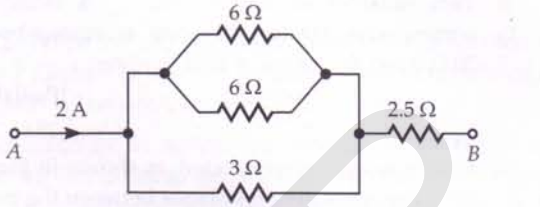


Fig. 3.69

20. In the circuit diagram shown in Fig. 3.70, a voltmeter reads 30 V when connected across 400 Ω resistance. Calculate what the same voltmeter reads when it is connected across 300 Ω resistance. **[IIT 90]**

(Ans. 22.5 V)

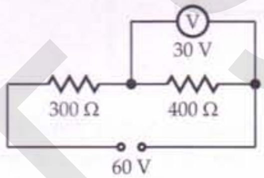


Fig. 3.70

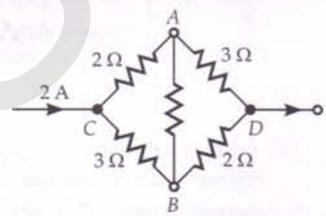


Fig. 3.71

21. Find the potential difference between points A and B i.e., $(V_A - V_B)$ in the network shown in Fig. 3.71. **[Punjab 93]** (Ans. 1 V)

22. In the circuit shown in Fig. 3.72, $R_1 = 4\Omega$, $R_2 = R_3 = 5\Omega$, $R_4 = 10\Omega$ and $\mathcal{E} = 6$ V. Work out the equivalent resistance of the circuit and the current in each resistor. **[CBSE D 11]** (Ans. 6 Ω, 1 A, 0.4 A, 0.2 A)

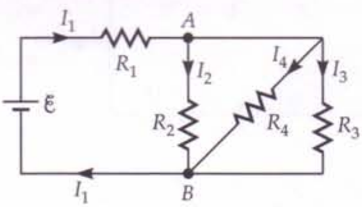


Fig. 3.72

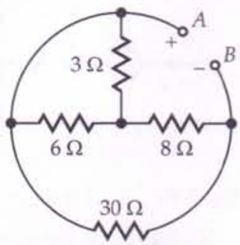


Fig. 3.73

23. Find the equivalent resistance between points A and B in Fig. 3.73.

(Ans. 7.5 Ω)

24. Letter A as shown in Fig. 3.74 has resistances on each side of arm. Calculate the total resistance between two ends of the legs.

[Himachal 93]

(Ans. 28.75 Ω)

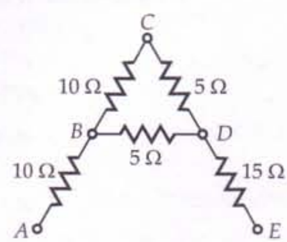


Fig. 3.74

25. Find the resistance between the points (*i*) *A* and *B* and (*ii*) *A* and *C* of the network shown in Fig. 3.75.
[Ans. (*i*) 27.5 Ω (*ii*) 30 Ω]

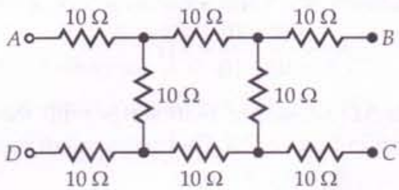


Fig. 3.75

26. A combination of four resistances is shown in Fig. 3.76. Calculate the potential difference between the points *P* and *Q*, and the values of currents flowing in the different resistances.
(Ans. 14.4 V, 0.8 A, 1.6 A)

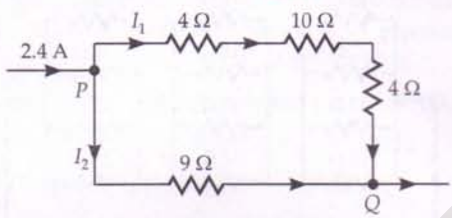


Fig. 3.76

27. In Fig. 3.77, *X*, *Y* and *Z* are ammeters and *Y* reads 0.5 A. (*i*) What are the readings in ammeters *X* and *Z* ? (*ii*) What is the total resistance of the circuit ?
[Ans. (*i*) 1.5 A, 1.0 A (*ii*) 4 Ω]

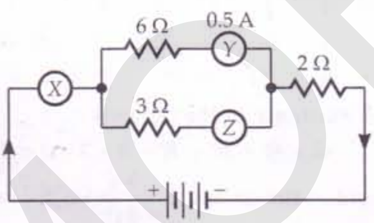


Fig. 3.77

28. In the circuit shown in Fig. 3.78, the terminal voltage of the battery is 6.0 V. Find the current *I* through the 18 Ω resistor. (Ans. 0.25 A)

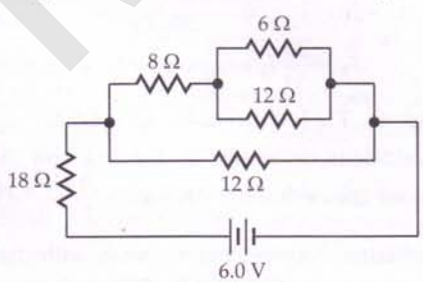


Fig. 3.78

29. In the circuit shown in Fig. 3.79, the battery has an emf of 12.0 V and an internal resistance of 5 *R*/ 11. If the ammeter reads 2.0 A, what is the value of *R* ?
(Ans. 6 Ω)

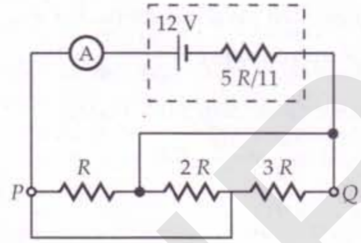


Fig. 3.79

30. Find the ammeter reading in the circuit shown in Fig. 3.80. (Ans. 3 A)

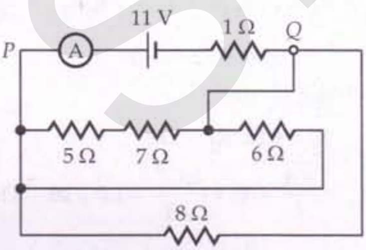


Fig. 3.80

HINTS

1. (*i*) When parallel combination of 1Ω and 2Ω resistors is connected in series with 3 Ω resistor, the equivalent resistance is

$$R = R_p + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

$$= \frac{1\times 2}{1+2} + 3 = \frac{2}{3} + 3 = \frac{11}{3}\,\Omega.$$

(*ii*) When parallel combination of 2Ω and 3Ω resistors is connected in series with 1Ω resistor, the equivalent resistance is

$$R = \frac{R_2 R_3}{R_2 + R_3} + R_1 = \frac{2\times 3}{2+3} + 1 = \frac{6}{5} + 1 = \mathbf{\frac{11}{5}}\,\Omega.$$

3. Total resistance $= 5 + \frac{6}{n} = 7\,\Omega$, so $n = 3$.

4. Resistance per unit length of the wire

$$= \frac{2.2}{2} = 1.1\,\Omega\text{m}^{-1}$$

Let R' be the resistance that should be connected in parallel to resistance, $R = 2.20\,\Omega$, so that effective resistance, $R_p = 2.0\,\Omega$. Then

$$\frac{1}{R'} = \frac{1}{R_p} - \frac{1}{R} = \frac{1}{2} - \frac{1}{2.2} = \frac{0.1}{2.2} = \frac{1}{22} \quad \therefore \; R' = 22\,\Omega$$

Length of the wire needed $= \frac{22}{1.1} = \mathbf{20\ m.}$

5. When the wire of 15 Ω resistance is stretched to double its original length, its resistance becomes

$$R' = n^2 R = (2)^2 \times 15 = 60\,\Omega$$

Resistance of each half part = 60/2 = 30 Ω

When the two parts are connected in parallel, their equivalent resistance $= \dfrac{30 \times 30}{30 + 30} = 15\,\Omega$

Current drawn from 3.0 V battery,

$$I = \frac{V}{R} = \frac{3.0}{15} = \mathbf{0.2\ A}.$$

6. $R_1 + R_2 = 9\,\Omega$ ...(1)

$$\frac{R_1 R_2}{R_1 + R_2} = 2$$

or $R_1 R_2 = 2(R_1 + R_2) = 2 \times 9 = 18$

$\therefore\ R_1 - R_2 = \sqrt{(R_1 + R_2)^2 - 4 R_1 R_2}$

$= \sqrt{81 - 72} = 3\,\Omega$ ...(2)

On solving (1) and (2),

$$R_1 = \mathbf{6\,\Omega},\ R_2 = \mathbf{3\,\Omega}$$

7. (*i*) $\rho_a = \dfrac{RA}{l} = 40 \times \dfrac{10^{-7}}{40} = \mathbf{1.0 \times 10^{-7}\ \Omega m}$

and $\rho_b = 20 \times \dfrac{10^{-7}}{40} = \mathbf{5.0 \times 10^{-8}\ \Omega m}$

(*ii*) Total resistance, $R = R_a + R_b = 40 + 20 = 60\,\Omega$

The current in the wires, $I = \dfrac{V}{R} = \dfrac{60}{60} = 1.0$ A.

∴ Potential differences between the ends of wires *a* and *b* are

$$V_a = I \times R_a = 1.0 \times 40 = 40\ \text{V}$$

and $V_b = I \times R_b = 1.0 \times 20 = 20$ V

Electric fields in the two wires are

$$E_a = \frac{V_a}{l_a} = \frac{40}{40} = \mathbf{1.0\ Vm^{-1}}$$

and $E_b = \dfrac{V_b}{l_b} = \dfrac{20}{40} = \mathbf{0.5\ Vm^{-1}}$

(*iii*) The current in each wire is the same. Also, the area of cross-section of each wire is same. Hence the current-density in each wire is

$$J_A = J_B = \frac{I}{A} = \frac{1.0}{10^{-7}} = \mathbf{1.0 \times 10^7\ Am^{-2}}.$$

8. $R = \dfrac{(4+4) \times 4}{(4+4) + 4} = \dfrac{32}{12} = \mathbf{2.67\,\Omega}.$

9. Let the two resistances be *R* and 4 R. Then

$$R_p = \frac{R \times 4R}{R + 4R} = 20\,\Omega \quad \text{or} \quad \frac{4}{5}R = 20\,\Omega$$

∴ $R = \mathbf{25\,\Omega}$ and $4R = \mathbf{100\,\Omega}$.

10. Resistance in branch *ADC*,

$$R_1 = 3 + 7 = 10\,\Omega$$

This resistance is in parallel with the 10 Ω resistance in branch *AC*. Their total resistance is

$$R_2 = \frac{10 \times 10}{10 + 10} = 5\,\Omega$$

This 5 Ω resistance is in series with the 9 Ω resistance in branch *AB*. Their equivalent resistance is

$$R_3 = 5 + 9 = 14\,\Omega$$

This 14 Ω resistance is in parallel with the 5 Ω resistance in branch *BC*. Hence the equivalent resistance between *B* and *C* is

$$R = \frac{14 \times 5}{14 + 5} = \mathbf{\frac{70}{19}}\,\Omega.$$

11. The circuit diagram is shown in Fig. 3.81.

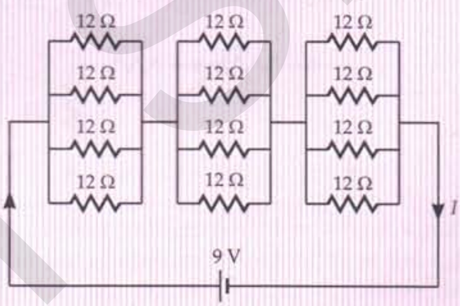


Fig. 3.81

Effective resistance *R*′ of four resistances of 12 Ω each connected in parallel is given by

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12}$$

or $R' = 3\,\Omega$

Total resistance of the network,

$$R = R' + R' + R' = 3 + 3 + 3 = \mathbf{9\,\Omega}$$

Current in the circuit, $I = \dfrac{9\ \text{V}}{9\,\Omega} = 1$ A

Current through each resistor

$$= \frac{1}{4} I = \frac{1}{4} \times 1 = \mathbf{0.25\ A}.$$

12. P.D. across 20 Ω = P.D. across 30 Ω

or $I_1 \times 20 = I_2 \times 30$

or $I_2 = \dfrac{20}{30} I_1 = \dfrac{20}{30} \times 2.4 = \mathbf{1.6\ A}$

and $I = I_1 + I_2 = 2.4 + 1.6 = \mathbf{4.0\ A}.$

13. Equivalent resistance of the 5 Ω and 20 Ω resistances connected in parallel $= \dfrac{5 \times 20}{5 + 20} = 4\,\Omega$. This resistance is connected in series with the rheostat whose minimum and maximum resistances are 0 Ω and 30 Ω.

When the rheostat is adjusted at the minimum resistance of $0\,\Omega$, current will be maximum.

$$I_{max} = \frac{6\text{ V}}{4\,\Omega} = \mathbf{1.5\ A}$$

When the rheostat is adjusted at the maximum resistance of $30\,\Omega$, current will be minimum.

$$I_{min} = \frac{6\text{ V}}{(4+30)\,\Omega} = \mathbf{0.18\ A.}$$

14. (*i*) When switch S is open, resistances of $5\,\Omega$ and $10\,\Omega$ are in series.

Current, $I = \frac{3\text{ V}}{(5+10)\,\Omega} = \mathbf{0.2\ A.}$

(*ii*) When switch S is closed, no current flows through $10\,\Omega$ resistance.

$\therefore$ Current, $I = \frac{3\text{ V}}{5\,\Omega} = \mathbf{0.6\ A.}$

15. Refer to Fig. 3.82. Clearly

$BC = CD = BD = 10$ cm

$AB = DE = 30$ cm

$\therefore \quad R_2 = R_3 = R_5 = 10\,\Omega$

and $\quad R_1 = R_4 = 30\,\Omega$

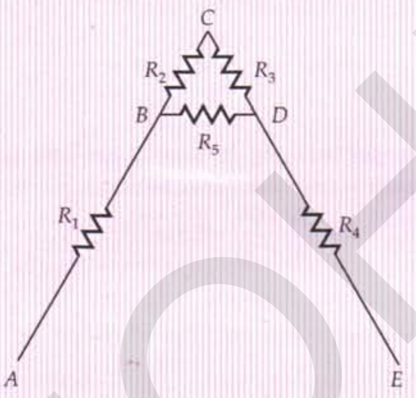


Fig. 3.82

Series combination of R_2 and R_3 is in parallel with R_5. Their equivalent resistance

$$= \frac{(10+10)\times 10}{(10+10)+10} = \frac{200}{30} = \frac{20}{3}\,\Omega = 6.67\,\Omega$$

This resistance is in series with R_1 and R_4. So the net resistance is

$$R = 30 + 6.67 + 30 = \mathbf{66.67\,\Omega.}$$

16. (*a*) $R = 8 + \frac{8\times 8}{8+8} = \mathbf{12\,\Omega.}$

(*b*) $R = 5 + \frac{10\times 5}{10+5} + 5 = \mathbf{\frac{40}{3}}\,\Omega.$

(*c*) $R = \frac{(3+3)\,3}{(3+3)+3} = \mathbf{2\,\Omega.}$

(*d*) All the three resistances are connected in parallel between points A and B.

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10} \quad \text{or} \quad R = \mathbf{\frac{10}{3}}\,\Omega.$$

(*e*) The given network is equivalent to the network shown in Fig. 3.83.

$$\therefore \quad R = 10 + \frac{10\times 15}{10+15} = \mathbf{16\,\Omega.}$$

Fig. 3.83

(*f*) Resistance in branch $ADC = 2 + 4 = 6\,\Omega$. This resistance is in parallel with $6\,\Omega$ resistance in arm AC.

Their equivalent resistance

$$= \frac{6\times 6}{6+6} = 3\,\Omega$$

The series combination of this $3\,\Omega$ resistance and $7\,\Omega$ resistance in arm BC is in parallel with $10\,\Omega$ resistance in arm AB.

$$\therefore \quad R = \frac{10\times 10}{10+10} = \mathbf{5\,\Omega.}$$

17. The corresponding equivalent circuit diagrams are given below :

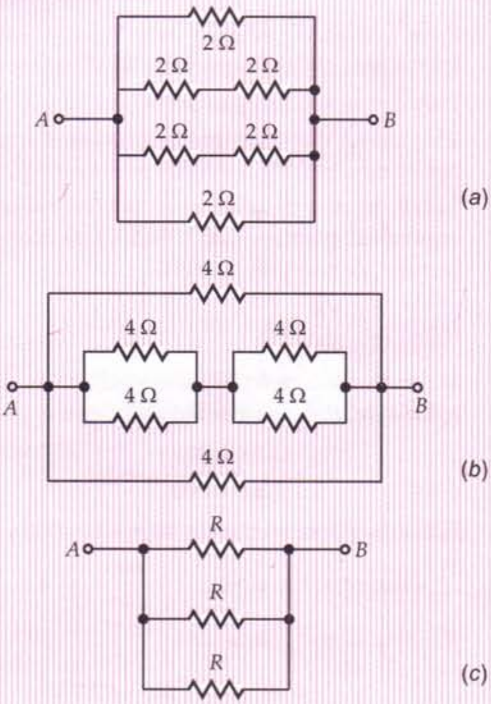
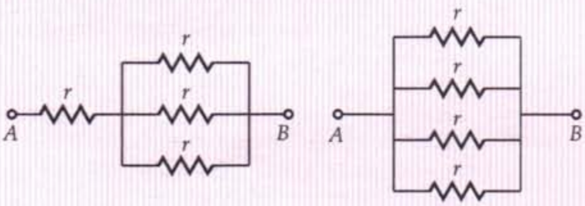


Fig. 3.84

18. (*a*) The equivalent network for 3.68(*a*) is shown in Fig. 3.85(*a*).

$$\therefore \quad R = r + \frac{r}{3} = \frac{\mathbf{4}}{\mathbf{3}}\mathbf{r}.$$



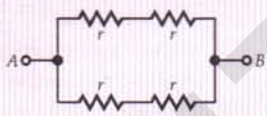
(*a*) (*b*)

Fig. 3.85

(*b*) The equivalent network for 3.68(*b*) is shown in Fig. 3.85(*b*).

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{4}{r} \quad \text{or} \quad R = \frac{\mathbf{r}}{\mathbf{4}}.$$

(*c*) The current divides symmetrically in the two upper and the two lower resistances. So the resistances in the vertical arm are ineffective. The given network reduces to the equivalent network shown in Fig. 3.86.



$$R = \frac{2r \times 2r}{2r + 2r} = \mathbf{r}.$$

Fig. 3.86

19. $R = \frac{3 \times 3}{3 + 3} + 2.5 = 1.5 + 2.5 = 4.0\,\Omega$

$V = RI = 4.0 \times 2 = \mathbf{8.0\ V.}$

20. P.D. across 400 Ω resistance = 30 V

P.D. across 300 Ω resistance = 60 − 30 = 30 V

This shows that potential drop is same across both resistances.

Let R be the resistance of the voltmeter. Then equivalent resistance of R and 400 Ω connected in parallel should also be 300 Ω.

$$\therefore \quad \frac{R \times 400}{R + 400} = 300 \quad \text{or} \quad R = 1200\,\Omega$$

When the voltmeter is connected across the 300 Ω resistance, their equivalent resistance is given by

$$R' = \frac{1200 \times 300}{1200 + 300} = 240\,\Omega$$

Total resistance in the circuit = 240 + 400 = 640 Ω

$$\therefore \text{ Current in the circuit, } I = \frac{60}{640} = \frac{3}{32}\text{ A}$$

Reading of the voltmeter

$$= IR' = \frac{3}{32} \times 240 = \mathbf{22.5\ V.}$$

21. Current through each branch = 2 / 2 = 1 A

$$V_C - V_A = 1 \times 2 = 2\text{ V}$$
$$V_C - V_B = 1 \times 3 = 3\text{ V}$$
$$V_A - V_B = (V_C - V_B) - (V_C - V_A) = 3 - 2 = \mathbf{1\ V.}$$

22. $$\frac{1}{R_{AB}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{5} + \frac{1}{5} + \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore \quad R_{AB} = 2\,\Omega$$
$$R = R_1 + R_{AB} = 4 + 2 = 6\,\Omega$$
$$I_1 = \frac{\mathcal{E}}{R} = \frac{6\text{V}}{6\Omega} = 1\text{ A}$$
$$V_{AB} = I_1 R_{AB} = 1 \times 2 = 2\text{V}$$
$$\therefore \quad I_2 = I_3 = \frac{2}{5}\text{ A} = \mathbf{0.4\ A} \text{ and } I_4 = \frac{2}{10} = \mathbf{0.2\ A.}$$

23. The equivalent circuit is shown in Fig. 3.87. The effective resistance between points C and D

$$= \frac{3 \times 6}{3 + 6} + 8 = 10\,\Omega$$

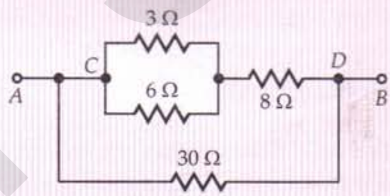


Fig. 3.87

Now the 10 Ω and 30 Ω resistances are in parallel. The equivalent resistance between points A and B

$$= \frac{10 \times 30}{10 + 30} = \mathbf{7.5\,\Omega.}$$

24. Proceed as in Example 51 on page 3.32.

25. (*i*) The equivalent circuit is shown in Fig. 3.88.

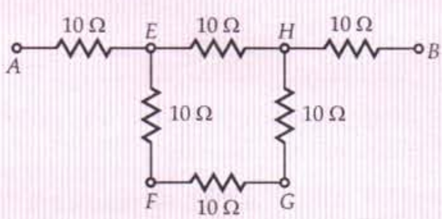


Fig. 3.88

Resistance of the arm $EFGH$ = 10 + 10 + 10 = 30 Ω

This resistance is parallel to the 10 Ω resistance of arm EH.

Equivalent resistance between points E and H

$$= \frac{10 \times 30}{10 + 30} = 7.5\,\Omega$$

Hence total resistance between points A and B

$$= 10 + 7.5 + 10 = \mathbf{27.5\,\Omega}$$

(*ii*) The equivalent circuit is shown in Fig. 3.89.

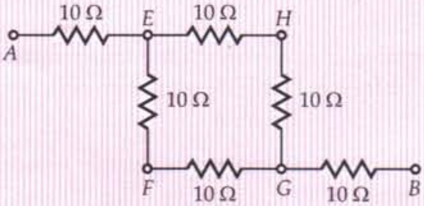


Fig. 3.89

Resistance of arm *EHG* $= 10 + 10 = 20\,\Omega$

Resistance of arm *EFG* $= 10 + 10 = 20\,\Omega$

These two $20\,\Omega$ resistances are in parallel.

$\therefore$ Effective resistance between points *E* and *G*

$$= \frac{20 \times 20}{20 + 20} = 10\,\Omega$$

Hence total resistance between points *A* and *C*

$$= 10 + 10 + 10 = \mathbf{30\,\Omega}.$$

26. The resistances of $4\,\Omega$, $10\,\Omega$ and $4\,\Omega$ are in series. Their equivalent resistance $= 18\,\Omega$. This is in parallel with $9\,\Omega$ resistance.

Equivalent resistance between *P* and *Q*,

$$R = \frac{18 \times 9}{18 + 9} = 6\,\Omega$$

P.D. between *P* and $Q = IR = 2.4 \times 6 = \mathbf{14.4\ V}$

$$\therefore \quad I_2 = \frac{V}{R} = \frac{14.4}{9} = \mathbf{1.6\ A}$$

and $\quad I_1 = 2.4 - 1.6 = \mathbf{0.8\ A.}$

27. P.D. across $6\,\Omega$ = P.D. across $3\,\Omega$

$$6 \times 0.5 = 3 \times I_2$$

Current through *Z*,

$$I_2 = \mathbf{1.0\ A}$$

Current through $X = 0.5 + 1.0 = \mathbf{1.5\ A}$

Total resistance $= \dfrac{6 \times 3}{6 + 3} + 2 = \mathbf{4\,\Omega}.$

28. Total resistance in the upper branch

$$= 8 + \frac{6 \times 12}{6 + 12} = 12\,\Omega$$

Total resistance in the circuit,

$$R = 18 + \frac{12 \times 12}{12 + 12} = 18 + 6 = 24\,\Omega$$

Current through $18\,\Omega$ resistor $= \dfrac{6}{24} = \mathbf{0.25\ A.}$

29. The resistances *R*, 2 *R* and 3 *R* are in parallel between the points *P* and *Q*. Their equivalent resistance *R'* is given by

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{3R} = \frac{11}{6R} \quad \text{or} \quad R' = \frac{6R}{11}$$

Now $6R/11$ and $5R/11$ are in series.

$\therefore$ Total resistance of the circuit $= \dfrac{6R}{11} + \dfrac{5R}{11} = R$

Resistance, $R = \dfrac{\mathcal{E}}{I} = \dfrac{12}{2} = \mathbf{6\,\Omega.}$

30. The resistances of $(5 + 7) = 12\,\Omega$, $6\,\Omega$ and $8\,\Omega$ are in parallel between points *P* and *Q*. Their equivalent resistance *R'* is given by

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{6} + \frac{1}{8} = \frac{3}{8} \quad \text{or} \quad R' = \frac{8}{3}\,\Omega$$

R' is in series with $1\,\Omega$ resistance.

$\therefore$ Total resistance $= \dfrac{8}{3} + 1 = \dfrac{11}{3}\,\Omega$

Current, $I = \dfrac{\mathcal{E}}{R} = \dfrac{11}{11/3} = \mathbf{3\ A.}$

3.19 INTERNAL RESISTANCE OF A CELL

34. *What is internal resistance of a cell ? On what factors does it depend ?*

Internal resistance. When the terminals of a cell are connected by a wire, an electric current flows in the wire from positive terminal of the cell towards the negative terminal. But inside the electrolyte of the cell, the positive ions flow from the lower to the higher potential (or negative ions from the higher to the lower potential) against the background of other ions and neutral atoms of the electrolyte. So the electrolyte offers some resistance to the flow of current inside the cell.

The resistance offered by the electrolyte of a cell to the flow of current between its electrodes is called ***internal resistance*** *of the cell.*

The internal resistance of a cell depends on following factors :

1. Nature of the electrolyte.
2. It is directly proportional to the concentration of the electrolyte.
3. It is directly proportional to the distance between the two electrodes.
4. It varies inversely as the common area of the electrodes immersed in the electrolyte.
5. It increases with the decrease in temperature of the electrolyte.

The internal resistance of a freshly prepared cell is usually low but its value increases as we draw more and more current from it.

3.20 RELATION BETWEEN INTERNAL RESISTANCE, EMF AND TERMINAL POTENTIAL DIFFERENCE OF A CELL

35. *Define terminal potential difference of a cell. Derive a relation between the internal resistance, emf and terminal potential difference of a cell. Draw (i) $\mathcal{E}$ vs. R (ii) V vs. R (iii) V vs. I graphs for a cell and explain their significance.*

Terminal potential difference. *The potential drop across the terminals of a cell when a current is being drawn from it is called its terminal potential difference (V).*

Relation between r, $\mathcal{E}$ and V. Consider a cell of emf $\mathcal{E}$ and internal resistance r connected to an external resistance R, as shown in Fig. 3.90. Suppose a constant current I flows through this circuit.

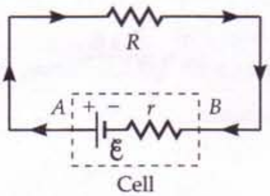


Fig. 3.90 Cell of emf $\mathcal{E}$ and internal resistance r.

By definition of emf,

$\mathcal{E}$ = Work done by the cell in carrying a unit charge along the closed circuit

= Work done in carrying a unit charge from A to B against external resistance R

+ Work done in carrying a unit charge from B to A against internal resistance r

or $\mathcal{E} = V + V'$

By Ohm's law,

$$V = IR \quad \text{and} \quad V' = Ir$$

$\therefore$ $\mathcal{E} = IR + Ir = I(R + r)$

Hence the current in the circuit is

$$I = \frac{\mathcal{E}}{R + r}$$

Thus to determine the current in the circuit, the internal resistance r combines in series with external resistance R.

The terminal p.d. of the cell that sends current I through the external resistance R is given by

$$V = IR = \frac{\mathcal{E}R}{R + r}$$

Also $V = \mathcal{E} - V' = \mathcal{E} - Ir$

or terminal p.d. = emf – potential drop across the internal resistance

Again, from the above equation, we get

$$r = \frac{\mathcal{E} - V}{I} = \frac{\mathcal{E} - V}{V/R} = \left(\frac{\mathcal{E} - V}{V}\right)R.$$

Special Cases

(*i*) When cell is on open circuit, *i.e.*, $I = 0$, we have

$$V_{open} = \mathcal{E}$$

Thus the potential difference across the terminals of the cell is equal to its emf when no current is being drawn from the cell.

(*ii*) A real cell has always some internal resistance r, so when current is being drawn from cell, we have

$$V < \mathcal{E}$$

Thus the potential difference across the terminals of the cell in a closed circuit is always less than its emf.

Characteristic curves for a cell. When a cell of emf $\mathcal{E}$ and internal resistance r is connected across a variable load resistance R, its functioning can be represented by the following three graphs :

(*i*) **$\mathcal{E}$ versus R graph.** The emf of a cell is equal to the terminal p.d. of the cell when no current is drawn from it. Hence emf $\mathcal{E}$ is independent of R and $\mathcal{E}$-R graph is a straight line, as shown in Fig. 3.91(*a*)

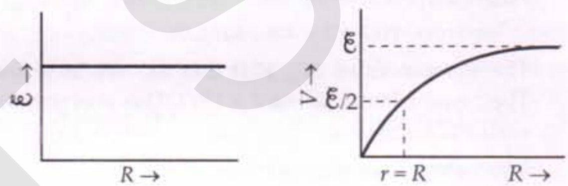


Fig. 3.91 (*a*) $\mathcal{E}$ vs. R graph for a cell. (*b*) V vs. R graph for a cell.

(*ii*) **V versus R graph.** In a closed circuit, the terminal p.d of the cell is

$$V = IR = \left(\frac{\mathcal{E}}{R + r}\right)R = \frac{\mathcal{E}}{1 + \frac{r}{R}}$$

As R increases, V also increases.

When $R \to 0$, $V = 0$

When $R = r$, $V = \mathcal{E}/2$

When $R \to \infty$, $V = \mathcal{E}$

Hence V-R graph is as shown in Fig. 3.91(*b*).

(*iii*) **V versus I graph.** As $V = \mathcal{E} - Ir$

$\Rightarrow V = -rI + \mathcal{E} \quad \Leftrightarrow \quad y = mx + c$

Hence, the graph between V and I is a straight line with a –ve slope, as shown in Fig. 3.91(*c*)

For point A, $I = 0$

Hence,

$V_A = \mathcal{E}$

= intercept on the y-axis

For point B, $V = 0$

$\therefore$ $\mathcal{E} = I_B r$

Hence, $r = \dfrac{\mathcal{E}}{I_B}$

= negative of the slope of V-I graph.

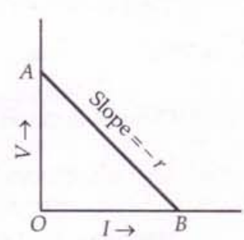


Fig. 3.91 (*c*) V vs. I graph for a cell.

Examples based on

EMF, Internal Resistance, Terminal Potential Difference and Grouping of Cells

Formulae Used

1. EMF of a cell, $\mathcal{E} = \frac{W}{q}$
2. For a cell of internal resistance r, the emf is
$$\mathcal{E} = V + Ir = I(R + r)$$
3. Terminal p.d. of a cell, $V = IR = \frac{\mathcal{E} R}{R + r}$
4. Terminal p.d. when a current is being drawn from the cell,
$$V = \mathcal{E} - Ir$$
5. Terminal p.d. when the cell is being charged,
$$V = \mathcal{E} + Ir$$
6. Internal resistance of a cell, $r = R\left[\frac{\mathcal{E} - V}{V}\right]$

Units Used

EMF $\mathcal{E}$ and terminal p.d. V are in volt (V), internal resistance r and external resistance R in Ω and current I in ampere (A).

Example 73. *For driving a current of 3 A for 5 minutes in an electric circuit, 900 J of work is to be done. Find the emf of the source in the circuit.*

Solution. The amount of charge that flows through the circuit in 5 minutes is

$$q = I \times t = 3 \times 5 \times 60 = 900 \text{ C}$$

As emf is the work done in flowing a unit charge in the closed circuit, therefore

$$\mathcal{E} = \frac{W}{q} = \frac{900 \text{ J}}{900 \text{ C}} = \mathbf{1.0 \text{ V}}.$$

Example 74. *A voltmeter of resistance 998 Ω is connected across a cell of emf 2 V and internal resistance 2 Ω. Find the p.d. across the voltmeter, that across the terminals of the cell and percentage error in the reading of the voltmeter.*

Solution. Here $\mathcal{E} = 2$ V, $r = 2\ \Omega$

Resistance of voltmeter,

$R = 998\ \Omega$

Current in the circuit is

$$I = \frac{\mathcal{E}}{R + r} = \frac{2 \text{ V}}{(998 + 2)\Omega} = 2 \times 10^{-3} \text{ A}$$

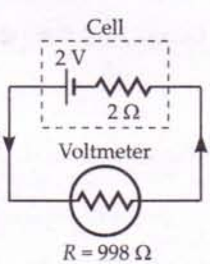


Fig. 3.92

The p.d. across the voltmeter is

$$V = IR = 2 \times 10^{-3} \times 998 = \mathbf{1.996 \text{ V}}$$

The same will be the p.d. across the terminals of the cell. The voltmeter used to measure the emf of the cell will read 1.996 volt. Hence the percentage error is

$$\frac{\mathcal{E} - V}{\mathcal{E}} \times 100 = \frac{2 - 1.996}{2} \times 100 = 0.2\%.$$

Example 75. *In the circuit shown in Fig. 3.93, the voltmeter reads 1.5 V, when the key is open. When the key is closed, the voltmeter reads 1.35 V and ammeter reads 1.5 A. Find the emf and the internal resistance of the cell.*

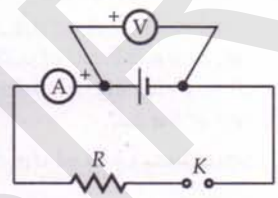


Fig. 3.93

Solution. When the key is open, the voltmeter reads almost the emf of the cell.

$$\therefore \quad \mathcal{E} = \mathbf{1.5 \text{ V}}$$

When the key is closed, voltmeter reads the terminal potential difference V.

$$V = 1.35 \text{ V}, \quad I = 1.5 \text{ A}, \quad r = ?$$

$$r = \frac{\mathcal{E} - V}{I} = \frac{1.5 - 1.35}{1.5} = \mathbf{0.1\ \Omega}.$$

Example 76. *A cell of emf 2 V and internal resistance 0.1 Ω is connected to a 3.9 Ω external resistance. What will be the p.d. across the terminals of the cell ?* [CBSE D 01C]

Solution. Here $\mathcal{E} = 2$ V, $r = 0.1\ \Omega$, $R = 3.9\ \Omega$

Current, $$I = \frac{\mathcal{E}}{R + r} = \frac{2}{3.9 + 0.1} = 0.5 \text{ A}$$

P.D. across the terminals of the cell,

$$V = IR = 0.5 \times 3.9 = \mathbf{1.95 \text{ V}}.$$

Example 77. *The reading on a high resistance voltmeter when a cell is connected across it is 2.2 V. When the terminals of the cell are also connected to a resistance of 5 Ω, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell.* [CBSE OD 10]

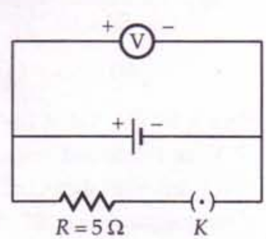


Fig. 3.94

Solution. Here $\mathcal{E} = 2.2$ V, $R = 5\ \Omega$, $V = 1.8$ V

Internal resistance,

$$r = R\left(\frac{\mathcal{E} - V}{V}\right) = 5\left(\frac{2.2 - 1.8}{1.8}\right)\Omega = \mathbf{1.1\ \Omega}.$$

Example 78. *A dry cell of emf 1.6 V and internal resistance 0.10 Ω is connected to a resistance of R ohm. The current drawn from the cell is 2.0 A. Find the voltage drop across R.*

Solution. Here $\mathcal{E} = 1.6$ V, $r = 0.10\ \Omega$, $I = 2.0$ A

Voltage drop across R will be

$$V = \mathcal{E} - Ir$$
$$= 1.6 - 2.0 \times 0.10 = \mathbf{1.4\ V.}$$

Example 79. *A battery of e.m.f. '$\mathcal{E}$', and internal resistance 'r', gives a current of 0.5 A with an external resistor of 12 ohm and a current of 0.25 A with an external resistor of 25 ohm. Calculate (i) internal resistance of the cell and (ii) emf of the cell.* [CBSE D 02 ; OD 13C]

Solution. EMF of the cell, $\mathcal{E} = I(R + r)$

In *first case*, $\mathcal{E} = 0.5\,(12 + r)$

in *second case*, $\mathcal{E} = 0.25\,(25 + r)$

$\therefore \quad 0.5\,(12 + r) = 0.25\,(25 + r)$

On solving, we get $r = \mathbf{1\ \Omega}$

Hence $\mathcal{E} = 0.5\,(12 + 1) = \mathbf{6.5\ V.}$

Example 80. *A battery of emf 3 volt and internal resistance r is connected in series with a resistor of 55 Ω through an ammeter of resistance 1 Ω. The ammeter reads 50 mA. Draw the circuit diagram and calculate the value of r.* [CBSE D 95 ; Haryana 02]

Solution. Total resistance

$$= 55 + 1 + r\,\Omega = 56 + r\,\Omega$$

Current

$$= 50 \text{ mA}$$
$$= 50 \times 10^{-3} \text{ A}$$

emf = 3 V

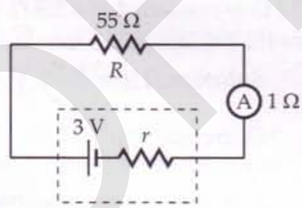


Fig. 3.95

$$\text{Resistance} = \frac{\text{emf}}{\text{Current}}$$

$$56 + r = \frac{3}{50 \times 10^{-3}} = 60$$

$$r = 60 - 56 = \mathbf{4\ \Omega.}$$

Example 81. *(a) A car has a fresh storage battery of emf 12 V and internal resistance $5.0 \times 10^{-2}\ \Omega$. If the starter motor draws a current of 90 A, what is the terminal voltage of the battery when the starter is on ?*
(b) After long use, the internal resistance of the storage battery increases to 500 Ω. What maximum current can be drawn from the battery ? Assume the emf of the battery to remain unchanged.
(c) If the discharged battery is charged by an external emf source, is the terminal voltage of the battery during charging greater or less than its emf 12 V ? [NCERT]

Solution. (*a*) Here $\mathcal{E} = 12$ V, $I = 90$ A,

$$r = 5.0 \times 10^{-2}\,\Omega$$

$\therefore$ Terminal voltage,

$$V = \mathcal{E} - Ir = 12 - 4.5 = \mathbf{7.5\ V.}$$

(*b*) The maximum current can be drawn from a battery by shorting it.

Then $V = 0$

and $$I_{max} = \frac{\mathcal{E}}{r} = \frac{12}{500} \text{ A} = \mathbf{24\ mA.}$$

Clearly, the battery is useless for starting the car and must be charged again.

(*c*) During discharge of the accumulator, the current *inside* the cells (of the accumulator) is opposite to what it is when the accumulator discharges. That is, during charging, current flows from the + ve to – ve terminal *inside* the cells. Consequently, during charging

$$V = \mathcal{E} + Ir$$

Hence V must be greater than 12 V during charging.

Example 82. A battery of emf 12.0 *V and internal resistance 0.5 Ω is to be charged by a battery charger which supplies 110 V d.c. How much resistance must be connected in series with the battery to limit the charging current to 5.0 A ? What will be the p.d. across the terminals of the battery during charging ?*

Solution. For charging, the positive terminal of the charger is connected to the positive terminal of the battery.

$\therefore$ Net emf $\mathcal{E}' = 110 - 12.0 = 98$ V

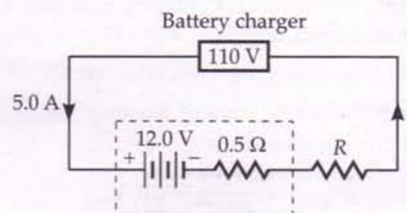


Fig. 3.96

If R is the series resistor, then the charging current will be

$$I = \frac{\mathcal{E}'}{R + r} = \frac{98}{R + 0.5} \text{ A}$$

Given $I = 5.0$ A, therefore

$$\frac{98}{R + 0.5} = 5.0 \quad \text{or} \quad R = 19.1\,\Omega$$

The terminal p.d. of the battery during charging is

$$V = \mathcal{E} + Ir = 12.0 + 5.0 \times 0.5 = \mathbf{14.5\ V}$$

If the series resistor R were not included in the charging circuit, the charging current would be 98/0.5 = 196 A, which is dangerously high.

Example 83. *A cell of emf 1.5 V and internal resistance 0.5 Ω is connected to a (non-linear) conductor whose V-I graph is shown in Fig. 3.97(a). Obtain graphically the current drawn from the cell and its terminal voltage.*

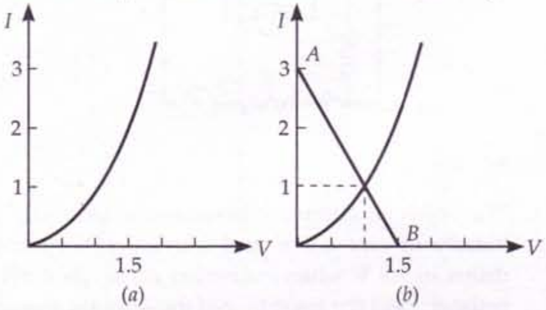


Fig. 3.97

Solution. Here $\mathcal{E} = 1.5$ V, $r = 0.5\,\Omega$

Terminal voltage of the cell, $V = \mathcal{E} - Ir$

For different currents, terminal voltages can be determined as follows :

$I = 0$, $\quad V = 1.5 - 0 = 1.5$ V

$I = 1$ A, $\quad V = 1.5 - 1 \times 0.5 = 1.0$ V

$I = 2$ A, $\quad V = 1.5 - 2 \times 0.5 = 0.5$ V

$I = 3$ A, $\quad V = 1.5 - 3 \times 0.5 = 0$

The V-I graph for the cell is a straight line AB, as shown in Fig. 3.97(b). This straight line graph intersects the given non-linear V-I graph at current = 1 A and at voltage = 1 V.

∴ Current drawn from the cell = **1 A**

Terminal voltage of the cell = **1 V.**

Example 84. *Potential differences across terminals of a cell were measured (in volt) against different currents (in ampere) flowing through the cell. A graph was drawn which was a straight line ABC, as shown in Fig. 3.98. Determine from the graph*

(*i*) *emf of the cell*

(*ii*) *maximum current obtained from the cell, and*

(*iii*) *internal resistance of the cell.* [CBSE D 11C]

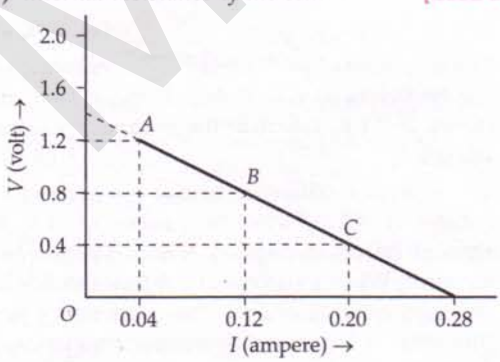


Fig. 3.98

Solution. (*i*) The potential difference corresponding to zero current equals the emf of the cell.

∴ EMF of the cell, $\mathcal{E} = \mathbf{1.4\ V.}$

(*ii*) Maximum current is drawn from the cell when the terminal potential difference is zero.

∴ $\quad I_{max} = \mathbf{0.28\ A.}$

(*iii*) Internal resistance,

$$r = \frac{\mathcal{E}}{I_{max}} = \frac{1.4\ \text{V}}{0.28\ \text{A}} = \mathbf{5\,\Omega.}$$

Example 85. *Find the current drawn from a cell of emf 1 V and internal resistance 2/3 Ω connected to the network given below.* [CBSE D 01C]

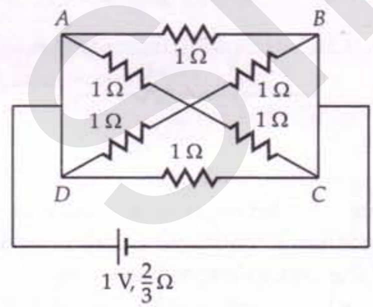


Fig. 3.99

Solution. The equivalent circuit is shown below.

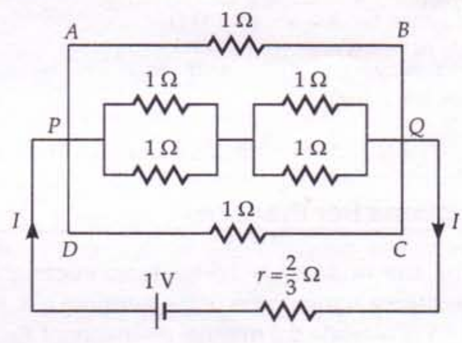


Fig. 3.100

Resistance in arm $AB = 1\,\Omega$

Resistance in arm $PQ = \dfrac{1 \times 1}{1+1} + \dfrac{1 \times 1}{1+1} = \dfrac{1}{2} + \dfrac{1}{2} = 1\,\Omega.$

Resistance in arm $DC = 1\,\Omega$

These three resistances are connected in parallel. Their equivalent resistance R is given by

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{3}{1} \quad \text{or} \quad R = \frac{1}{3}\,\Omega$$

Current drawn from the cell,

$$I = \frac{\mathcal{E}}{R + r} = \frac{1\ \text{V}}{\left(\frac{1}{3} + \frac{2}{3}\right)\Omega} = \mathbf{1\ A.}$$

Example 86. *A uniform wire of resistance 12 Ω is cut into three pieces in the ratio 1 : 2 : 3 and the three pieces are connected to form a triangle. A cell of emf 8 V and internal resistance 1 Ω is connected across the highest of the three resistors. Calculate the current through each part of the circuit.* [CBSE OD 13C]

Solution. In Fig. 3.101, $R_{AB} = 2\ \Omega$, $R_{BC} = 4\ \Omega$ and $R_{AC} = 6\ \Omega$.

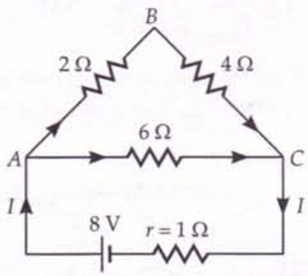


Fig. 3.101

The series combination of 2 Ω and 4 Ω (of equivalent resistance 6 Ω) is in parallel with the 6 Ω resistance. The equivalent resistance is

$$R = \frac{6 \times 6}{6 + 6} = 3\ \Omega$$

Current, $$I = \frac{\mathcal{E}}{R + r} = \frac{8\text{ V}}{(3+1)\ \Omega} = 2\text{ A}$$

The resistances R_{BAC} and R_{BC} of the parallel branches are equal.

$\therefore \quad I_{ABC} = I_{AC} = \mathbf{1\ A.}$

Problems For Practice

1. The emf of a cell is 1.5 V. On connecting a 14 Ω resistance across the cell, the terminal p.d. falls to 1.4 V. Calculate the internal resistance of the cell. **[Haryana 01]** (Ans. 1Ω)
2. The potential difference across a cell is 1.8 V when a current of 0.5 A is drawn from it. The p.d. falls to 1.6 V when a current of 1.0 A is drawn. Find the emf and the internal resistance of the cell. (Ans. 2.0 V, 0.4 Ω)
3. The potential difference of a cell in an open circuit is 6 V, which falls to 4 V when a current of 2 A is drawn from the cell. Calculate the emf and the internal resistance of the cell. (Ans. 6 V, 1Ω)
4. In the circuit shown in Fig. 3.102, the resistance of the ammeter *A* is negligible and that of the voltmeter *V* is very high. When the switch *S* is open, the reading of voltmeter is 1.53 V. On closing the switch *S*, the reading of ammeter is 1.00 A and that of the voltmeter drops to 1.03 V. Calculate : (*i*) emf of the cell (*ii*) value of *R* (*iii*) internal resistance of the cell. [Ans. (*i*) 1.53 V (*ii*) 1.03 Ω (*iii*) 0.50 Ω]

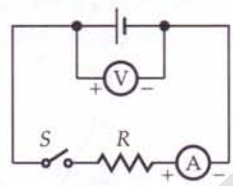


Fig. 3.102

5. The potential difference between the terminals of a battery of emf 6.0 V and internal resistance 1 Ω drops to 5.8 V when connected across an external resistor. Find the resistance of the external resistor. (Ans. 29 Ω)
6. The potential difference between the terminals of a 6.0 V battery is 7.2 V when it is being charged by a current of 2.0 A. What is the internal resistance of the battery ? (Ans. 0.6 Ω)
7. A battery of emf 2 V and internal resistance 0.5 Ω is connected across a resistance of 9.5 Ω. How many electrons pass through a cross-section of the resistance in 1 second ? (Ans. 1.25×10^{18})
8. A cell of emf $\mathcal{E}$ and internal resistance *r* is connected across a variable load resistor *R*. It is found that when $R = 4\ \Omega$, the current is 1A and when *R* is increased to 9 Ω, the current reduces to 0.5 A. Find the values of the emf $\mathcal{E}$ and internal resistance *r*. **[CBSE D 15]** (Ans. 5 V, 1 Ω)
9. The emf of a battery is 4.0 V and its internal resistance is 1.5 Ω. Its potential difference is measured by a voltmeter of resistance 1000 Ω. Calculate the percentage error in the reading of emf shown by voltmeter. (Ans. 0.15 %)
10. The emf of a battery is 6 V and its internal resistance is 0.6 Ω. A wire of resistance 2.4 Ω is connected to the two ends of the battery, calculate (*a*) current in the circuit and (*b*) the potential difference between the two terminals of the battery in closed circuit. (Ans. 2 A, 4.8 V)
11. The two poles of a cell of emf 1.5 V are connected to the two ends of a 10 Ω coil. If the current in the circuit is 0.1 A, calculate the internal resistance of the cell. (Ans. 5 Ω)
12. The potential difference across the terminals of a battery is 8.5 V, when a current of 3 A flows through it from its negative terminal to the positive terminal. When a current of 2 A flows through it in the opposite direction, the terminal potential difference is 11 V. Find the internal resistance of the battery and its emf. (Ans. 0.5 Ω, 10 V)

13. In the circuit shown in Fig. 3.103, a potential difference of 3 V is required between the points A and B. Find the value of resistance R_1. (Ans. $3\,\Omega$)

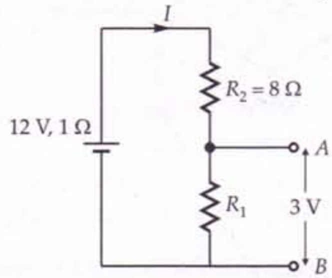


Fig. 3.103

HINTS

1. $r = R\left(\frac{\mathcal{E} - V}{V}\right) = 14\left(\frac{1.5 - 1.4}{1.4}\right) = \mathbf{1\,\Omega}.$

2. EMF of a cell, $\mathcal{E} = V + Ir$

   When $I = 0.5$ A, $V = 1.8$ V $\therefore$ $\mathcal{E} = 1.8 + 0.5\,r$ ...(1)

   When $I = 1.0$ A, $V = 1.6$ V $\therefore$ $\mathcal{E} = 1.6 + 1.0\,r$ ...(2)

   Solving (1) and (2), we get

   $$\mathcal{E} = \mathbf{2.0\ V} \quad \text{and} \quad r = \mathbf{0.4\,\Omega}.$$

3. $\mathcal{E}$ = P.D. measured in open circuit = **6 V**

   $r = \frac{\mathcal{E} - V}{I} = \frac{6 - 4}{2} = \mathbf{1\,\Omega}.$

4. (*i*) $\mathcal{E}$ = P.D. measured in open circuit = **1.53 V.**

   (*ii*) $R = \frac{V}{I} = \frac{1.03}{1.00} = \mathbf{1.03\,\Omega}.$

   (*iii*) $r = R\left(\frac{\mathcal{E} - V}{V}\right) = 1.03\left(\frac{1.53 - 1.03}{1.03}\right) = \mathbf{0.50\,\Omega}.$

5. $R = \frac{rV}{\mathcal{E} - V} = \frac{1 \times 5.8}{6.0 - 5.8} = \frac{5.8}{0.2} = \mathbf{29\,\Omega}.$

6. During charging, $V = \mathcal{E} + Ir$

   $\therefore$ $7.2 = 6.0 + 2 \times r$ or $r = \mathbf{0.6\,\Omega}.$

7. $I = \frac{\mathcal{E}}{R + r} = \frac{2}{9.5 + 0.5} = 0.2$ A

   $n = \frac{It}{e} = \frac{0.2 \times 1}{1.6 \times 10^{-19}} = \mathbf{1.25 \times 10^{18}}.$

8. Proceed as in Example 79 on page 3.48.

9. Proceed as in Example 74 on page 3.47.

10. Current, $I = \frac{\mathcal{E}}{R + r} = \frac{6}{2.4 + 0.6} = \mathbf{2\ A}.$

    P.D. between the two terminals of the battery is

    $$V = IR = 2 \times 2.4\ \text{V} = \mathbf{4.8\ V}.$$

11. As $I = \frac{\mathcal{E}}{R + r}$ $\therefore$ $0.1 = \frac{1.5}{10 + r}$ or $r = 5\,\Omega.$

12. When current flows through the cell from its negative to positive terminal,

    $$V = \mathcal{E} - Ir$$

    or $\quad 8.5 = \mathcal{E} - 3r$ ...(*i*)

    When current flows through the cell from its positive to negative terminal, p.d. across r adds to its emf. So

    $$V = \mathcal{E} + Ir$$

    or $\quad 11 = \mathcal{E} + 2r$ ...(*ii*)

    On solving equations (*i*) and (*ii*), we get

    $$r = \mathbf{0.5\,\Omega} \text{ and } \mathcal{E} = \mathbf{10\ V}.$$

13. Current in the main circuit,

    $$I = \frac{\mathcal{E}}{R_1 + R_2 + r}$$

    Since a potential difference of 3 V is required across R_1, therefore

    $$IR_1 = 3 \text{ volt}$$

    or $\quad \frac{\mathcal{E}R_1}{R_1 + R_2 + r} = 3$ or $\frac{12\,R_1}{R_1 + 8 + 1} = 3$

    or $\quad R_1 = 3\,\Omega.$

3.21 COMBINATIONS OF CELLS IN SERIES AND PARALLEL

36. *Why do we often use a combination of cells ?*

Combination of cells. A single cell provides a feeble current. In order to get a higher current in a circuit, we often use a combination of cells, two or more cells. *A combination of cells is called a* ***battery***. The battery cells are used in torches, transistor sets, automobiles, etc. Cells can be joined in series, parallel or in a mixed way.

37. *What do you mean by a series combination of cells ? Two cells of different emfs and internal resistances are connected in series. Find expressions for the equivalent emf and equivalent internal resistance of the combination.*

Cells in series. *When the negative terminal of one cell is connected to the positive terminal of the other cell and so on, the cells are said to be connected in series.*

As shown in Fig. 3.104, suppose two cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 are connected in series between points A and C. Let I be the current flowing through the series combination.

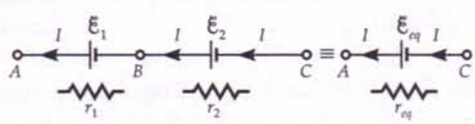


Fig. 3.104 A series combination of two cells is equivalent to a single cell of emf $\mathcal{E}_{eq}$ and internal resistance r_{eq}

Let V_A, V_B and V_C be the potentials at points A, B and C respectively. The potential differences across the terminals of the two cells will be

$$V_{AB} = V_A - V_B = \mathcal{E}_1 - Ir_1$$

and
$$V_{BC} = V_B - V_C = \mathcal{E}_2 - Ir_2$$

Thus the potential difference between the terminals A and C of the series combination is

$$V_{AC} = V_A - V_C = (V_A - V_B) + (V_B - V_C)$$
$$= (\mathcal{E}_1 - Ir_1) + (\mathcal{E}_2 - Ir_2)$$

or
$$V_{AC} = (\mathcal{E}_1 + \mathcal{E}_2) - I(r_1 + r_2)$$

If we wish to replace the series combination by a single cell of emf $\mathcal{E}_{eq}$ and internal resistance r_{eq}, then

$$V_{AC} = \mathcal{E}_{eq} - Ir_{eq}$$

Comparing the last two equations, we get

$$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2 \quad \text{and} \quad r_{eq} = r_1 + r_2$$

We can extend the above rule to a series combination of any number of cells :

1. The equivalent emf of a series combination of n cells is equal to the sum of their individual emfs.
$$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + ... + \mathcal{E}_n$$
2. The equivalent internal resistance of a series combination of n cells is equal to the sum of their individual internal resistances.
$$r_{eq} = r + r_2 + r_3 + ... + r_n$$
3. The above expression for $\mathcal{E}_{eq}$ is valid when the n cells assist each other *i.e.*, the current leaves each cell from the positive terminal. However, if one cell of emf $\mathcal{E}_2$, say, is turned around 'in opposition' to other cells, then
$$\mathcal{E}_{eq} = \mathcal{E}_1 - \mathcal{E}_2 + \mathcal{E}_3 + ... + \mathcal{E}_n.$$

38. *What do you mean by a parallel combination of cells ? Two cells of different emfs and internal resistances are connected in parallel with one another. Find the expressions for the equivalent emf and equivalent internal resistance of the combination.*

Cells in parallel. *When the positive terminals of all cells are connected to one point and all their negative terminals to another point, the cells are said to be connected in parallel.*

As shown in Fig. 3.105, suppose two cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 are connected in parallel between two points. Suppose the currents I_1 and I_2 from the positive terminals of the two cells flow towards the junction B_1, and current I flows out. Since as much charge flows in as flows out, we have

$$I = I_1 + I_2$$

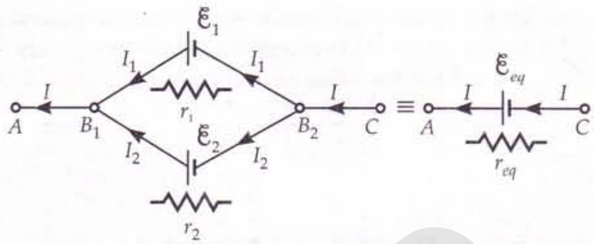


Fig. 3.105 A parallel combination of two cells is equivalent to a single cell of emf $\mathcal{E}_{eq}$ and internal resistance r_{eq}.

As the two cells are connected in parallel between the same two points B_1 and B_2, the potential difference V across both cells must be same.

The potential difference between the terminals of first cell is

$$V = V_{B_1} - V_{B_2} = \mathcal{E}_1 - I_1 r_1$$

$$\therefore \quad I_1 = \frac{\mathcal{E}_1 - V}{r_1}$$

The potential difference between the terminals of $\mathcal{E}_2$ is

$$V = V_{B_1} - V_{B_2} = \mathcal{E}_2 - I_2 r_2$$

$$\therefore \quad I_2 = \frac{\mathcal{E}_2 - V}{r_2}$$

Hence
$$I = I_1 + I_2 = \frac{\mathcal{E}_1 - V}{r_1} + \frac{\mathcal{E}_2 - V}{r_2}$$
$$= \left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

or
$$V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 r_2} - I$$

or
$$V = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2} - I\,\frac{r_1 r_2}{r_1 + r_2}$$

If we wish to replace the parallel combination by a single cell of emf $\mathcal{E}_{eq}$ and internal resistance r_{eq}, then

$$V = \mathcal{E}_{eq} - Ir_{eq}$$

Comparing the last two equations, we get

$$\mathcal{E}_{eq} = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2} \quad \text{and} \quad r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

We can express the above results in a simpler way as follows :

$$\frac{\mathcal{E}_{eq}}{r_{eq}} = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}$$

and
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

For a parallel combination of n cells, we can write

$$\frac{\mathcal{E}_{eq}}{r_{eq}} = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \ldots\ldots + \frac{\mathcal{E}_n}{r_n}$$

and $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots + \frac{1}{r_n}.$$

39. *Derive the condition for obtaining maximum current through an external resistance connected across a series combination of cells.*

Condition for maximum current from a series combination of cells. As shown in Fig. 3.106, suppose n similar cells each of emf $\mathcal{E}$ and internal resistance r be connected in series. Let R be the external resistance.

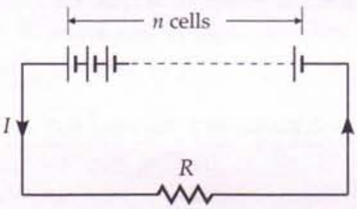


Fig. 3.106 A series combination of n cells.

Total emf of n cells in series

$$= \text{Sum of emfs of all cells} = n\mathcal{E}$$

Total internal resistance of n cells in series

$$= r + r + r + \ldots\ldots n \text{ terms} = nr$$

Total resistance in the circuit $= R + nr$

The current in the circuit is

$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$= \frac{n\mathcal{E}}{R + nr}$$

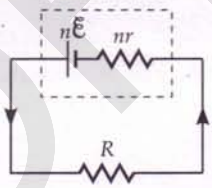


Fig. 3.107 Equivalent circuit.

Special Cases

(*i*) If $R >> nr$, then

$$I = \frac{n\mathcal{E}}{R}$$

$= n$ times the current ($\mathcal{E}/R$) that can be drawn from one cell.

(*ii*) If $R << nr$, then

$$I = \frac{n\mathcal{E}}{nr} = \frac{\mathcal{E}}{r}$$

= the current given by a single cell

Thus, *when external resistance is much higher than the total internal resistance, the cells should be connected in series to get maximum current.*

40. *Derive the condition for obtaining maximum current through an external resistance connected to a parallel combination of cells.*

Condition for maximum current from a parallel combination of cells. As shown in Fig. 3.108, suppose m cells each of emf $\mathcal{E}$ and internal resistance r be connected in parallel between points A and B. Let R be the external resistance.

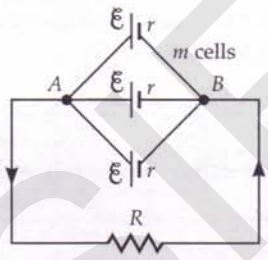


Fig. 3.108 A parallel combination of m cells.

Since all the m internal resistances are connected in parallel, their equivalent resistance R' is given by

$$\frac{1}{R'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \ldots\ldots m \text{ terms} = \frac{m}{r}$$

or $$R' = \frac{r}{m}$$

Total resistance in the circuit

$$= R + R' = R + \frac{r}{m}$$

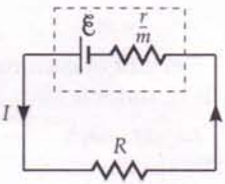


Fig. 3.109 Equivalent circuit.

As the only effect of joining m cells in parallel is to get a single cell of larger size with the same chemical materials, so

total emf of parallel combination

$$= \text{emf due to single cell} = \mathcal{E}$$

$\therefore$ The current in the circuit is

$$I = \frac{\mathcal{E}}{R + r/m} = \frac{m\mathcal{E}}{mR + r}$$

Special Cases

(*i*) If $R << \frac{r}{m}$, then

$$I = \frac{m\mathcal{E}}{r} = m \text{ times the current due to a single cell.}$$

(*ii*) If $R >> \frac{r}{m}$, then

$$I = \frac{\mathcal{E}}{R} = \text{the current given by a single cell.}$$

Thus, *when external resistance is much smaller than the net internal resistance, the cells should be connected in parallel to get maximum current.*

41. *Derive the condition for obtaining maximum current through an external resistance connected across a mixed grouping of cells.*

Mixed grouping of cells. *In this combination, a certain number of identical cells are joined in series, and all such rows are then connected in parallel with each other.*

As shown in Fig. 3.110, suppose n cells, each of emf $\mathcal{E}$ and internal resistance r, are connected in series in each row and m such rows are connected in parallel across the external resistance R.

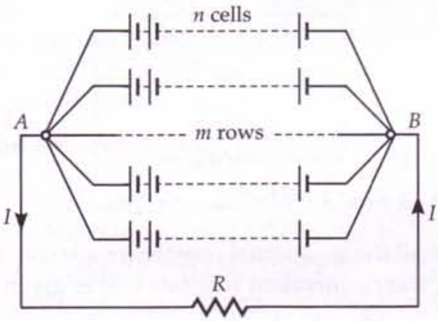


Fig. 3.110 Mixed grouping of cells.

Total number of cells

$= mn$

Net emf of each row of n cells in series $= n\mathcal{E}$

As m such rows are connected in parallel, so net emf of the combination $= n\mathcal{E}$

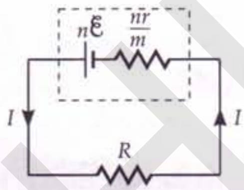


Fig. 3.111 Equivalent circuit.

Net internal resistance of each row of n cells $= nr$

As m such rows are connected in parallel, so the total internal resistance r' of the combination is given by

$$\frac{1}{r'} = \frac{1}{nr} + \frac{1}{nr} + \frac{1}{nr} + \ldots\ldots m \text{ terms} = \frac{m}{nr}$$

or $$r' = \frac{nr}{m}$$

Total resistance of the circuit

$$= R + r' = R + \frac{nr}{m}$$

The current through the external resistance R,

$$I = \frac{\text{Total emf}}{\text{Total resistance}} = \frac{n\mathcal{E}}{R + nr/m}$$

$$= \frac{mn\mathcal{E}}{mR + nr}$$

Clearly, the current I will be maximum if the denominator *i.e.*, $(mR + nr)$ is minimum.

Now

$$mR + nr = (\sqrt{mR})^2 + (\sqrt{nr})^2$$
$$= (\sqrt{mR})^2 + (\sqrt{nr})^2 - 2\sqrt{mR}\sqrt{nr} + 2\sqrt{mR}\sqrt{nr}$$
$$= (\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnRr}$$

As the perfect square cannot be negative, so $mR + nr$ will be minimum if

i.e., $$\sqrt{mR} - \sqrt{nr} = 0$$

or $$mR = nr$$

or $$R = \frac{nr}{m}$$

or External resistance

= Total internal resistance of the cells.

Thus, *in a mixed grouping of cells, the current through the external resistance will be maximum if the external resistance is equal to the total internal resistance of the cells.*

Examples based on Grouping of Cells

Formulae Used

1. For n cells in series, $I = \dfrac{n\mathcal{E}}{R + nr}$
2. For n cells in parallel, $I = \dfrac{n\mathcal{E}}{nR + r}$
3. For mixed grouping, $I = \dfrac{mn\mathcal{E}}{mR + nr}$

   where n = no. of cells in series in one row,

   m = no. of rows of cells in parallel.
4. For maximum current, the external resistance must be equal to the total internal resistance.

   i.e., $\dfrac{nr}{m} = R$

   or $nr = mR$

Units Used

EMF and terminal p.d. are in volt (V), internal resistance (r) and external resistance R in Ω, current in ampere (A).

Example 87. *(a) Three cells of emf 2.0 V, 1.8 V and 1.5 V are connected in series. Their internal resistances are 0.05 Ω, 0.7 Ω and 1 Ω respectively. If the battery is connected to an external resistor of 4 Ω via a very low resistance ammeter, what would be the reading in the ammeter ?*

(b) If the three cells above were joined in parallel, would they be characterised by a definite emf and internal resistance (independent of external circuit) ? If not, how will you obtain currents in different branches ? [NCERT]

Solution. (*a*) The circuit diagram is shown in Fig. 3.112.

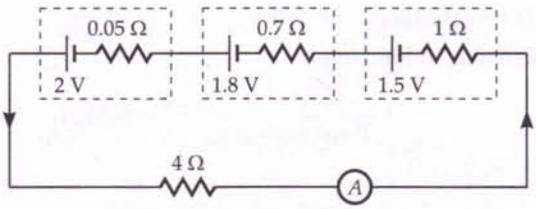


Fig. 3.112

As the three cells have been connected in series to an external resistor of 4 Ω, therefore

Total emf $= (2.0 + 1.8 + 1.5)\text{ V} = 5.3\text{ V}$

Total resistance

$$= (0.05 + 0.7 + 1 + 4)\,\Omega = 5.75\,\Omega$$

Current, $I = \dfrac{\text{emf}}{\text{resistance}}$

$$= \frac{5.3}{5.75}\text{ A} = \mathbf{0.92\text{ A}}.$$

(*b*) No, there is no formula for emf and internal resistance of non-similar cells, joined in parallel. For this situation, we must use Kirchhoff's laws.

Example 88. *A cell of emf 1.1 V and internal resistance 0.5 Ω is connected to a wire of resistance 0.5 Ω. Another cell of the same emf is connected in series but the current in the wire remains the same. Find the internal resistance of the second cell.*

Solution. *In first case* :

Total emf, $\mathcal{E} = 1.1\text{ V}$

Total resistance, $R = 0.5 + 0.5 = 1\,\Omega$

$\therefore$ Current, $I = \dfrac{\mathcal{E}}{R} = \dfrac{1.1}{1} = 1.1\text{ A}$

In *second case* :

Total emf,

$$\mathcal{E} = 1.1 + 1.1 = 2.2\text{ V}$$

Total resistance,

$$R = 0.5 + 0.5 + r = (1 + r)\,\Omega$$

where r is the internal resistance of the second cell.

$\therefore$ Current, $I = \dfrac{2.2}{1 + r} = 1.1$ or $r = \mathbf{1\,\Omega}$.

Example 89. *Two identical cells of emf 1.5 V each joined in parallel provide supply to an external circuit consisting of two resistances of 17 Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell.* [CBSE D 95C]

Solution. Here $\mathcal{E} = 1.5\text{ V},\ V = 1.4\text{ V}$

Resistance of external circuit = Total resistance of two resistances of 17 Ω connected in parallel

or $$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{17 \times 17}{17 + 17}\,\Omega = 8.5\,\Omega$$

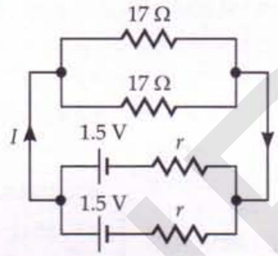


Fig. 3.113

Let r' be the total internal resistance of the two cells. Then

$$r' = R\left[\frac{\mathcal{E} - V}{V}\right] = 8.5\left[\frac{1.5 - 1.4}{1.4}\right]\Omega = 0.6\,\Omega.$$

As the two cells of internal resistance $r\,\Omega$ each have been connected in parallel, therefore.

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} \quad \text{or} \quad \frac{1}{0.6} = \frac{2}{r}$$

or $$r = 0.6 \times 2 = \mathbf{1.2\,\Omega}.$$

Example 90. *Four identical cells, each of emf 2 V, are joined in parallel providing supply of current to external circuit consisting of two 15 Ω resistors joined in parallel. The terminal voltage of the cells, as read by an ideal voltmeter is 1.6 volt. Calculate the internal resistance of each cell.* [CBSE D 02]

Solution. As shown in Fig. 3.114, four cell are connected in parallel to the parallel combination of two 15 Ω resistors.

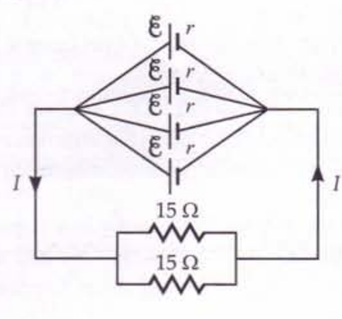


Fig. 3.114

Here $\mathcal{E} = 2\text{ V},\ V = 1.6\text{ V}$

The external resistance provided by two 15 Ω resistors connected in parallel is

$$R = \frac{15 \times 15}{15 + 15} = 7.5\,\Omega$$

If r' is the total internal resistance of the four cells connected in parallel, then

$$r' = R\left(\frac{\mathcal{E} - V}{V}\right) = 7.5\left(\frac{2 - 1.6}{1.6}\right) = \frac{15}{8}\,\Omega$$

If r is the internal resistance of each cell, then

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{4}{r}$$

or $$r = 4r' = 4 \times \frac{15}{8} = \mathbf{7.5\,\Omega}.$$

Example 91. *In the circuit diagram given in Fig. 3.115, the cells E_1 and E_2 have emfs 4 V and 8 V and internal resistances 0.5 Ω and 1.0 Ω respectively. Calculate the current in each resistance.* [CBSE D 15C]

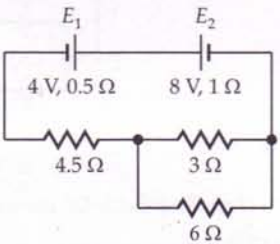


Fig. 3.115

Solution. Effective emf of the circuit

$$= \mathcal{E}_2 - \mathcal{E}_1 = 8 - 4 = 4\text{ V}$$

Total resistance of the circuit

$$= 1 + 0.5 + 4.5\,\Omega + \frac{3 \times 6}{3 + 6} = 8\,\Omega$$

∴ Current in the circuit, $I = \frac{4}{8} = 0.5$ A

Current through 4.5 Ω resistance $= I = \mathbf{0.5\ A}$

P.D. across the parallel combination of 3 Ω and 6 Ω resistances is

$$V = R'I = \frac{3 \times 6}{3 + 6} \times 0.5 = 1\text{ V}$$

Current through 3 Ω resistance $= \frac{1\text{ V}}{3\,\Omega} = \mathbf{\frac{1}{3}\ A}$

Current through 6 Ω resistance $= \frac{1\text{ V}}{6\,\Omega} = \mathbf{\frac{1}{6}\ A.}$

Example 92. *In Fig. 3.116, $\mathcal{E}_1$ and $\mathcal{E}_2$ are respectively 2.0 V and 4.0 V and the resistances r_1, r_2 and R are respectively 1.0 Ω, 2.0 Ω and 5.0 Ω. Calculate the current in the circuit. Also calculate (i) potential difference between the points b and a, (ii) potential difference between a and c.*

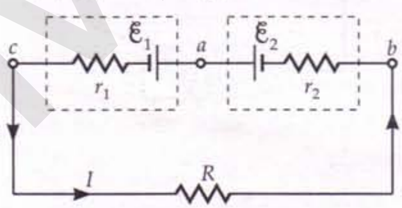


Fig. 3.116

Solution. As emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are opposing each other and $\mathcal{E}_2 > \mathcal{E}_1$, so

$$\text{Net emf} = \mathcal{E}_2 - \mathcal{E}_1 = 4 - 2 = 2\text{ V}.$$

This emf sends circuit I in the anticlockwise direction.

Total resistance $= R + r_1 + r_2 = 5 + 1 + 2 = 8\,\Omega$

Current in the circuit

$$= \frac{\text{Net emf}}{\text{Total resistance}} = \frac{2}{8} = \mathbf{0.25\ A.}$$

(*i*) Current inside the cell $\mathcal{E}_2$ flows from –ve to +ve terminal, so the terminal p.d. of this cell is

$$V_a - V_b = \mathcal{E}_2 - Ir_2$$
$$= 4.0 - 0.25 \times 2.0 = \mathbf{3.5\ V.}$$

(*ii*) Current inside the cell $\mathcal{E}_1$ flows from +ve to –ve terminal. Hence the terminal p.d. of this cell is

$$V_a - V_c = \mathcal{E}_1 + Ir_1$$
$$= 2.0 + 0.25 \times 1.0 = \mathbf{2.25\ V.}$$

Example 93. *In the two electric circuits shown in Fig. 3.117, determine the readings of ideal ammeter (A) and the ideal voltmeter (V).* [CBSE D 15C]

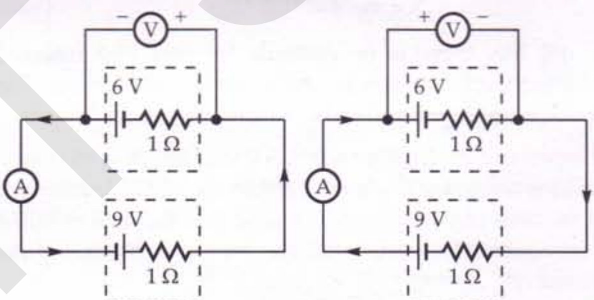


Fig. 3.117 (*a*) (*b*)

Solution. In the circuit (*a*)

Total emf = 15 V, Total resistance = 2 Ω

Current, $I = \frac{15\text{ V}}{2\,\Omega} = 7.5$ A

As the current I flows from –ve to +ve terminal inside the cell of 6 V, the terminal p.d. of the cells is

$$V = \mathcal{E} - Ir = 6 - 7.5 \times 1 = -1.5\text{ V}$$

∴ Reading of ammeter = **7.5 A,**

Reading of voltmeter = **– 1.5 V.**

In the circuit (*b*)

Net emf = 9 – 6 = 3 V, Total resistance = 2 Ω

Current, $I = \frac{3\text{ V}}{2\,\Omega} = 1.5$ A

As the current I flows from +ve to –ve terminal inside the 6 V cell, so the terminal p.d. of the cell is

$$V = \mathcal{E} + Ir = 6 + 1.5 \times 1 = 7.5\text{ V}$$

∴ Reading of ammeter = **1.5 A,**

Reading of voltmeter = **7.5 V.**

Example 94. *A network of resistances is connected to a 16 V battery with internal resistance of 1Ω, as shown in Fig. 3.118.*

(*a*) *Compute equivalent resistance of the network,*

(*b*) *Obtain the current in each resistor, and*

(*c*) *Obtain the voltage drops* V_{AB}, V_{BC} *and* V_{CD}.

[NCERT ; CBSE F 10]

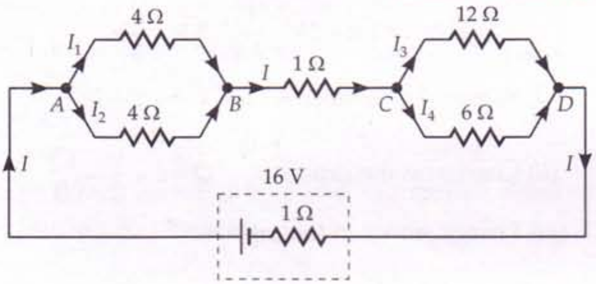


Fig. 3.118

Solution. (*a*) As the two 4 Ω resistances are in parallel, their equivalent resistance is

$$R_1 = \frac{4 \times 4}{4 + 4} = 2\,\Omega$$

Also, the 12 Ω and 6 Ω resistances are in parallel, their equivalent resistance is

$$R_2 = \frac{12 \times 6}{12 + 6} = 4\,\Omega$$

Now the resistances R_1, R_2 and 1 Ω are in series. Hence the equivalent resistance of the network is

$$R = R_1 + R_2 + 1 = 2 + 4 + 1 = 7\,\Omega.$$

(*b*) The total current in the circuit is

$$I = \frac{\mathcal{E}}{R + r} = \frac{16}{7 + 1} = \mathbf{2\ A}$$

The potential difference between *A* and *B* is

$$V_{AB} = 4\,I_1 = 4\,I_2$$

$$\therefore \quad I_1 = I_2$$

But $\quad I_1 + I_2 = I = 2$ A

$$\therefore \quad I_1 = I_2 = \mathbf{1\ A}$$

The potential difference between *C* and *D* is

$$V_{CD} = 12\,I_3 = 6\,I_4 \quad i.e., \quad I_4 = 2\,I_3$$

But $\quad I_3 + I_4 = I = 2$ A

or $\quad I_3 + 2\,I_3 = 2$ A

$$\therefore \quad I_3 = \frac{2}{3}\ \mathbf{A} \quad \text{and} \quad I_4 = \frac{4}{3}\ \mathbf{A}.$$

(*c*)
$$V_{AB} = 4 \times I_1 = 4 \times 1 = \mathbf{4\ V},$$
$$V_{BC} = 1 \times I = 1 \times 2 = \mathbf{2\ V},$$
$$V_{CD} = 12 \times I_3 = 12 \times \frac{2}{3} = \mathbf{8\ V}.$$

Example 95. *A 20 V battery of internal resistance 1 Ω is connected to three coils of 12 Ω, 6 Ω and 4 Ω in parallel, a resistor of 5 Ω and a reversed battery (emf = 8 V and internal resistance = 2 Ω), as shown in Fig. 3.119. Calculate the current in each resistor and the terminal potential difference across each battery.* [CBSE OD 01C]

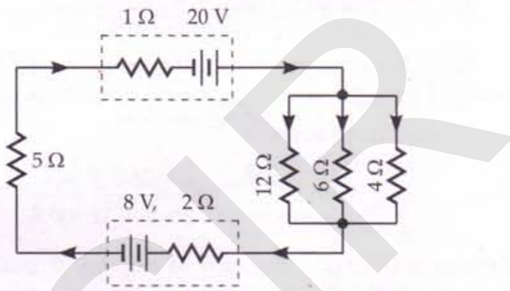


Fig. 3.119

Solution. Equivalent resistance R' of 12 Ω, 6 Ω, 4 Ω resistances connected in parallel is given by

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{6}{12} = \frac{1}{2}$$

or $\quad R' = 2\,\Omega$

Total resistance $= 1 + 5 + 2 + 2 = 10\,\Omega$

Net emf $= 20 - 8 = 12$

Current in the circuit, $I = \frac{12}{10} = \mathbf{1.2\ A}$

So the current through each battery and 5 Ω resistor is 1.2 A.

P.D. across the parallel combination of three resistors is

$$V' = IR' = 1.2 \times 2 = 2.4\ \text{V}$$

$\therefore$ Current in 2 Ω coil $= \frac{2.4}{12} = \mathbf{0.2\ A}$

Current in 6 Ω coil $= \frac{2.4}{6} = \mathbf{0.4\ A}$

Current in 4 Ω coil $= \frac{2.4}{4} = \mathbf{0.6\ A.}$

Terminal p.d. across 20 V battery,

$$V = \mathcal{E} - Ir = 20 - 1.2 \times 1 = \mathbf{18.8\ V}$$

Terminal p.d. across 8 V battery,

$$V' = \mathcal{E}' + I\,r' = 8 + 1.2 \times 2 = \mathbf{10.4\ V.}$$

Example 96. *36 cells each of internal resistance 0.5 Ω and emf 1.5 V each are used to send current through an external circuit of 2 Ω resistance. Find the best mode of grouping them and the current through the external circuit.*

Solution. Here $\mathcal{E} = 1.5$ V, $r = 0.5\,\Omega$, $R = 2\,\Omega$

Total number of cells, $mn = 36$ ...(1)

For maximum current in the mixed grouping,

$$\frac{nr}{m} = R \quad \text{or} \quad \frac{n \times 0.5}{m} = 2 \qquad ...(2)$$

Multiplying equations (1) and (2), we get

$$0.5\,n^2 = 72 \quad \text{or} \quad n^2 = 144$$

$$\therefore \quad n = 12 \quad \text{and} \quad m = \frac{36}{12} = 3$$

Thus for maximum current there should be three rows in parallel, each containing 12 cells in series.

∴ Maximum current

$$= \frac{mn\mathcal{E}}{mR + nr} = \frac{36 \times 1.5}{3 \times 2 + 12 \times 0.5} = \mathbf{4.0\ A.}$$

Example 97. *12 cells, each of emf 1.5 V and internal resistance of 0.5 Ω, are arranged in m rows each containing n cells connected in series, as shown. Calculate the values of n and m for which this combination would send maximum current through an external resistance of 1.5 Ω.* [CBSE Sample Paper 08]

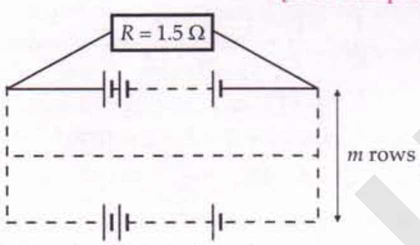


Fig. 3.120

Solution. For maximum current through the external resistance,

External resistance

= Total internal resistance of the cells

$$\text{or} \quad R = \frac{nr}{m}$$

$$\therefore \quad 1.5 = \frac{n \times 0.5}{\frac{12}{n}} \qquad [mn = 12]$$

$$\text{or} \quad 36 = n^2 \quad \text{or} \quad n = \mathbf{6} \quad \text{and} \quad m = \mathbf{2}.$$

Example 98. *In the given circuit in the steady state, obtain the expressions for (i) the potential drop (ii) the charge and (iii) the energy stored in the capacitor, C.* [CBSE F 15]

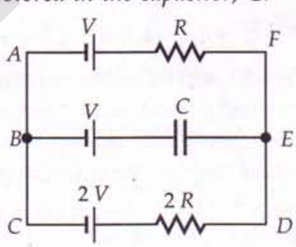


Fig. 3.121

Solution. In the steady state (when the capacitor is fully charged), no current flows through the branch BE.

Net emf $= 2V - V = V$

Net resistance $= 2R + R = 3R$

$$\therefore \text{ Current in the circuit, } I = \frac{V}{3R}$$

Potential difference across BE

$$= 2V - I \times 2R = 2V - \frac{V}{3R} \times 2R = \frac{4}{3}V$$

(*i*) Potential difference across $C = \frac{4}{3}V - V = \mathbf{\frac{V}{3}}$.

(*ii*) Charge on the capacitor, $Q = C \times \frac{V}{3} = \mathbf{\frac{CV}{3}}$.

(*iii*) Energy stored in the capacitor

$$= \frac{1}{2}C\left(\frac{V}{3}\right)^2 = \mathbf{\frac{CV^2}{18}}.$$

Problems For Practice

1. Three identical cells, each of emf 2 V and internal resistance 0.2 Ω are connected in series to an external resistor of 7.4 Ω. Calculate the current in the circuit. (Ans. 0.75 A)
2. Three identical cells each of emf 2 V and unknown internal resistance are connected in parallel. This combination is connected to a 5 Ω resistor. If the terminal voltage across the cells is 1.5 V, what is the internal resistance of each cell ? **[CBSE OD 99]** (Ans. 5 Ω)
3. Two cells connected in series have electromotive force of 1.5 V each. Their internal resistances are 0.5 Ω and 0.25 Ω respectively. This combination is connected to a resistance of 2.25 Ω. Calculate the current flowing in the circuit and the potential difference across the terminals of each cell. (Ans. 1.0 A, 1.0 V, 1.25 V)
4. When 10 cells in series are connected to the ends of a resistance of 59 Ω, the current is found to be 0.25 A, but when the same cells after being connected in parallel are joined to the ends of a 0.05 Ω, the current is 25 A. Calculate the internal resistance and emf of each cell. (Ans. 0.1 Ω, 1.5 V)
5. Find the minimum number of cells required to produce an electric current of 1.5 A through a resistance of 30 Ω. Given that the emf of each cell is 1.5 V and internal resistance 1.0 Ω. (Ans. 120 cells, 60 cells in one row and two rows in parallel)
6. Two identical cells, whether joined together in series or in parallel give the same current, when connected to an external resistance of 1 Ω. Find the internal resistance of each cell. **[ISCE 95]** (Ans. 1 Ω)

7. A set of 4 cells, each of emf 2 V and internal resistance 1.5 Ω are connected across an external load of 10 Ω with 2 rows, 2 cells in each branch. Calculate the current in each branch and potential difference across 10 Ω. **[Karnataka 91C]**

(Ans. 0.175 A, 3.5 V)

HINTS

1. $I = \dfrac{n\mathcal{E}}{R + nr} = \dfrac{3 \times 2}{7.4 + 3 \times 0.2} = \dfrac{6}{8} = \mathbf{0.75\ A.}$

2. Here $\mathcal{E} = 2V,\ V = 1.5\ V,\ R = 5\,\Omega$

   If r' is the total internal resistance of the three cells connected in parallel, then

   $$r' = R\left[\frac{\mathcal{E} - V}{V}\right] = 5\left[\frac{2 - 1.5}{1.5}\right] = \frac{5}{3}\,\Omega$$

   But $\dfrac{1}{r'} = \dfrac{1}{r} + \dfrac{1}{r} + \dfrac{1}{r}$ or $\dfrac{3}{5} = \dfrac{3}{r}$ $\therefore$ $r = \mathbf{5\,\Omega.}$

3. Total resistance, $R = 0.5 + 0.25 + 2.25 = 3.0\,\Omega$

   Total emf, $\mathcal{E} = 1.5 + 1.5 = 3.0\ V$

   Current in the circuit, $I = \dfrac{\mathcal{E}}{R} = \dfrac{3.0}{3.0} = \mathbf{1.0\ A}$

   P.D. across first cell,

   $V_1 = \mathcal{E} - Ir_1 = 1.5 - 1.0 \times 0.5 = \mathbf{1.0\ V}$

   P.D. across second cell,

   $V_2 = \mathcal{E} - Ir_2 = 1.5 - 1.0 \times 0.25 = \mathbf{1.25\ V.}$

4. *For series combination.* The current is

   $$\frac{10\,\mathcal{E}}{59 + 10\,r} = 0.25\ A$$

   For parallel combination. The current is

   $$\frac{10\,\mathcal{E}}{10 \times 0.05 + r} = 25\ A$$

   On solving the above two equations,

   $r = \mathbf{0.1\,\Omega}$ and $\mathcal{E} = \mathbf{1.5\ V.}$

5. As $\dfrac{nr}{m} = R$ $\therefore$ $\dfrac{n \times 1}{m} = 30$ or $n = 30\,m$

   $I = \dfrac{n\mathcal{E}}{2R}$ or $1.5 = \dfrac{n \times 1.5}{2 \times 30}$ or $n = \mathbf{60}$

   $\therefore$ $m = 60/30 = \mathbf{2}$ and $mn = \mathbf{120}$.

6. When the cells are connected in series, (Fig. 3.122), the current in the circuit is

   $$I_s = \frac{2\mathcal{E}}{1 + 2r}$$

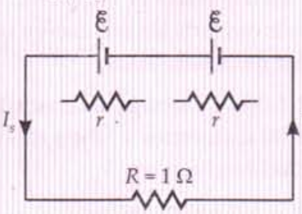


Fig. 3.122

When the cells are connected in parallel (Fig. 3.123), the current in the circuit is

$$I_p = \frac{\mathcal{E}}{1 + \dfrac{r \times r}{r + r}} = \frac{2\mathcal{E}}{2 + r}$$

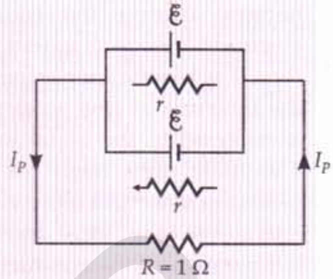


Fig. 3.123

Given $I_s = I_p$ $\therefore$ $\dfrac{2\mathcal{E}}{1 + 2r} = \dfrac{2\mathcal{E}}{2 + r}$

or $1 + 2r = 2 + r$ or $r = \mathbf{1\,\Omega.}$

7. The circuit diagram is shown below.

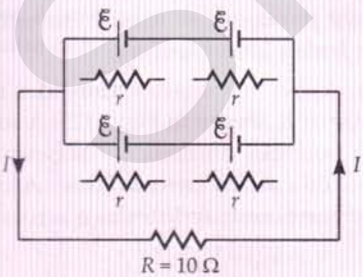


Fig. 3.124

Here $\mathcal{E} = 2\ V,\ r = 1.5\,\Omega,\ R = 10\,\Omega,\ n = 2,\ m = 2$

$$I = \frac{n\mathcal{E}}{R + \dfrac{nr}{m}} = \frac{2 \times 2}{10 + \dfrac{2 \times 1.5}{2}} = \frac{4}{11.5} = 0.35\ A.$$

The two branches are identical.

$\therefore$ Current in each branch $= \dfrac{0.35}{2} = \mathbf{0.175\ A.}$

Potential difference across R

$= IR = 0.35 \times 10 = \mathbf{3.5\ V.}$

3.22 HEATING EFFECT OF CURRENT

42. *What is heating effect of current ? Explain the cause of heating effect of current.*

Heating effect of current. Consider a purely resistive circuit *i.e.,* a circuit which consists of only some resistors and a source of emf. The energy of the source gets dissipated entirely in the form of heat produced in the resistors. *The phenomenon of the production of heat in a resistor by the flow of an electric current through it is called heating effect of current or Joule heating.*

Cause of heating effect of current. When a potential difference is applied across the ends of a conductor, its free electrons get accelerated in the opposite direction of the applied field. But the speed of the electrons does not increase beyond a constant drift speed. This is because during the course of their motion, the electrons collide frequently with the positive metal ions. The kinetic energy gained by the electrons during the intervals of free acceleration between collisions is transferred to the metal ions at the time of collision. The metal ions begin to vibrate about their mean positions more and more violently. The average kinetic energy of the ions increases. This increases the temperature of the conductor. Thus the conductor gets heated due to the flow of current. Obviously, the electrical energy supplied by the source of emf is converted into heat.

3.23 HEAT PRODUCED BY ELECTRIC CURRENT : JOULE'S LAW

43. *Obtain an expression for the heat developed in a resistor by the passage of an electric current through it. Hence state Joule's law of heating.*

Heat produced in a resistor. Consider a conductor AB of resistance R, shown in Fig. 3.125. A source of emf maintains a potential difference V between its ends A and B and sends a steady current I from A to B. Clearly, $V_A > V_B$ and the potential difference across AB is

$$V = V_A - V_B > 0$$

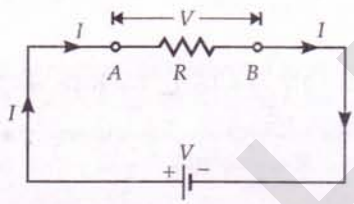


Fig. 3.125 Heat produced in a resistor.

The amount of charge that flows from A to B in time t is

$$q = It$$

As the charge q moves through a decrease of potential of magnitude V, its potential energy decreases by the amount,

$$U = \text{Final P.E at } B - \text{ Initial P.E. at } A$$
$$= qV_B - qV_A = -q(V_A - V_B) = -qV < 0$$

If the charges move through the conductor without suffering collisions, their kinetic energy would change so that the total energy is unchanged. By conservation of energy, the change in kinetic energy must be

$$K = -U = qV = It \times V = VIt > 0$$

Thus, in case, charges were moving freely through the conductor under the action of the electric field, their kinetic energy would increase as they move. However, we know that on the average, the charge carriers or electrons do not move with any acceleration but with a steady drift velocity. This is because of the collisions of electrons with ions and atoms during the course of their motion. The kinetic energy gained by the electrons is shared with the metal ions. These ions vibrate more vigorously and the conductor gets heated up. The amount of energy dissipated as heat in conductor in time t is

$$H = VIt \text{ joule} = I^2Rt \text{ joule} = \frac{V^2t}{R} \text{ joule}$$

or $$H = \frac{VIt}{4.18} \text{ cal} = \frac{I^2Rt}{4.18} \text{ cal} = \frac{V^2t}{4.18\,R} \text{ cal}$$

The above equations are known as ***Joule's law of heating***. *According to this law, the heat produced in a resistor is*

1. *directly proportional to the square of current for a given R,*
2. *directly proportional to the resistance R for a given I,*
3. *inversely proportional to the resistance R for a given V,* and
4. *directly proportional to the time for which the current flows through the resistor.*

For Your Knowledge

- The equation : $W = VIt$ is applicable to the conversion of electrical energy into any other form, but the equation : $H = I^2Rt$ is applicable only to the conversion of electrical energy into heat energy in an ohmic resistor.
- Joule's law of heating holds good even for a.c. circuits. Only current and voltage have to be replaced by their rms values.
- If the circuit is purely resistive, the energy expended by the source entirely appears as heat. But if the circuit has an active element like a motor, then a part of the energy supplied by the source goes to do useful work and the rest appears as heat.

3.24 ELECTRIC POWER

44. *Define the term electric power and state its SI unit.*

Electric power. *The rate at which work is done by a source of emf in maintaining an electric current through a circuit is called electric power of the circuit. Or, the rate at which an appliance converts electric energy into other forms of energy is called its electric power.*

If a current I flows through a circuit for time t at a constant potential difference V, then the work done or energy consumed is given by

$$W = VIt \text{ joule}$$

∴ Electric power,

$$P = \frac{W}{t} = VI = I^2 R = \frac{V^2}{R}$$

or Electric power = current × voltage.

SI unit of electric power. The SI unit of electric power is ***watt* (W)**. *The power of an appliance is one watt if it consumes energy at the rate of 1 joule per second.* Or, the power of a circuit is one watt if 1 ampere of current flows through it on applying a potential difference of 1 volt across it.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}} = \frac{1 \text{ joule}}{1 \text{ coulomb}} \times \frac{1 \text{ coulomb}}{1 \text{ second}}$$

or $1 \text{ watt} = 1 \text{ volt} \times 1 \text{ ampere}$

The bigger units of electric power are ***kilowatt*** (kW) and ***megawatt*** (MW).

$$1 \text{ kW} = 1000 \text{ W and } 1 \text{ MW} = 10^6 \text{ W}$$

The commercial unit of power is ***horse power*** (hp)

$$1 \text{ hp} = 746 \text{ W}.$$

3.25 ELECTRIC ENERGY

45. *Define the term electric energy. State its SI and commercial units.*

Electric energy. *The total work done (or the energy supplied) by the source of emf in maintaining an electric current in a circuit for a given time is called electric energy consumed in the circuit.* It depends upon the power of the appliance used in the circuit and the time for which this power is maintained.

Electric energy,

$$W = P.t = VIt \text{ joule} = I^2 Rt \text{ joule}$$

The SI unit of electric energy is ***joule*** (J).

$$1 \text{ joule} = 1 \text{ volt} \times 1 \text{ ampere} \times 1 \text{ second}$$
$$= 1 \text{ watt} \times 1 \text{ second}$$

Commercial unit of electric energy. The commercial unit of electric energy is ***kilowatt hour*** or ***Board of Trade*** (B.O.T.) ***unit***. *One kilowatt hour is defined as the electric energy consumed by an appliance of 1 kilowatt in one hour.*

$$1 \text{ kilowatt hour} = 1 \text{ kilowatt} \times 1 \text{ hour}$$
$$= 1000 \text{ watt} \times 3600 \text{ s}$$
$$= 3{,}600{,}000 \text{ joules}$$

or $1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

The electric metres installed in our houses measure the electrical energy consumed in kilowatt hours.

Another common unit of electric energy is ***watt hour***. *It is the electric energy consumed by an appliance of one watt in one hour.*

$$1 \text{ watt hour} = 1 \text{ watt} \times 1 \text{ hour} = 3.6 \times 10^3 \text{ J}$$

3.26 POWER RATING

46. *What is meant by the power rating of a circuit element ? Briefly explain how can we measure the electric power of an electric lamp ?*

Power rating. *The power rating of an electrical appliance is the electrical energy consumed per second by the appliance when connected across the marked voltage of the mains.* If a voltage V applied across a circuit element of resistance R sends current I through it, then power rating of the element will be

$$P = \frac{V^2}{R} = I^2 R = VI \text{ watt}$$

Measurement of electric power. To measure the electric power of an appliance, say an electric lamp, we connect a battery and an ammeter in series with the electric lamp and a voltmeter in parallel with it, as shown in Fig. 3.126. Suppose the voltmeter reads V volts and the ammeter reads I amperes, then power rating of the electric lamp will be

$$P = VI \text{ watt}$$

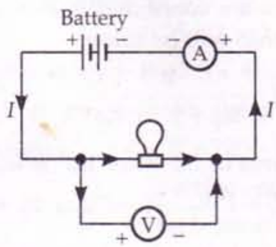


Fig. 3.126 To measure electric power of an electric lamp.

3.27 POWER CONSUMPTION IN A COMBINATION OF APPLIANCES

47. *Prove that the reciprocal of the total power consumed by a series combination of appliances is equal to the sum of the reciprocals of the individual powers of the appliances.*

Power consumed by a series combination of appliances. As shown in Fig. 3.127, consider a series combination of three bulbs of powers P_1, P_2 and P_3 ; which have been manufactured for working on the same voltage V.

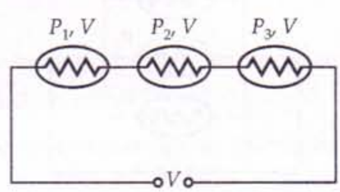


Fig. 3.127 Series combination of bulbs.

The resistances of the three bulbs will be

$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2},\ R_3 = \frac{V^2}{P_3}$$

As the bulbs are connected in series, so their equivalent resistance is

$$R = R_1 + R_2 + R_3$$

If P is the effective power of the combination, then

$$\frac{V^2}{P} = \frac{V^2}{P_1} + \frac{V^2}{P_2} + \frac{V^2}{P_3}$$

or
$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$$

Thus *for a series combination of appliances, the reciprocal of the effective power is equal to the sum of the reciprocals of the individual powers of the appliances.*

Clearly, when N bulbs of same power P are connected in series,

$$P_{eff} = \frac{P}{N}$$

As the bulbs are connected in series, the current I through each bulb will be same.

$$I = \frac{V}{R_1 + R_2 + R_3}$$

The brightness of the three bulbs will be

$$P_1' = I^2 R_1,\ P_2' = I^2 R_2,\ P_3' = I^2 R_3$$

As $R \propto \frac{1}{P}$, the bulb of lowest wattage (power) will have maximum resistance and it will glow with maximum brightness. When the current in the circuit exceeds the safety limit, the bulb of lowest wattage will be fused first.

48. *Prove that when electrical appliances are connected in parallel, the total power consumed is equal to the sum of the powers of the individual appliances.*

Power consumed by a parallel combination of appliances. As shown in Fig. 3.128, consider a parallel combination of three bulbs of powers P_1, P_2 and P_3, which have been manufactured for working on the same voltage V.

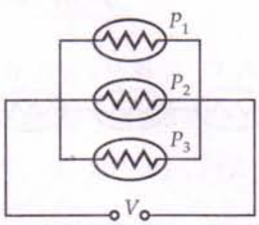


Fig. 3.128 Parallel combination of bulbs.

The resistances of the three bulbs will be

$$R_1 = \frac{V^2}{P_1}, \qquad R_2 = \frac{V^2}{P_2}, \qquad R_3 = \frac{V^2}{P_3}$$

As the bulbs are connected in parallel, their effective resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Multiplying both sides by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$$

or
$$P = P_1 + P_2 + P_3$$

Thus *for a parallel combination of appliances, the effective power is equal to the sum of the powers of the individual appliances.*

If N bulbs, each of power P, are connected in parallel, then

$$P_{eff} = NP$$

The brightness of the three bulbs will be

$$P_1 = \frac{V^2}{R_1},\ P_2 = \frac{V^2}{R_2},\ P_3 = \frac{V^2}{R_3}.$$

As the resistance of the highest wattage (power) bulb is minimum, it will glow with maximum brightness. If the current in the circuit exceeds the safety limit, the bulb with maximum wattage will be fused first. For this reason, the appliances in houses are connected in parallel.

3.28 EFFICIENCY OF A SOURCE OF EMF

49. *Define efficiency of a source of emf. Write an expression for it.*

Efficiency of a source of emf. *The efficiency of a source of emf is defined as the ratio of the output power to the input power.* Suppose a source of emf $\mathcal{E}$ and internal resistance r is connected to an external resistance R. Then its efficiency will be

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{VI}{\mathcal{E}I} = \frac{V}{\mathcal{E}} = \frac{IR}{I(R+r)}$$

or
$$\eta = \frac{R}{R+r}$$

50. (*a*) *A battery of emf $\mathcal{E}$ and internal resistance r is connected across a pure resistive device (e.g., an electric heater or an electric bulb) of resistance R. Show that the power output of the device is maximum when there is a perfect 'matching' between the external resistance and the source resistance (i.e., where $R = r$). Determine the maximum power output.*

(*b*) *What is power output of the source above if the battery is shorted ? What is the power dissipation inside the battery in that case ?* [NCERT]

Maximum power theorem. *It states that the output power of a source of emf is maximum when the external resistance in the circuit is equal to the internal resistance of the source.*

Let emf of the battery $= \mathcal{E}$

Internal resistance $= r$

Resistance of the device $= R$

$\therefore$ Current through the device,

$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$= \frac{\mathcal{E}}{R+r}$$

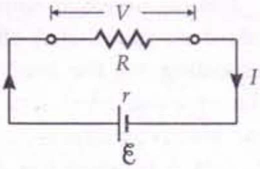


Fig. 3.129

$\therefore$ Power output of the resistive device will be

$$P = I^2 R = \left(\frac{\mathcal{E}}{R+r}\right)^2 R$$

$$= \frac{\mathcal{E}^2 R}{(R+r)^2} = \frac{\mathcal{E}^2 R}{(R-r)^2 + 4Rr} \quad ...(i)$$

Obviously, the power output will be maximum when

$$R - r = 0 \quad \text{or} \quad R = r$$

Thus, the power output of the device is maximum when there is a perfect matching between the external resistance and the resistance of the source, *i.e.*, when $R = r$. This proves maximum power theorem.

Maximum power output of the source is

$$P_{max} = \frac{\mathcal{E}^2 r}{(r+r)^2} = \frac{\mathcal{E}^2}{4r} \quad \text{[Putting } R = r \text{ in Eq. } (i)]$$

(*b*) When the battery is shorted, R becomes zero, therefore, power output = 0. In this case, entire power of the battery is dissipated as heat inside the battery due to its internal resistance.

Power dissipation inside the battery

$$= I^2 r = \left(\frac{\mathcal{E}}{r}\right)^2 r = \frac{\mathcal{E}^2}{r}.$$

51. *Show that the efficiency of a battery when delivering maximum power is only 50%.*

Maximum efficiency of a source of emf. For a source of emf,

Input power $= \mathcal{E} I$

Output power $= VI$

$$\therefore \text{Efficiency} \quad \eta = \frac{VI}{\mathcal{E}I} = \frac{V}{\mathcal{E}} = \frac{IR}{I(R+r)} = \frac{R}{R+r}$$

When the source delivers maximum power, $R = r$

$$\therefore \quad \eta = \frac{r}{r+r} = \frac{1}{2} = 50\%$$

Thus the efficiency of a source of emf is just 50% when it is delivering maximum power.

3.29 EFFICIENCY OF AN ELECTRIC DEVICE

52. *Define efficiency of an electric device. Write an expression for the efficiency of an electric motor.*

Efficiency of an electric device. *The efficiency of an electric device is defined as the ratio of the output power to the input power*

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

For an electric motor, we can write

$$\eta = \frac{\text{Output mechanical power}}{\text{Input electric power}}$$

Here, input electrical power

= Output mechanical power + Power lost as heat

53. (*a*) *An electric motor runs on a d.c. source of emf $\mathcal{E}$ and internal resistance r. Show that the power output of the source is maximum when the current drawn by the motor is $\mathcal{E}/2r$.*

(*b*) *Show that power output of electric motor is maximum when the back emf is one-half the source emf, provided the resistance of the windings of the motor is negligible.*

(*c*) *Compare and contrast carefully the situation in this exercise with that in Q.50(a) above.* [NCERT]

(*a*) **Output power from a source connected to an electric motor.** Let the current drawn by the motor be I. Then

Power output of the source, $P = \mathcal{E}I - I^2 r$

P is maximum when $\frac{dP}{dI} = 0$

or $\quad \mathcal{E} - 2Ir = 0 \quad$ or $\quad I = \frac{\mathcal{E}}{2r}$

Hence *the power output of the source is maximum when the current drawn by the motor is $\mathcal{E}/2r$.*

(*b*) Here, emf of source $= \mathcal{E}$

Internal resistance of source $= r$

Back emf of motor $= \mathcal{E}'$

Resistance of motor $= R \simeq 0$

As the external resistance R is negligible, therefore current in the circuit $= \frac{\mathcal{E} - \mathcal{E}'}{r}$.

And power output of the motor

$$= \text{Power output of the source}$$
$$= \mathcal{E}I - I^2 r$$

From part (*a*), this is maximum when

$$I = \frac{\mathcal{E}}{2r} \quad \text{or} \quad \frac{\mathcal{E} - \mathcal{E}'}{r} = \frac{\mathcal{E}}{2r} \quad \text{or} \quad \mathcal{E}' = \frac{\mathcal{E}}{2}$$

Hence *the power output of electric motor is maximum when the back emf is one-half the source emf.*

(*c*) The condition in Q. 50(*a*) is for a passive resistor in which the entire electric energy is converted into heat while the condition in Q. 53(*a*) is for a non-passive resistor (*e.g.*, electric motor) in which the supplied electric energy changes partly into heat and partly into mechanical work. So the former is a special case of the latter.

3.30 APPLICATIONS OF HEATING EFFECT OF CURRENT

54. *Discuss some practical applications of the heating effect of current.*

Applications of heating effect of current. Some of the important applications of Joule heating are as follows :

1. ***Household heating appliances.*** Many electrical appliances used in daily life are based on the heating effect of current such as room heater, electric toaster, electric iron, electric oven, electric kettle, geyser, etc. The designing of these devices requires the selection of a proper resistor. The resistor should have high resistance so that most of the electric power is converted into heat. In most of the household heating appliances, *nichrome element* is used because of the following reasons :

(*i*) Its melting point is high
(*ii*) Its resistivity is large
(*iii*) It is tensile, *i.e.*, it can be easily drawn into wires.
(*iv*) It is not easily oxidised by the oxygen of the air when heated.

2. ***Incandescent electric bulb.*** It is an important application of Joule heating in producing light. It consists of a filament of fine metallic wire enclosed in a glass bulb filled with chemically inactive gases like nitrogen and argon. The filament material should have high resistivity and high melting point. Therefore, tungsten (melting point 3380°C) is used for bulb filament. When current is passed through the filament, it gets heated to a high temperature and emits light. Most of the power consumed by the filament is converted into heat and only a small part of it appears as light. A bulb gives nearly 1 candela of light energy for the consumption of every watt of electric power.

3. ***Electric fuse.*** It is a safety device used to protect electrical appliances from strong currents. A fuse wire must have high resistivity and low melting point. It is usually made from an alloy of tin (63%) and lead (37%). It is put in series with the live wire of the circuit. When the current exceeds the safety limit, the fuse wire melts and breaks the circuit. The electric installations are thus saved from getting damaged.

The fuse wire of suitable current rating (1 A, 2 A, 3 A, 5 A, 10 A etc.) should be used in the circuit depending on the load in the circuit. For example, when we use an electric iron of 1 kW electric power with electric mains of 220 V, a current of (1000/220) A *i.e.*, 4.54 A flows in the circuit. This requires a fuse of 5 A rating.

4. ***Electric arc.*** It consists of two carbon rods with a small gap between their pointed ends. When a high potential difference of 40 – 60 V is applied between the two rods, very intense light is emitted by the gap. We know that $E = -dV/dr$. Clearly, E will be large if the gap is small. When the electric field exceeds the dielectric strength of air, ionisation of air occurs. This causes a big spark to pass across the gap.

5. ***Other devices.*** Many other devices are based on the heating effect of current such as electric welding, thermionic valves, hotwire ammeters and voltmeters.

55. *Explain why is electric power transmitted at high voltages and low currents to distant places.*

High voltage power transmission. Electric power is transmitted from power stations to homes and factories through transmission cables. These cables have resistance. Power is wasted in them as heat. Let us see how can we minimise this power loss.

Suppose power P is delivered to a load R via transmission cables of resistance R_t. If V is the voltage across load R and I the current through it, then

$$P = VI$$

The power wasted in transmission cables is

$$P_t = I^2 R_t = \frac{P^2 R_t}{V^2}$$

Thus the power wasted in the transmission cables is inversely proportional to the square of voltage. Hence to minimise the power loss, electric power is transmitted to distant places at high voltages and low currents. These voltages are stepped down by transformers before supplying to homes and factories.

For Your Knowledge

- The emission of light by a substance when heated to a high temperature is called *incandescence*.
- A heater wire is made from a material of large resistivity and high melting point while a fuse wire is made from a material of large resistivity and low melting point.
- The load in an electric circuit refers to the current drawn by the circuit from the supply line. If the current in a circuit exceeds the safe value, we say that the circuit is overloaded.
- The temperature upto which a wire gets heated (*i.e.*, steady state temperature θ) is directly proportional to the square of the current and is inversely proportional to the cube of its radius but is independent of its length.

$$\theta \propto \frac{I^2}{r^3}$$

- When the resistances are connected in *series*, the current I through each resistance is same. Consequently,

$$P \propto R \qquad (\because P = I^2R)$$
and
$$V \propto R \qquad (\because V = IR)$$

Hence in a *series* combination of resistances, the potential difference, power consumed and hence heat produced will be *larger in the higher resistance*.

- When the resistances are connected in parallel, the potential difference V is same across each resistance. Consequently,

$$P \propto \frac{1}{R} \qquad \left(\because P = \frac{V^2}{R}\right)$$
and
$$I \propto \frac{1}{R} \qquad \left(\because I = \frac{V}{R}\right)$$

Hence in a *parallel* combination of resistances, the current, power consumed and hence heat produced will be *larger in the smaller resistance*.

Examples based on Heating Effect of Current, Electric Power and Electric Energy

Formulae Used

1. Heat produced by electric current,
$$H = I^2Rt \text{ joule} = \frac{I^2Rt}{4.18} \text{ cal}$$
or $$H = VIt \text{ joule} = \frac{VIt}{4.18} \text{ cal}$$
2. Electric power, $P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$
3. Electric energy, $W = Pt = VIt = I^2Rt$

Units Used

Current I is in ampere, resistance R in ohm, time t in second, power P in watt, electric energy in joule or in kWh.

Example 99. *An electric current of 4.0 A flows through a 12 Ω resistor. What is the rate at which heat energy is produced in the resistor ?* [NCERT]

Solution. Here $I = 4$ A, $R = 12\ \Omega$

Rate of production of heat energy,
$$P = I^2R = 4^2 \times 12 = \mathbf{192\ W.}$$

Example 100. *How many electrons flow through the filament of a 120 V and 60 W electric lamp per second ? Given $e = 1.6 \times 10^{-19}$ C.*

Solution. Here $P = 60$ W, $V = 120$ V, $t = 1$ s
$$I = \frac{P}{V} = \frac{60}{120} = 0.5 \text{ A}$$
But $$I = \frac{q}{t} = \frac{ne}{t}$$
∴ No. of electrons flowing per second is
$$n = \frac{It}{e} = \frac{0.5 \times 1}{1.6 \times 10^{-19}} = \mathbf{3.125 \times 10^{18}}.$$

Example 101. *A heating element is marked 210 V, 630 W. What is the current drawn by the element when connected to a 210 V d.c. mains ? What is the resistance of the element ?* [NCERT]

Solution. Here $P = 630$ W, $V = 210$ V

Current drawn, $I = \frac{P}{V} = \frac{630}{210} = \mathbf{3\ A.}$

Resistance of the element, $R = \frac{V}{I} = \frac{210}{3} = \mathbf{70\ \Omega.}$

Example 102. *A 10 V storage battery of negligible internal resistance is connected across a 50 Ω resistor made of alloy manganin. How much heat energy is produced in the resistor in 1 h ? What is the source of this energy ?* [NCERT]

Solution. Here $V = 10$ V, $R = 50\ \Omega$, $t = 1$ h $= 3600$ s

Heat energy produced in 1 h is
$$H = \frac{V^2t}{R} = \frac{10 \times 10 \times 3600}{50} = \mathbf{7200\ J.}$$

The source of this energy is the chemical energy stored in the battery.

Example 103. *An electric motor operates on a 50 V supply and draws a current of 12 A. If the motor yields a mechanical power of 150 W, what is the percentage efficiency of the motor ?* [NCERT]

Solution. Input power $= VI = 50 \times 12 = 600$ W

Output power $= 150$ W

Efficiency of motor
$$= \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{150 \times 100}{600}$$
$$= \mathbf{25\%}.$$

Example 104. *An electric motor operating on a 50 V d.c. supply draws a current of 12 A. If the efficiency of the motor is 30%, estimate the resistance of the windings of the motor.* [NCERT]

Solution. Here $V = 50$ V, $I = 12$ A, $\eta = 30\%$

As the efficiency of electric motor is 30%, therefore, power dissipated as heat is

$$P = 70\% \text{ of } VI = \frac{70}{100} \times 50 \times 12 \text{ W} = 420 \text{ W}$$

But power dissipated as heat, $P = I^2 R$

$\therefore \quad I^2 R = 420$

or $\quad R = \frac{420}{I^2} = \frac{420}{144} = \mathbf{2.9\ \Omega}.$

Example 105. *(a) A nichrome heating element across 230 V supply consumes 1.5 kW of power and heats up to a temperature of 750°C. A tungsten bulb across the same supply operates at a much higher temperature of 1600°C in order to be able to emit light. Does it mean that the tungsten bulb necessarily consumes greater power ? (b) Which of the two has greater resistance : a 1 kW heater or a 100 W tungsten bulb, both marked for 230 V ?* [NCERT]

Solution. (*a*) No, the steady temperature acquired by a resistor depends not only on the power consumed but also its characteristics such as surface area, emissivity, etc., which determine its power loss due to radiation.

(*b*) Here $V = 230$ V, $P_1 = 1$ kW $= 1000$ W, $P_2 = 100$ W

$$R_1 = \frac{V^2}{P_1} = \frac{230 \times 230}{1000}\ \Omega = \mathbf{52.9\ \Omega}$$

$$R_2 = \frac{V^2}{P_2} = \frac{230 \times 230}{100}\ \Omega = \mathbf{529\ \Omega}$$

Thus the 100 W bulb has a greater resistance.

Example 106. *An electric power station (100 MW) transmits power to a distant load through long and thin cables. Which of the two modes of transmission would result in lesser power wastage : power transmission of : (i) 20,000 V or (ii) 200 V ?* [NCERT]

Solution. Let R be the resistance of transmission cables.

Here $\quad P = 100 \text{ MW} = 100 \times 10^6$ W

(*i*) $V_1 = 20{,}000$ V

$$\therefore \text{ Current, } I_1 = \frac{P}{V_1} = \frac{100 \times 10^6}{20{,}000} = 5000 \text{ A}$$

Rate of heat dissipation at 20,000 V is

$$P_1 = I_1^2 R = (5000)^2 R = 25 \times 10^6 R \text{ watt.}$$

(*ii*) $V_2 = 200$ V

$$\therefore \text{ Current, } I_2 = \frac{100 \times 10^6}{200} = 5 \times 10^5 \text{ A}$$

Rate of heat dissipation at 200 V is

$$P_2 = I_2^2 R = (5 \times 10^5)^2 R = 25 \times 10^{10} R \text{ watt}$$

Clearly, $P_1 < P_2$

Hence there will be lesser power wastage when the power is transmitted at **20,000 V.**

Example 107. *Two ribbons are given with the following particulars :*

Ribbon	*A*	*B*
Alloy	Constantan	Nichrome
Length (m)	8.456	4.235
Width (mm)	1.0	2.0
Thickness (mm)	0.03	0.06
Temp. coefficient of resistivity (°C^{-1})	Negligible	Negligible
Resistivity (Ωm)	4.9×10^{-7}	1.1×10^{-6}

For a fixed voltage supply, which of the two ribbons corresponds to a greater rate of heat production ? [NCERT]

Solution. Since $R = \rho \frac{l}{A}$

$\therefore$ Resistance of constantan ribbon,

$$R_1 = \frac{4.9 \times 10^{-7} \times 8.456}{1.0 \times 10^{-3} \times 0.03 \times 10^{-3}}\ \Omega = 138.1\ \Omega$$

Let V be the fixed supply voltage. Then the rate of production of heat in constantan ribbon,

$$P_1 = \frac{V^2}{R_1} = \frac{V^2}{138.1} \text{ watt}$$

Resistance of nichrome ribbon,

$$R_2 = \frac{1.1 \times 10^{-6} \times 4.235}{2.0 \times 10^{-3} \times 0.06 \times 10^{-3}}\ \Omega = 38.8\ \Omega$$

Rate of production of heat in nichrome ribbon,

$$P_2 = \frac{V^2}{R_2} = \frac{V^2}{38.8} \text{ watt}$$

Clearly ***nichrome ribbon*** has greater rate of production of heat because of its lesser resistance.

Example 108. *A heater coil is rated 100 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel, to the same source of 200 V. Calculate the energy liberated per second in the new combination.* [CBSE OD 2000]

Solution. Resistance of heater coil,

$$R = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400\ \Omega$$

Resistance of either half part $= 200\ \Omega$

Equivalent resistance when both parts are connected in parallel,

$$R' = \frac{200 \times 200}{200 + 200} = 100\ \Omega$$

Energy liberated per second when combination is connected to a source of 200 V

$$= \frac{V^2}{R'} = \frac{200 \times 200}{100} = \mathbf{400\ J.}$$

Example 109. *An electric bulb is marked 100 W, 230 V. If the supply voltage drops to 115 V, what is the heat and light energy produced by the bulb in 20 min? Calculate the current flowing through it.* [NCERT ; CBSE F 94]

Solution. If the resistance of the bulb be *R*, then

Rate of production of heat and light energy,

$$P = \frac{V^2}{R}$$

$$\therefore \quad R = \frac{V^2}{P} = \frac{230 \times 230}{100} = 529\ \Omega$$

When the voltage drops to $V' = 115$ V, the total heat and light energy produced by the bulb in 20 min will be

$$H = P \times t = \frac{V'^2}{R} \times t$$

$$= \frac{115 \times 115}{529} \times 20 \times 60 = 30{,}000\ \text{J} = \mathbf{30\ kJ.}$$

Current, $I = \frac{V'}{R} = \frac{115}{529} = \mathbf{\frac{5}{23}\ A.}$

Example 110. *An electric bulb rated for 500 W at 100 V is used in circuit having a 200 V supply. Calculate the resistance R that must be put in series with the bulb, so that the bulb delivers 500 W.* [IIT 87]

Solution. Resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{100 \times 100}{500} = 20\ \Omega$$

Current through the bulb,

$$I = \frac{V}{R} = \frac{100}{20} = 5\ \text{A}$$

For the same power dissipation, the current through bulb must be 5 A.

When the bulb is connected to 200 V supply, the safe resistance of the circuit should be

$$R' = \frac{V'}{I} = \frac{200}{5} = 40\ \Omega$$

$\therefore$ Resistance required to be put in series with the bulb is

$$R' - R = 40 - 20 = \mathbf{20\ \Omega.}$$

Example 111. *The maximum power rating of a 20 Ω resistor is 2.0 kW. (That is, this is the maximum power the resistor can dissipate (as heat) without melting or changing in some other undesirable way). Would you connect this resistor directly across a 300 V d.c. source of negligible internal resistance? Explain your answer.* [Haryana 97C ; NCERT]

Solution. Maximum power rating of the given 20 Ω resistor,

$$P' = 2.0\ \text{kW}$$

When connected to 300 V d.c. supply, the power consumption or rate of production of heat would be

$$P = \frac{V^2}{R} = \frac{300 \times 300}{20}\ \text{W} = 4500\ \text{W} = 4.5\ \text{kW}$$

This power consumption exceeds the maximum power rating of the resistor. Hence the 20 Ω resistor must not be connected directly across the 300 V d.c. source. For doing so, a small resistance of 10 Ω should be connected in series with it.

Example 112. *An electric heater and an electric bulb are rated 500 W, 220 V and 100 W, 220 V respectively. Both are connected in series to a 220 V d.c. mains. Calculate the power consumed by (i) the heater and (ii) electric bulb.* [CBSE D 97]

Solution. Resistances of heater and bulb are

$$R_1 = \frac{V^2}{P_1} = \frac{220 \times 220}{500} = \frac{484}{5} = 96.8\ \Omega$$

$$R_2 = \frac{V^2}{P_2} = \frac{220 \times 220}{100} = 484\ \Omega$$

Total resistance of series combination is

$$R_1 + R_2 = 96.8 + 484 = 580.8\ \Omega$$

Current, $I = \frac{V}{R} = \frac{220}{580.8} \simeq 0.38\ \text{A}$

(*i*) Power consumed by heater is

$$P_1 = I^2 R_1 = 0.38^2 \times 96.8 = \mathbf{13.8\ W.}$$

(*ii*) Power consumed by bulb,

$$P_2 = I^2 R_2 = 0.38^2 \times 484 = \mathbf{69.89\ W.}$$

Example 113. *Two heaters are marked 200 V, 300 W and 200 V, 600 W. If the heaters are combined in series and the combination connected to a 200 V d.c. supply, which heater will produce more heat?* [NCERT]

Solution. Resistances of the two heaters are

$$R_1 = \frac{V^2}{P_1} = \frac{200 \times 200}{300} = \frac{400}{3}\ \Omega$$

$$R_2 = \frac{V^2}{P_2} = \frac{200 \times 200}{600} = \frac{200}{3}\ \Omega$$

For series combination,

$$R_1 + R_2 = \frac{600}{3} = 200\,\Omega$$

$\therefore$ Current, $I = \frac{V}{R} = \frac{200}{200} = 1$ A

Power dissipations in the two heaters are

$$P_1' = I^2 R_1 = 1^2 \times \frac{400}{3} = \frac{400}{3}\text{ W}$$

$$P_2' = I^2 R_2 = 1^2 \times \frac{200}{3} = \frac{200}{3}\text{ W}$$

$\therefore \quad P_1' = 2P_2'$

The first heater (of 300 W) produces more heat than the second heater.

Example 114. *In a part of the circuit shown in the Fig. 3.130, the rate of heat dissipation in 4 Ω resistor is 100 J / s. Calculate the heat dissipated in the 3 Ω resistor in 10 seconds.* [CBSE Sample Paper 03]

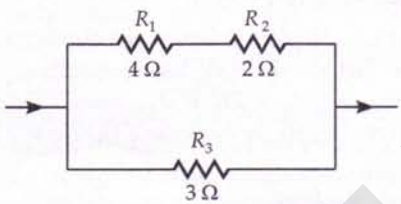


Fig. 3.130

Solution. Let I_1 be the current through the series combination of R_1 and R_2 and I_2 be the current through R_3.

P.D. across $(R_1 + R_2)$ = P.D. across R_3

$\therefore \quad (4 + 2)\,I_1 = 3\,I_2$ or $I_2 = 2\,I_1$

Rate of heat dissipation in 4 Ω resistor

$$= I_1^2 R_1 = I_1^2 \times 4 = 100\text{ Js}^{-1}$$

$$\therefore \quad I_1 = \sqrt{\frac{100}{4}} = \sqrt{25} = 5\text{ A}$$

and $\quad I_2 = 2\,I_1 = 10$ A

Heat dissipated in 3 Ω resistor in 10 s

$$= I_2^2 R_2 t = (10)^2 \times 3 \times 10 = \mathbf{3000\ J.}$$

Example 114. *The resistance of each of the three wires, shown in Fig. 3.131, is 4 Ω. This combination of resistors is connected to a source of emf ε. The ammeter shows a reading of 1 A. Calculate the power dissipated in the circuit.* [CBSE F 03]

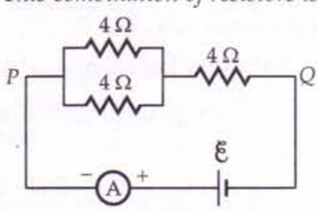


Fig. 3.131

Solution. Total resistance between the points P and Q,

$$R = \frac{4 \times 4}{4 + 4} + 4 = 2 + 4 = 6\,\Omega$$

Current in the circuit, $I = 1$ A

Power dissipated in the circuit,

$$P = I^2 R = 1^2 \times 6 = \mathbf{6\ W.}$$

Example 116. *A house is fitted with 20 lamps of 60 W each, 10 fans consuming 0.5 A each and an electric kettle of resistance 110 Ω. If the energy is supplied at 220 V and costs 75 paise per unit, calculate the monthly bill for running appliances for 6 hours a day. Take 1 month = 30 days.*

Solution. Power of 20 lamps of 60 W each

$$= 20 \times 60 = 1200\text{ W}$$

Power consumed by 10 fans at 0.5 A current

$$= 10 \times VI = 10 \times 220 \times 0.5 = 1100\text{ W}$$

Power consumed by electric kettle of 110 Ω resistance

$$= \frac{V^2}{R} = \frac{220 \times 220}{110} = 440\text{ W}$$

Total power of the appliances

$$= 1200 + 1100 + 440 = 2740\text{ W} = 2.74\text{ kW}$$

Total time for which appliances are used

$$= 6 \times 30 = 180\text{ h}$$

Total energy consumed

$$= P\,.\,t = 2.74\text{ kW} \times 180\text{ h}$$

$$= 493.2\text{ kWh or units}$$

$\therefore$ Monthly bill = 493.2 × 0.75 = **₹369.90.**

Example 117. *There are two electric bulbs rated 60 W, 110 V and 100 W, 110 V. They are connected in series with a 220 V d.c. supply. Will any bulb fuse ? What will happen if they are connected in parallel with the same supply ?*

Solution. Currents required by the two bulbs for the normal glowness are

$$I_1 = \frac{P_1}{V} = \frac{60}{110} = 0.55\text{ A}$$

and $$I_2 = \frac{P_2}{V} = \frac{100}{110} = 0.91\text{ A}$$

The resistances of the two bulbs are

$$R_1 = \frac{V}{I_1} = \frac{110}{0.55} = 202\,\Omega$$

and $$R_2 = \frac{V}{I_2} = \frac{110}{0.91} = 121\,\Omega$$

When the bulbs are connected in series across the 220 V supply, the current through each bulb will be

$$I = \frac{V}{R_1 + R_2} = \frac{220}{202 + 121} = 0.68\text{ A}$$

As $I_1 < I$ and $I_2 > I$, so that 60 W bulb will fuse while the 100 W bulb will light up dim.

When the bulbs are joined in parallel, their equivalent resistance is

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{202 \times 121}{202 + 121} = 76\,\Omega$$

Current drawn from the 220 V supply will be

$$I' = \frac{V}{R'} = \frac{220}{76} \simeq 3\text{ A}$$

In the two bulbs of resistances $R_1\,(\simeq 202\,\Omega)$ and $R_2\,(=120\,\Omega)$, the current of 3 A will split up into roughly 1 A and 2 A respectively. Hence both the bulbs will fuse.

Example 118. *The resistance of a 240 V and 200 W electric bulb when hot is 10 times the resistance when cold. Find its resistance at room temperature. If the working temperature of the filament is 2000°C, find the temperature coefficient of the filament.*

Solution. Resistance of the hot bulb is given by

$$R' = \frac{V^2}{P} = \frac{240 \times 240}{200} = 288\,\Omega$$

Resistance of bulb at room temperature,

$$R = \frac{R'}{10} = \frac{288}{10} = \mathbf{28.8\,\Omega}$$

Since $R' = R(1 + \alpha t)$

$\therefore$ $288 = 28.8(1 + \alpha \times 2000)$

or $\alpha = \frac{9}{2000}\,°C^{-1} = \mathbf{4.5 \times 10^{-3}\,°C^{-1}}.$

Example 119. *A thin metallic wire of resistance 100 Ω is immersed in a calorimeter containing 250 g of water at 10°C and a current of 0.5 ampere is passed through it for half an hour. If the water equivalent of the calorimeter is 10 g, find the rise of temperature.*

Solution. Here $m = 250$ g, $I = 0.5$ A,

$t = 30$ min $= 1800$ s, $w = 10$ g

$\therefore$ Heat produced

$= I^2 Rt = (0.5)^2 \times 100 \times 1800\text{ J} = 45000\text{ J}$

Heat gained by water and calorimeter

$= (m + w)\,c\,\theta = (250 + 10) \times 1 \times \theta$ cal

$= 260 \times 4.2\ \theta$ joule

$\therefore$ $260 \times 4.2 \times \theta = 45000$

Rise in temperature, $\theta = \frac{45000}{260 \times 4.2} = \mathbf{41.2°C}.$

Example 120. *A copper electric kettle weighing 1000 g contains 900 g of water at 20°C. It takes 12 minutes to raise the temperature to 100°C. If electric energy is supplied at 210 V, calculate the strength of the current, assuming that 10% heat is wasted. Specific heat of copper is 0.1.*

Solution. Water equivalent of copper kettle is

$$w = \text{Mass} \times \text{Specific heat} = 1000 \times 0.1 = 100\text{ g}$$

Also $m = 900$ g,

$$\theta = \theta_2 - \theta_1 = 100 - 20 = 80°C$$

Heat required,

$$H = (m + w)\,c\,\theta = (900 + 100) \times 1 \times 80 = 80{,}000\text{ cal}$$

Heat produced

$$= \frac{VIt}{4.2} = \frac{210 \times I \times 12 \times 60}{4.2}\text{ cal} = 36000\,I\text{ cal}$$

Useful heat

$$= 90\%\text{ of }36000\,I$$

$$= \frac{90 \times 36000\,I}{100} = 32400\,I\text{ cal}$$

$\therefore$ $32400\,I = 80{,}000$

Current, $I = \frac{80000}{32400} = \mathbf{2.469\ A.}$

Example 121. *A coil of enamelled copper wire of resistance 50 Ω is embedded in a block of ice and a potential difference of 210 V applied across it. Calculate the rate at which ice melts. Latent heat of ice is 80 cal per gram.*

Solution. Here $R = 50\,\Omega$, $V = 210$ V, $t = 1$ s,

$L = 80$ cal g^{-1}

Heat produced,

$$H = \frac{V^2 t}{4.2\,R} = \frac{210 \times 210 \times 1}{4.2 \times 50} = 210\text{ cal}$$

Suppose m gram of ice melts per second. Then

$mL = H$

or $m = \frac{H}{L} = \frac{210}{80} = \mathbf{2.62\ g\ s^{-1}}.$

Example 122. *An electric kettle has two heating coils, when one of the coils is switched on, the kettle begins to boil in 6 minutes and when the other is switched on, the boiling begins in 8 minutes. In what time will the boiling begin if both the coils are switched on simultaneously (i) in series and (ii) in parallel ?* [IIT]

Solution. Let R_1 and R_2 be the resistances of the two coils, V the supply voltage and H, the heat required to boil the water.

For the first coil, $H = \frac{V^2 t_1}{JR_1} = \frac{V^2 \times 6 \times 60}{4.2\,R_1}$ cal

For the second coil, $H = \frac{V^2 t_2}{JR_2} = \frac{V^2 \times 8 \times 60}{4.2\,R_2}$ cal

$$\therefore \quad \frac{V^2 \times 6 \times 60}{4.2\,R_1} = \frac{V^2 \times 8 \times 60}{4.2\,R_2}$$

or $\frac{R_2}{R_1} = \frac{8}{6} = \frac{4}{3}$

(*i*) When the coils are connected in series,

effective resistance = $R_1 + R_2$.

Let the boiling occur in time t_1 min.

Then

$$\frac{V^2 t_1 \times 60}{4.2(R_1 + R_2)} = H = \frac{V^2 \times 6 \times 60}{4.2\ R_1}$$

or $$t_1 = 6\left(\frac{R_1 + R_2}{R_1}\right) = 6\left(1 + \frac{R_2}{R_1}\right)$$

$$= 6\left(1 + \frac{4}{3}\right) \text{min} = \mathbf{14\ min}.$$

(*ii*) When the two coils are connected in parallel,

$$\text{effective resistance} = \frac{R_1 R_2}{R_1 + R_2}$$

Let the boiling occur in time t_2 min. Then

$$\frac{V^2 t_2 \times 60}{4.2\left(\frac{R_1 R_2}{R_1 + R_2}\right)} = H = \frac{V^2 \times 6 \times 60}{4.2\ R_1}$$

or $$t_2 = 6 \times \frac{R_1 R_2}{(R_1 + R_2) R_1} = 6 \times \frac{1}{\left(1 + \frac{R_1}{R_2}\right)}$$

$$= 6 \times \frac{1}{\left(1 + \frac{3}{4}\right)} \text{min} = \mathbf{3.43\ min.}$$

Example 123. *The heater coil of an electric kettle is rated at 2000 W, 200 V. How much time will it take in raising the temperature of 1 litre of water from 20°C to 100°C, assuming that only 80% of the total heat energy produced by the heater coil is used in raising the temperature of water. Density of water = 1 g cm^{-3} and specific heat of water = 1 cal g^{-1} °C^{-1}.*

Solution. Here $P = 2000$ W,

Volume of water = 1 litre = 1000 cm^3

Mass of water,

m = Volume × density

$= 1000\ \text{cm}^3 \times 1\ \text{g cm}^{-3} = 1000\ \text{g}$

Rise in temperature,

$\theta = \theta_2 - \theta_1 = 100 - 20 = 80°\text{C}$

Heat gained by water

$= mc\,\theta = 1000 \times 1 \times 80 = 80{,}000$ cal

Let t be the time taken to increase the temperature from 20° to 100°C.

Then total heat produced by heating coil

$= Pt = 2000\ t$ joule

Useful heat produced

$$= 80\%\ 2000\ t = \frac{80 \times 2000\ t}{100}\ \text{J}$$

$$= \frac{80 \times 2000\ t}{100 \times 4.2}\ \text{cal}$$

Useful heat produced = Heat gained by water

$$\frac{80 \times 2000\ t}{100 \times 4.2} = 80000$$

or $$t = \frac{80000 \times 100 \times 4.2}{80 \times 2000} = \mathbf{210\ s.}$$

Example 124. *One kilowatt electric heater is to be used with 220 V d.c. supply. (i) What is the current in the heater? (ii) What is its resistance? (iii) What is the power dissipated in the heater? (iv) How much heat in calories is produced per second? (v) How many grams of water at 100°C will be converted per minute into steam at 100°C, with the heater? Assume that the heat losses due to radiation are negligible. Latent heat of steam = 540 cal per gram.* [IIT]

Solution. Here $P = 1$ kW $= 1000$ W, $V = 220$ V

(*i*) Current, $I = \frac{P}{V} = \frac{1000}{220} = \mathbf{4.55\ A.}$

(*ii*) Resistance, $R = \frac{V^2}{P} = \frac{220 \times 220}{1000} = \mathbf{48.4\ \Omega.}$

(*iii*) Power dissipated in heater = **1000 W.**

(*iv*) Heat produced per second,

$$H = \frac{VIt}{J} = \frac{P.t}{J} = \frac{1000 \times 1}{4.2} = \mathbf{240\ cal\ s^{-1}}.$$

(*v*) Heat produced per minute,

$H = 240 \times 60 = 14400$ cal

We know that 540 cal of heat convert 1 g water at 100°C into steam at 100°C.

∴ Mass of water converted into steam

$$= \frac{14400}{540} = \mathbf{26.67\ g.}$$

Example 125. *The walls of a closed cubical box of edge 50 cm are made of a material of thickness 1 mm and thermal conductivity 4×10^{-4} cal s^{-1} cm^{-1} °C^{-1}. The interior of the box maintained at 100°C above the outside temperature by a heater placed inside the box and connected across a 400 V d.c. source. Calculate the resistance of the heater.* [IIT]

Solution. Here, $K = 4 \times 10^{-4}\ \text{cal s}^{-1}\text{cm}^{-1}\,°\text{C}^{-1}$,

$\theta_2 - \theta_1 = 100°\text{C}$, $d = 1\ \text{mm} = 0.1\ \text{cm}$

Surface area of the six faces of the cubical box,

$A = 6 \times (50 \times 50) = 15000\ \text{cm}^2$

The amount of heat conducted out per second through the walls of the cubical box is

$$H_1 = \frac{KA(\theta_2 - \theta_1)}{d} = \frac{4 \times 10^{-4} \times 15000 \times 100}{0.1}$$

$$= 6000 \text{ cal} = 6000 \times 4.2 \text{ J}$$

If R is the resistance of the heater, then heat produced per second

$$H_2 = I^2 Rt = \frac{V^2}{R} = \frac{(400)^2}{R} \qquad [t = 1\text{s}]$$

Temperature inside the box will be maintained by the heater if

$$H_1 = H_2 \quad \text{or} \quad \frac{(400)^2}{R} = 6000 \times 4.2$$

or $$R = \frac{400 \times 400}{6000 \times 4.2} = \mathbf{6.35\ \Omega}$$

Example 126. *A 10 V battery of negligible internal resistance is charged by a 200 V d.c. supply. If the resistance in the charging circuit is 38 Ω, what is the value of charging current ?* [NCERT]

Solution. As the battery emf opposes the charging emf, therefore,

net emf $= 200 - 10 = 190$ V

Charging current,

$$I = \frac{\text{Net emf}}{\text{Resistance}} = \frac{200 - 10}{38} = \mathbf{5\ A.}$$

Example 127. *A dry cell of emf 1.6 V and internal resistance 0.10 ohm is connected to a resistor of resistance R ohm. If the current drawn from the cell is 2 A, then*

(*i*) *what is the voltage drop across R ?*

(*ii*) *what is the energy dissipation in the resistor ?*

Solution. Here $\mathcal{E} = 1.6$ V, $r = 0.10\ \Omega$, $I = 2.0$ A

$$R + r = \frac{\mathcal{E}}{I} = \frac{1.6}{2.0} = 0.8\ \Omega$$

$$R = 0.8 - 0.10 = 0.70\ \Omega$$

(*i*) Voltage drop across R,

$$V = IR = 2 \times 0.70 = \mathbf{1.4\ V.}$$

(*ii*) Rate of energy dissipation inside the resistor

$$= VI = 1.4 \times 2.0 = \mathbf{2.8\ W.}$$

Example 128. *A dry cell of emf 1.5 V and internal resistance 0.10 Ω is connected across a resistor in series with a very low resistance ammeter. When the circuit is switched on, the ammeter reading settles to a steady value of 2.0 A. What is the steady*

(*a*) *rate of chemical energy consumption of the cell,*

(*b*) *rate of energy dissipation inside the cell,*

(*c*) *rate of energy dissipation inside the resistor,*

(*d*) *power output of the source ?* [NCERT]

Solution. Here $\mathcal{E} = 1.5$ V, $r = 0.10\ \Omega$, $I = 2.0$ A

(*a*) Rate of chemical energy consumption of the cell

$$= \mathcal{E}I = 1.5 \text{ V} \times 2.0 \text{ A} = \mathbf{3.0\ W.}$$

(*b*) Rate of energy dissipation inside the cell

$$= I^2 r = (2)^2 \times 0.10 \text{ W} = \mathbf{0.40\ W.}$$

(*c*) Rate of energy dissipation inside the resistor

$$= \mathcal{E}I - I^2 r = 3.0 - 0.40 = \mathbf{2.6\ W.}$$

(*d*) Power output of the source

$$= \text{Power input to the external circuit}$$

$$= \mathcal{E}I - I^2 r = \mathbf{2.6\ W.}$$

Example 129. *A series battery of 10 lead accumulators, each of emf 2 V and internal resistance 0.25 ohm, is charged by a 220 V d.c. mains. To limit the charging current, a resistance of 47.5 ohm is used in series in the charging circuit. What is (a) the power supplied by the mains and (b) power dissipated as heat ? Account for the difference of power in (a) and (b).* [CBSE Sample Paper 98]

Solution. emf of the battery $= 10 \times 2 = 20$ V

Internal resistance of the battery

$$= 10 \times 0.25 = 2.5\ \Omega$$

Total resistance $= r + R = 2.5 + 47.5 = 50.0\ \Omega$

As the battery emf opposes the charging emf,

$\therefore$ Effective emf $= \mathcal{E} - V = 220 - 20 = 200$ V

$$\text{Charging current} = \frac{\text{Effective emf}}{\text{Total resistance}} = \frac{200}{50} = 4 \text{ A}$$

(*a*) Power supplied by the mains

$$= VI = 220 \times 4 = \mathbf{880\ W.}$$

(*b*) Power dissipated as heat

$$= I^2(R + r) = 4^2 \times 50 = \mathbf{800\ W.}$$

The difference of power $= 880 - 800 = 80$ W, is stored in the battery in the form of chemical energy.

Example 130. *A series battery of 6 lead accumulators each of emf 2.0 V and internal resistance 0.50 Ω is charged by a 100 V d.c. supply. What series resistance should be used in the charging circuit in order to limit the current to 8.0 A ? Using the required resistor, obtain (a) the power supplied by the d.c. source (b) the power supplied by the d.c. energy stored in the battery in 15 min.* [NCERT]

Solution. Here $\mathcal{E} = 2.0$ V, $r = 0.50\ \Omega$, $V = 100$ V, $I = 8.0$ A

As the battery emf opposes the charging emf,

$\therefore$ Effective emf $= 100 - 2.0 \times 6 = 88$ V

Let the required series resistance be of $R\ \Omega$. Then

total resistance $= (0.50 \times 6 + R)\ \Omega = (3 + R)\ \Omega$

Now $I = \dfrac{\text{Total emf}}{\text{Total resistance}}$

$\therefore \quad 8 = \dfrac{88}{3+R}$

or $\quad 24 + 8R = 88 \quad$ or $\quad R = \dfrac{64}{8}\,\Omega = 8\,\Omega.$

(*a*) Power supplied by d.c. source

$= VI = 100\text{ V} \times 8\text{ A} = \mathbf{800\ W}.$

(*b*) Power dissipated as heat

$= I^2(R + r) = 8^2(8 + 0.50 \times 6)\text{ W}$

$= 64 \times 11\text{ W} = \mathbf{704\ W}.$

(*c*) Power supplied by the d.c. energy stored in the battery in 15 min

$= (800 - 704)\text{W} \times 15\text{ min}$

$= 96\text{ W} \times 900\text{ s} = \mathbf{86400\ J}.$

Example 131. *Power from a 64 V d.c. supply goes to charge a battery of 8 lead accumulators each of emf 2.0 V and internal resistance 1/8 Ω. The charging current also runs an electric motor placed in series with the battery. If the resistance of the windings of the motor is 7.0 Ω and the steady supply current is 3.5 A, obtain*

(*a*) *the mechanical energy yielded by the motor,*

(*b*) *the chemical energy, stored in the battery during charging in 1 h.* [NCERT]

Solution. emf of the battery,

$\mathcal{E}_b = 2.0 \times 8\text{ V} = 16\text{ V}$

d.c. supply voltage, $\mathcal{E}_s = 64\text{ V}$

Internal resistance of the battery,

$r = \dfrac{1}{8} \times 8\,\Omega = 1\,\Omega$

Resistance of motor, $R = 7.0\,\Omega$

Let back emf of motor $= \mathcal{E}_m$

Both the back emf $\mathcal{E}_m$ of the motor and the emf ε_b of the battery act in the opposite direction of the supply emf $\mathcal{E}_s$. Therefore, net current in the circuit must be

$$I = \frac{\text{Net emf}}{\text{Net resistance}} = \frac{\mathcal{E}_s - \mathcal{E}_b - \mathcal{E}_m}{r + R}$$

or $\quad 3.5 = \dfrac{64 - 16 - \mathcal{E}_m}{8}$

or $\quad \mathcal{E}_m = 48 - 28 = 20\text{ V}.$

(*a*) Mechanical energy yielded by motor in 1 h

$= \mathcal{E}_m \,.\, It = 20 \times 3.5 \times 3600\text{ J} = \mathbf{252000\ J}.$

(*b*) Chemical energy stored in the battery in 1 h

$= \mathcal{E}_b \,.\, It = 16 \times 3.5 \times 3600\text{ J} = \mathbf{201600\ J}.$

Example 132. *A 24 V battery of internal resistance 4.0 Ω is connected to a variable resistor. At what value of the current drawn from the battery is the rate of heat produced in the resistor maximum ?* [NCERT]

Solution. Here $\mathcal{E} = 24\text{ V}, \quad r = 4.0\,\Omega$

Let the variable resistor be R. The rate of heat produced in the resistor will be maximum when

External resistance = internal resistance

or $\quad R = 4\,\Omega$

$\therefore$ Required current,

$$I = \frac{\text{emf}}{\text{resistance}} = \frac{24}{4+4}\text{ A} = \mathbf{3.0\ A}.$$

Example 133. *4 cells of identical emf $\mathcal{E}$, internal resistance r, are connected in series to a variable resistor. The following graph shows the variation of terminal voltage of the combination with the current output.*

(*i*) *What is the emf of each cell used ?*

(*ii*) *For what current from the cells, does maximum power dissipation occur in the circuit ?*

(*iii*) *Calculate the internal resistance of each cell.* [CBSE OD 06C]

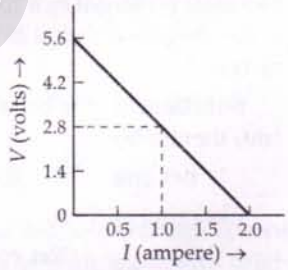


Fig. 3.132

Solution. When $I = 0$,

total emf = terminal voltage

$\therefore \quad 4\,\mathcal{E} = 5.6\text{ V}$

or $\quad \mathcal{E} = 1.4\text{ V}$

When $I = 1.0\text{ A},\ V = \dfrac{2.8}{4} = 0.7\text{ V}$

Internal resistance

$$r = \frac{\mathcal{E} - V}{I} = \frac{1.4 - 0.7}{1.0} = 0.7\,\Omega$$

The output power is maximum, when

external resistance = internal resistance = $4r$

$$I_{max} = \frac{\text{Total emf}}{\text{Total resistance}} = \frac{4\,\mathcal{E}}{4r + 4r}$$

$$= \frac{\mathcal{E}}{2r} = \frac{1.4}{2 \times 0.7} = \mathbf{1\ A}.$$

Example 134. *Two batteries, each of emf $\mathcal{E}$ and internal resistance r, are connected in parallel. If we take current from this combination in an external resistance R, then for what value of R maximum power will be obtained ? What will be this power ?*

Solution. The situation is shown in Fig. 3.133.

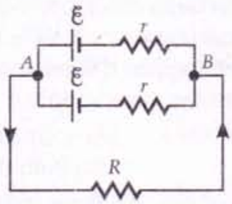


Fig. 3.133

Net emf of the parallel combination of two cells = ε

Total resistance in the circuit

$$= \frac{r \times r}{r + r} + R = \frac{r}{2} + R$$

Hence current in the circuit is

$$I = \frac{\varepsilon}{\frac{r}{2} + R} = \frac{2\,\varepsilon}{r + 2R}$$

Power dissipated in the resistance R is

$$P = I^2 R = \frac{(2\,\varepsilon)^2\, R}{(r + 2R)^2} = \frac{4\varepsilon^2 R}{(r - 2R)^2 + 8rR}$$

Power P will be maximum when the denominator has a minimum value. This happens when

$$(r - 2R)^2 = 0 \quad \text{or} \quad R = \frac{r}{2}$$

$$\therefore \quad P_{max} = \frac{(2\,\varepsilon)^2\, r/2}{(r + r)^2} = \frac{\varepsilon^2}{2r}.$$

Example 135. *Two wires made of tinned copper having identical cross-section ($=10^{-6}\,m^2$) and lengths 10 cm and 15 cm are to be used as fuses. Show that the fuses will melt at the same value of current in each case.* [NCERT]

Solution. The temperature of the wire increases up to a certain temperature θ where the heat produced per second by the current equals heat lost (by radiation) per second.

But heat produced by the current

$$= I^2 R = I^2 \rho \frac{l}{A} = \frac{I^2 \rho l}{\pi r^2}$$

If h is heat lost per second per unit surface area of the wire and if we ignore the heat loss from the end faces of the wire, then heat loss per second by the wire

$$= h \times \text{curved surface area of the wire}$$

$$= h \times 2\pi r l$$

When the steady state temperature is attained,

$$h \times 2\pi r l = \frac{I^2 \rho l}{\pi r^2}$$

or

$$h = \frac{I^2 \rho}{2\pi^2 r^3} \qquad ...(i)$$

Now h is independent of l and the values of r and ρ are same for both wires, hence steady state temperature θ will depend only on I *i.e.*, the two fuses will melt at the same values of current.

Example 136. *A fuse with a circular cross-sectional radius of 0.15 mm blows at 15 A. What should be the radius of cross-section of a fuse made of the same material which will blow at 30 A?* [NCERT]

Solution. Here $r_1 = 0.15$ mm, $I_1 = 15$ A, $r_2 = ?$

$$I_2 = 30 \text{ A}$$

From Eq. (*i*), the heat lost per second per unit surface area of the wire is

$$h = \frac{I^2 \rho}{2\pi^2 r^3}$$

∴ For a fuse wire of the given material and the given value of h,

$$r^3 \propto I^2$$

or

$$\frac{r_2^{\,3}}{r_1^{\,3}} = \frac{I_2^{\,2}}{I_1^{\,2}}$$

or

$$r_2^{\,3} = \frac{I_2^{\,2}}{I_1^{\,2}} \times r_1^{\,3} = \left(\frac{30}{15}\right)^2 \times (0.15)^3$$

$$\therefore \quad r_2 = (4)^{1/3} \times 0.15 \text{ mm}$$

$$= 1.5874 \times 0.15 \text{ mm} = \mathbf{0.24\ mm.}$$

Problems For Practice

1. Calculate the current flowing through a heater rated at 2 kW when connected to a 300 V d.c. supply. **[CBSE F 94 C]**
   (Ans. 6.67 A)
2. Calculate the amount of heat produced per second (in calories), when a bulb of 100 W – 220 V glows assuming that only 20% of electric energy is converted into light. $J = 4.2\ J\ cal^{-1}$. **[Haryana 01]**
   (Ans. 19.05 cal)
3. An electric heating element to dissipate 480 watts on 240 V mains is to be made from nichrome ribbon 1 mm wide and thickness 0.05 mm. Calculate the length of the ribbon required if the resistivity of nichrome is $1.1 \times 10^{-6}\ \Omega m$. (Ans. 5.45 m)
4. 100 W, 220 V bulb is connected to 110 V source. Calculate the power consumed by the bulb. **[Roorkee 86]**
   (Ans. 25 W)
5. How many electrons flow per second through an electric bulb rated 220 V, 100 W? **[BIT Ranchi 98]**
   (Ans. 2.84×10^{18})
6. An ammeter reads a current of 30 A when it is connected across the terminals of a cell of emf 1.5 V.

Neglecting the meter resistance, find the amount of heat produced in the battery in 10 seconds ?

(Ans. 107.14 cal)

7. A coil of resistance 100 Ω is connected across a battery of emf 6.0 V. Assume that the heat developed in the coil is used to raise its temperature. If the thermal capacity of coil is 4.0 JK^{-1}, how long would it take to raise the temperature of the coil by 15°C ? (Ans. 2.8 min)

8. A generator is supplying power to a factory by cables of resistance 20 Ω. If the generator is generating 50 kW power at 5000 V, what is the power received by the factory ? **[Punjab 96 C]**

(Ans. 48 kW)

9. Two bulbs are marked 220 V, 100 W and 220 V, 50 W respectively. They are connected in series to 220 V mains. Find the ratio of heats generated in them. (Ans. 1 : 2)

10. In a house having 220 V line, the following appliances are working : (*i*) a 60 W bulb (*ii*) a 1000 W heater (*iii*) a 40 W radio. Calculate (*a*) the current drawn by heater and (*b*) the current passing through the fuse line. **[MNREC 86]**

$\left(\text{Ans. } (a)\ \frac{50}{11}\text{ A } (b)\ 5\text{ A}\right)$

11. Three equal resistances connected in series across a source of emf consume 20 W. If the same resistances are connected in parallel across the same source of emf, what will be the power dissipated ? **[Punjab 99]**

(Ans. 180 W)

12. An electric heater consists of 20 m length of manganin wire of 0.23 m^2 cross-sectional area. Calculate the wattage of the heater when a potential difference of 200 V is applied across it. Resistivity of manganin $= 4.6 \times 10^{-7}\,\Omega$m. (Ans. 10^9 W)

13. A line having a total resistance of 0.2 Ω delivers 10 kW at 220 V to a small factory. Calculate the efficiency of the transmission. (Ans. 96%)

14. A motor operating on 120 V draws a current of 2 A. If the heat is developed in the motor at the rate of 9 cal s^{-1}, what is its efficiency ? (**Ans.** 84.425%)

15. A 500 W electric heater is designed to work with a 200 V line. If the voltage of the line drops to 160 V, then what will be the percentage loss of the heat developed ? (Ans. 36%)

16. A 50 W bulb is connected in a 200 V line. Determine the current flowing in it and its resistance. If 10% of the total power is converted into light, then what will be the rate of production of heat ?

Take $J = 4.2$ J cal^{-1}

(Ans. 0.25 A, 800 Ω, 10.7 cal s^{-1})

17. Two bulbs rated 25 W, 220 V and 100 W, 220 V are connected in series to a 440 V supply. (*i*) Show with necessary calculations which bulb if any will fuse. (*ii*) What will happen if the two bulbs are connected in parallel to the same supply ?

[Ans. (*i*) 25 W bulb will fuse
(*ii*) Both the bulbs will fuse]

18. A servo voltage stabiliser restricts the voltage output to 220 V ± 1%. If an electric bulb rated at 220 V, 100 W is connected to it, what will be the minimum and maximum power consumed by it ?

(Ans. 98.01 W, 102.01 W)

19. A room is lighted by 200 W, 124 V incandescent lamps fed by a generator whose output voltage is 130 V. The connecting wires from the generator to the user are made of aluminium wire of total length 150 m and cross-sectional area 15 mm^2. How many such lamps can be installed ? What is the total power consumed by the user ? Specific resistance of aluminium $= 2.9 \times 10^{-8}\,\Omega$m. (Ans. 12, 2.4 kW)

20. Two wires *A* and *B* of same material and mass, have their lengths in the ratio 1 : 2. On connecting them, one at a time to the same source of emf, the rate of heat dissipation in *B* is found to be 5 W. What is the rate of heat dissipation in *A* ? (Ans. 20 W)

21. Two electric bulbs rated as 100 W, 220 V and 25 W, 220 V are connected in series across 220 V line. Calculate (*i*) current through (*ii*) potential difference across and (*iii*) actual powers consumed in filament of each bulb.

$\left(\text{Ans. } (i)\ \frac{1}{11}\text{ A } (ii)\ 44\text{ V}, 176\text{ V}, \ (iii)\ 4\text{ W}, 16\text{ W}\right)$

22. The heater coil of an electric kettle is rated as 2000 W at 200 V. How much time will it take to heat one litre of water from 20°C to 100°C, assuming that entire electric energy liberated from the heater coil is utilised for heating water ? Also calculate the resistance of the coil. Density of water is 1 g cm^{-3}.

(Ans. 168 s, 20 Ω)

23. An electric kettle was marked 500 W, 230 V and was found to raise 1 kg of water at 15° C to the boiling point in 15 minutes. Calculate the heat efficiency of the kettle. (Ans. 79.3%)

24. A copper kettle weighing 1000 g holds 1900 g of water at 19°C. It takes 12 minutes to raise the temperature to 100°C. If energy is supplied at 210 V, calculate the strength of current, assuming that 10% of heat is wasted. Specific heat of copper = 0.1 cal $g^{-1}\,°C^{-1}$. (Ans. 5.0 A)

25. A 30 V storage battery is being charged by 120 V d.c. supply. A resistor has been connected in series with the battery to limit the charging current to 15 A.

Find the rate at which energy is dissipated in the resistor. If the total heat produced could be made available for heating water, how long would it take to bring 1 kg of water from 15°C to the boiling point ? Specific heat of water = $1\,\text{cal}\,g^{-1}\,°C^{-1}$ and 1 cal = 4.2 J. **[MNREC 84]**

(Ans. $1350\ Js^{-1}$, 264.4 s)

26. In the circuit shown in Fig. 3.134, each of the three resistors of 4 Ω can have a maximum power of 20 W (otherwise it will melt). What maximum power can the whole circuit take ? (Ans. 30 W)

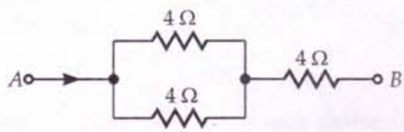


Fig. 3.134

27. Find the heat produced per minute in each of the resistors shown in Fig. 3.135. (Ans. 360 J, 720 J, 540 J)

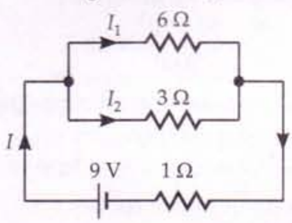


Fig. 3.135

28. Calculate the current drawn from the battery of emf 15 V and internal resistance 0.5 Ω in the circuit shown in Fig. 3.136. Also find the power dissipated in the 6 Ω resistor. **[IIT]**

(Ans. 1.0 A, 3.375 W)

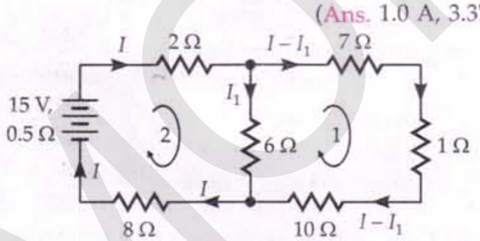


Fig. 3.136

29. In the circuit shown in Fig. 3.137, the heat produced by 4 Ω resistance due to current flowing through it is $40\ cal\ s^{-1}$. Find the rate at which heat is produced in 2 Ω resistance. (Ans. $80\ cal\ s^{-1}$)

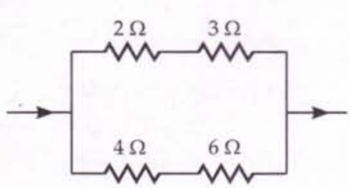


Fig. 3.137

30. The 2.0 Ω resistor shown in Fig. 3.138 is dipped into a calorimeter containing water. The heat capacity of the calorimeter together with water is $2000\ JK^{-1}$. (*a*) If the circuit is active for 30 minutes, what would be the rise in the temperature of the water ? (*b*) Suppose the 6.0 Ω resistor gets burnt. What would be the rise in the temperature of the water in the next 30 minutes ? (Ans. 5.8°C, 7.2°C)

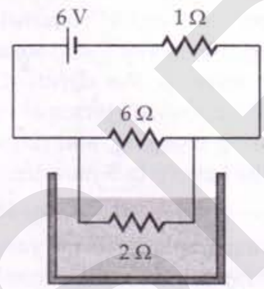


Fig. 3.138

31. Three resistors R_1, R_2 and R_3 each of 240 Ω are connected across a 120 V supply, as shown in Fig. 3.139. Find (*i*) the potential difference across each resistor and (*ii*) the total heat developed across the three resistors in 1 minute.

[Ans. (*i*) $V_1 = 80$ V, (*ii*) $V_2 = V_3 = 40$ V (*iii*) 2400 J]

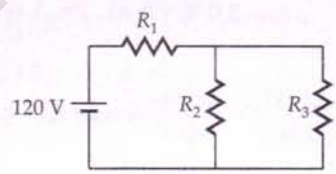


Fig. 3.139

32 A heating coil is connected in series with a resistance *R*. The coil is dipped in a liquid of mass 2 kg and specific heat $0.5\,\text{cal}\,g^{-1}\,°C^{-1}$. A potential difference of 200 V is applied and the temperature of the liquid is found to increase by 60°C in 20 minutes. If *R* is removed, the same rise in, temperature is reached in 15 minutes. Find the value of *R*. (Ans. 22.14 Ω)

33. A house is fitted with two electric lamps, each of 100 W ; one heater of resistance 110 Ω and two fans, each consuming 0.25 A. If electric energy is supplied at 200 V and each appliance works for 5 hours a day, find the monthly bill at the rate of Rs. 3.0 per kWh.

[Punjab 98C] (Ans. ₹298.65)

34. An electric kettle has two coils. When one coil is switched on, it takes 5 minutes to boil water and when second coil is switched on, it takes 10 minutes. How long will it take to boil water, when both the coils are used in series ? **[Punjab 01]**

(Ans. 15 minutes)

35. A series battery of 6 lead accumulators, each of emf 2.0 V and internal resistance 0.25 Ω is charged by a 230 V d.c. mains. To limit the charging current, a series resistance of 53 Ω is used in the charging circuit. What is (*i*) power supplied by the mains (*ii*) power dissipated as heat ? Account for the difference in the two cases. **[NCERT]**

(**Ans.** 920 W, 872 W)

36. A storage battery of emf 8 V, internal resistance 1 Ω, is being charged by a 120 V d.c. source, using a 15 Ω resistor in series in the circuit. Calculate (*i*) the current in the circuit, (*ii*) terminal voltage across the battery during charging, and (*iii*) chemical energy stored in the battery in 5 minutes. **[CBSE 01, 08]**

[**Ans.** (*i*) 7 A, (*ii*) 15 V, (*iii*) 16800 J]

37. The following graph shows the variation of terminal potential difference V, across a combination of three cells in series to a resistor, versus the current, i :
   (*i*) Calculate the emf of each cell
   (*ii*) For what current i, will the power dissipation of the circuit be maximum ? **[CBSE OD 08]**

(**Ans.** 2.0 V, 1.0 A)

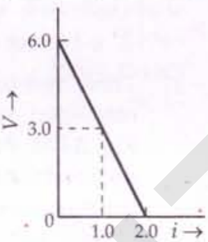


Fig. 3.140

HINTS

1. $I = \frac{P}{V} = \frac{2\text{ kW}}{300\text{ V}} = \frac{2000\text{ W}}{300\text{ V}} = \mathbf{6.67\ A.}$

2. Power of bulb, $P = 100$ W

$\therefore$ Electric energy consumed per second = 100 J

Amount of heat produced per second

$$= 80\% \text{ of } 100\text{ J} = 80\text{ J} = \frac{80}{4.2}\text{ cal} = \mathbf{19.05\ cal.}$$

3. Power, $P = \frac{V^2}{R}$ $\therefore$ $R = \frac{V^2}{P} = \frac{240 \times 240}{480} = 120\ \Omega$

Area of cross-section of the ribbon,

$$A = 0.05\text{ mm}^2 = 0.05 \times 10^{-6}\text{ m}^2$$

Required length,

$$l = \frac{RA}{\rho} = \frac{120 \times 0.05 \times 10^{-6}}{11 \times 10^{-6}}\text{ m} = \mathbf{5.45\ m.}$$

4. Here $P = 100$ W, $V = 220$ V

$\therefore$ Resistance of bulb, $R = \frac{V^2}{P} = \frac{220 \times 220}{100} = 484\ \Omega$

When the bulb is connected to 110 V source, the power consumed by the bulb is

$$P' = \frac{V'^2}{R} = \frac{110 \times 110}{484} = \mathbf{25\ W.}$$

5. $n = \frac{It}{e} = \frac{Pt}{Ve} = \frac{100 \times 1}{220 \times 1.6 \times 10^{-19}} = \mathbf{2.84 \times 10^{18}}.$

6. If r is the internal resistance of the cell, then

$$I = \frac{\varepsilon}{r} \quad \text{or} \quad r = \frac{\varepsilon}{I} = \frac{1.5}{30} = 0.05\ \Omega$$

$$\therefore \quad H = \frac{I^2 rt}{J} = \frac{(30)^2 \times 0.05 \times 10}{4.2} = \mathbf{107.14\ cal.}$$

7. Heat required by the coil = Thermal capacity × rise in temperature

$= 4.0 \times 15 = 60$ J

Rate of production of heat,

$$P = \frac{V^2}{R} = \frac{6 \times 6}{100} = 0.36\text{ Js}^{-1}$$

$$\therefore \text{ Required time} = \frac{60\text{ J}}{0.36\text{ Js}^{-1}} = \frac{60}{0.36 \times 60}\text{ min} = \mathbf{2.8\ min.}$$

8. Here $P = 50$ kW $= 50 \times 10^3$ W, $V = 5000$ V

Current supplied by generator,

$$I = \frac{P}{V} = \frac{50 \times 10^3}{5000} = 10\text{ A}$$

Power wasted as heat during transmission by cables of 20 Ω resistance,

$$P' = I^2 R = (10)^2 \times 20 = 2000\text{ W} = 2\text{ kW}$$

Power received by the factory

$= P' - P = 50 - 2 = \mathbf{48\ kW.}$

9. $R_1 = \frac{220 \times 220}{100} = 484\ \Omega,\ R_2 = \frac{220 \times 220}{50} = 968\ \Omega$

Ratio of heats produced when connected in series,

$$\frac{P_1}{P_2} = \frac{I^2 R_1}{I^2 R_2} = \frac{R_1}{R_2} = \frac{484}{968} = \mathbf{1 : 2}$$

10. (*a*) Current drawn by heater $= \frac{P_1}{V} = \frac{1000}{220} = \frac{\mathbf{50}}{\mathbf{11}}$ **A**

Current drawn by bulb $= \frac{P_2}{V} = \frac{60}{220} = \frac{3}{11}$ A

Current drawn by radio $= \frac{P_3}{V} = \frac{40}{220} = \frac{2}{11}$ A

(*b*) Current passing through fuse for the line

$$= \frac{50}{11} + \frac{3}{11} + \frac{2}{11} = \mathbf{5\ A.}$$

11. Let R be the resistance of each resistor and ε the emf of the source.

For series combination : $R_s = R + R + R = 3R$

$$\therefore \quad P = \frac{V^2}{R_s} \text{ or } 20 = \frac{V^2}{3R} \quad \therefore \quad \frac{V^2}{R} = 60\text{ W.}$$

For parallel combination : $R_p = R/3$

$$\therefore \quad P' = \frac{V^2}{R_p} = \frac{V^2}{R/3} = \frac{3V^2}{R} = 3 \times 60 = \mathbf{180\ W.}$$

12. First find $R = \rho \frac{l}{A}$ and then $P = \frac{V^2}{R}$.

13. Let P' be the power loss in the transmission line in the form of heat. Then

$$P' = I^2 R = \left(\frac{P}{V}\right)^2 R = \left(\frac{10{,}000}{220}\right)^2 \times 0.2$$

$$= 413.2 \text{ W} = 0.4132 \text{ kW}$$

Efficiency of transmission,

$$\eta = \frac{\text{Power delivered by line}}{\text{Power supplied to line}}$$

$$= \frac{\text{Power delivered}}{\text{Power delivered + Power loss}}$$

$$= \frac{10}{10 + 0.4132} = 0.96 = \mathbf{96\%}.$$

14. Power supplied to line $= VI = 120 \times 2 = 240$ W

Power loss in the form of heat

$$= 9 \text{ cal s}^{-1} = 9 \times 4.2 \text{ Js}^{-1} = 37.8 \text{ W}$$

Power delivered by line $= 240 - 37.8 = 202.2$ W

$$\text{Efficiency, } \eta = \frac{\text{Power delivered by line}}{\text{Power supplied to line}} = \frac{202.2}{240}$$

$$= 0.8425 = \mathbf{84.25\%}.$$

15. Here $P = 500$ W, $V = 200$ V

$$R = \frac{V^2}{P} = \frac{200 \times 200}{500} = 80\,\Omega$$

When the voltage drops to 160 V, rate of heat production is

$$P' = \frac{V'^2}{R} = \frac{160 \times 160}{80} = 320 \text{ W}$$

% Drop in heat production

$$= \frac{P - P'}{P} \times 100 = \frac{180 \times 100}{500} = \mathbf{36\%}.$$

17. Proceed as in Example 117 on page 3.68.

18. Resistance of the bulb, $R = \frac{(220)^2}{100} = 484\,\Omega$

Variation in voltage $= \pm 1\%$ of 220 V $= \pm 2.2$ V

Minimum voltage $= 220 - 2.2 = 217.8$ V

Minimum power $= \frac{(217.8)^2}{484} = \mathbf{98.01\ W.}$

Maximum voltage $= 220 + 2.2 = 222.2$ V

Maximum power $= \frac{(222.2)^2}{484} = \mathbf{102.01\ W.}$

19. Resistance of aluminium wire,

$$R = \frac{\rho l}{A} = \frac{2.9 \times 10^{-8} \times 150}{15 \times 10^{-6}} = 0.29\,\Omega$$

Current from the main line $= \frac{130 - 124}{0.29} = 20.69$ A

Current through each lamp $= \frac{200}{124} = 1.613$ A

$\therefore$ No. of bulbs which can be used $= \frac{20.69}{1.613} = 12.83.$

No. of bulbs that should be installed = **12.**

Power consumed $= 12 \times 200 = 2400$ W $=$ **2.4 kW.**

20. As the two wires are of same material and mass, their volumes must be equal.

$\therefore \quad a_1 l_1 = a_2 l_2 \quad$ or $\quad a_1 \times l = a_2 \times 2l \quad$ or $\quad a_1 = 2a_2$

If ε is the emf of the source, then rate of heat dissipation in wire B is

$$\frac{\varepsilon^2}{R_2} = 5 \quad \text{or} \quad \frac{\varepsilon^2}{\rho\, l_2 / a_2} = 5$$

$$\text{or} \quad \frac{\varepsilon^2}{\rho \,.\, 2l / a_2} = 5 \quad \text{or} \quad \frac{\varepsilon^2 a_2}{\rho l} = 10 \qquad [\because\ l_2 = 2l]$$

Rate of heat dissipation in wire A is

$$\frac{\varepsilon^2}{R_1} = \frac{\varepsilon^2}{\rho\, l / a_1} = \frac{\varepsilon^2}{\rho l} \,.\, 2a_2 = 2 \times 10 = \mathbf{20\ W.}$$

21. Proceed as in Example 123 on page 3.70.

22. $I = \frac{P}{V} = \frac{2000}{200} = 10$ A

Heat produced in time $t = \frac{VIt}{J} = \frac{200 \times 10 \times t}{4.2}$ cal

Heat gained by water $= mc\theta = 1000 \times 1 \times 80$ cal

$$\therefore \quad \frac{2000\,t}{4.2} = 1000 \times 80$$

$$\text{or} \quad t = \frac{1000 \times 80 \times 4.2}{2000} = \mathbf{168\ s.}$$

$$R = \frac{V^2}{P} = \frac{200 \times 200}{2000} = \mathbf{20\,\Omega.}$$

23. Heat absorbed by water

$$= 1 \times 4200 \times (100 - 15) = 4200 \times 85 \text{ J}$$

Heat produced by electric kettle

$$= Pt = 500 \times 15 \times 60 \text{ J}$$

$$\text{Heat efficiency} = \frac{4200 \times 85}{500 \times 15 \times 60} \times 100 = \mathbf{79.3\%}.$$

24. Proceed as in Example 120 on page 3.69.

25. Charging current, $I = \frac{120 - 30}{R} = 15$

$$\therefore \text{ Series resistor, } \quad R = \frac{90}{15} = 6\,\Omega$$

Rate of energy dissipation in the resistor,

$$P = I^2 R = (15)^2 \times 6 = \mathbf{1350\ Js^{-1}}.$$

Heat produced in resistor in time t = Heat absorbed by water

$$\therefore \quad 1350 \times t = 1 \times 4200 \times (100 - 15)$$

$$\text{or} \quad t = \frac{4200 \times 85}{1350} = \mathbf{264.4\ s.}$$

26. Let I be the current through a resistance of maximum power 20 W. Then

$$I^2R = 20 \quad \text{or} \quad I^2 \times 4 = 20 \quad \text{or} \quad I^2 = 5$$

Effective resistance between A and C,

$$R' = \frac{4 \times 4}{4+4} + 4 = 2 + 4 = 6\,\Omega$$

The maximum power that can be dissipated by the circuit,

$$P = I^2R' = 5 \times 6 = \mathbf{30\ W.}$$

27. The equivalent resistance of the circuit is

$$R = \frac{6 \times 3}{6+3} + 1 = 2 + 1 = 3\,\Omega$$

Current drawn from the battery is

$$I = \frac{9\text{V}}{3\,\Omega} = 3\text{ A}$$

As the current through 1Ω resistor is 3 A, so heat produced in this resistor in 1 minute (or 60 s) is

$$H = I^2\ Rt = 3^2 \times 1 \times 60 = \mathbf{540\ J}$$

Current through $6\,\Omega$ resistor,

$$I_1 = \frac{3}{6+3} \times 3 = 1\text{ A}$$

∴ Heat produced in $6\,\Omega$ resistor

$$= 1^2 \times 6 \times 60 = \mathbf{360\ J.}$$

Current through $3\,\Omega$ resistor,

$$I_2 = I - I_1 = 3 - 1 = 2\text{ A}$$

∴ Heat produced in $3\,\Omega$ resistor

$$= 2^2 \times 3 \times 60 = \mathbf{720\ J.}$$

28. The distribution of current is shown in Fig. 3.136. Applying Kirchhoff's second law to the loops 1 and 2, we get

$$(I - I_1) \times (7 + 1 + 10) - I_1 \times 6 = 0$$

and $$I_1 \times 6 + I \times (8 + 0.5 + 2) = 15$$

On solving the above two equations, we get

$$I_1 = 0.75\text{ A} \quad \text{and} \quad I = \mathbf{1.0\ A}$$

Power dissipated in the $6\,\Omega$ resistor which carries current I_1 is

$$P = I_1^2 R = (0.75)^2 \times 6 = \mathbf{3.375\ W.}$$

29. Resistance of the upper arm = 2 + 3 = $5\,\Omega$

Resistance of the lower arm = 4 + 6 = $10\,\Omega$

Let I be the total current in the circuit. Then current flowing through the upper arm will be

$$I_1 = \frac{I \times 10}{5 + 10} = \frac{2I}{3}$$

Current flowing through the lower arm.

$$I_2 = \frac{I \times 5}{5+10} = \frac{I}{3}$$

Heat produced per second in $2\,\Omega$ resistor,

$$P_1 \propto I_1^2 \times 2$$

Heat produced per second in $4\,\Omega$ resistor,

$$P_2 \propto I_2^2 \times 4$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{I_1^2 \times 2}{I_2^2 \times 4} = \frac{(2I/3)^2 \times 2}{(I/3) \times 4} = 2$$

or $$P_1 = 2\ P_2 = 2 \times 40 = \mathbf{80\ cal\ s^{-1}}.$$

30. (*a*) Total resistance in the circuit $= \dfrac{6 \times 2}{6+2} + 1 = \dfrac{5}{2}\,\Omega$

Total current, $$I = \frac{6\text{ V}}{(5/2)\,\Omega} = \frac{12}{5}\text{ A}$$

Current through $2\,\Omega$ resistance

$$= \frac{12}{5} \times \frac{6}{6+2} = 1.8\text{ A}$$

Heat produced in $2\,\Omega$ resistance in 30 minutes

$$= (1.8)^2 \times 2 \times 30 \times 60 = 11664\text{ J}$$

Rise in temperature

$$= \frac{11664\text{ J}}{2000\text{ JK}} \simeq \mathbf{5.8\ K}\ \textit{or}\ \mathbf{5.8°C.}$$

(*b*) When the $6\,\Omega$ resistor gets burnt,

$$\text{Current} = \frac{6\text{ V}}{(2+1)\,\Omega} = 2\text{ A}$$

Heat produced in $2\,\Omega$ resistor in 30 minutes

$$= (2)^2 \times 2 \times 30 \times 60 = 14400\text{ J}$$

Rise in temperature

$$= \frac{14400\text{ J}}{2000\text{ JK}^{-1}} = \mathbf{7.2\ K}\text{ or }\mathbf{7.2°C.}$$

31. (*i*) Total resistance of the circuit,

$$R = R_1 + \frac{R_2 \times R_3}{R_2 + R_3} = 240 + \frac{240 \times 240}{240 + 240} = 360\,\Omega$$

Current drawn from the battery,

$$I = \frac{V}{R} = \frac{120}{360} = \frac{1}{3}\text{ A}$$

P.D. across R_1, $V_1 = R_1 I = 240 \times \dfrac{1}{3} = \mathbf{80\ V.}$

As $R_2 = R_3$,

so current through each of these resistors

$$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}\text{ A}$$

P.D. across R_2 or R_3, $V_2 = V_3 = 240 \times \dfrac{1}{6} = \mathbf{40\ V.}$

(*ii*) Total heat developed in three resistors in 1 minute,

$$H = I^2Rt = \left(\frac{1}{3}\right)^2 \times 360 \times 60 = \mathbf{2400\ J.}$$

32 Here $m = 2$ kg = 2000 g, $c = 0.5$ cal g^{-1}°C^{-1}, $\theta = 60$°C, $t_1 = 20$ min, $t_2 = 15$ m, $R = ?$

∴ Heat gained by liquid

$$H = mc\,\theta = 2000 \times 0.5 \times 60 = 6 \times 10^4 \text{ cal}$$
$$= 6 \times 10^4 \times 4.2 \text{ J} = 2.52 \times 10^5 \text{ J}$$

Let r be the resistance of the heating coil. In the first case, the resistance R is in the circuit.

∴ Current, $I = \frac{V}{R+r}$

Heat dissipated in time t_1, $H_1 = \left(\frac{V}{R+r}\right)^2 rt_1$

In the second case, the resistance R is removed.

∴ Current, $I = \frac{V}{r}$

Heat dissipated in time t_2, $H_2 = \left(\frac{V}{r}\right)^2 rt_2 = \frac{V^2 t_2}{r}$

As the liquid is raised to same temperature in both cases, so

$$H = H_1 = H_2$$

or $\left(\frac{V}{R+r}\right)^2 rt_1 = \left(\frac{V}{r}\right)^2 rt_2$

or $\frac{r^2}{(R+r)^2} = \frac{t_2}{t_1} = \frac{15}{20} = \frac{3}{4}$

or $\frac{r}{R+r} = \frac{\sqrt{3}}{2}$ or $\frac{R+r}{r} = \frac{2}{\sqrt{3}}$

or $\frac{R}{r} + 1 = \frac{2}{\sqrt{3}}$

or $\frac{R}{r} = 1.155 - 1 = 0.155$ or $r = \frac{R}{0.155}$

As $H = H_2$

∴ $2.52 \times 10^5 = \frac{(200)^2 \times 15 \times 60 \times 0.155}{R}$

or $R = \frac{4 \times 10^4 \times 15 \times 60 \times 0.155}{2.52 \times 10^5} = \mathbf{22.14\,\Omega}.$

33. Proceed as in Example 116 on page 3.68.

34. Proceed as in Example 122 on page 3.69.

35. EMF of the battery = $6 \times 2.0 = 12$ V

Internal resistance of the battery = $6 \times 0.25 = 1.5\,\Omega$

Total resistance = $1.5 + 53 = 54.5\,\Omega$

Charging current

$$= \frac{\text{Effective emf}}{\text{Total resistance}} = \frac{230 - 12}{54.5} = 4.0 \text{ A}$$

(*i*) Power supplied by the mains

$$= VI = 230 \times 4.0 = \mathbf{920\ W}.$$

(*ii*) Power dissipated as heat

$$= I^2 (R + r) = (4)^2 \times (53 + 1.5) = \mathbf{872\ W}.$$

The difference : $920 - 872 = 48$ W, is the power stored in the accumulator in the form of chemical energy of its contents.

36. Total emf = $120 - 8 = 112$ V

Total resistance = $1 + 15 = 16\,\Omega$

(*i*) Current, $I = \frac{\text{Total emf}}{\text{Total resistance}} = \frac{112}{16} = \mathbf{7\ A.}$

(*ii*) Terminal voltage during charging,

$$V = \mathcal{E} + Ir = 8 + 7 \times 1 = \mathbf{15\ V.}$$

(*iii*) Chemical energy stored in the battery in 5 minutes

$$= \mathcal{E}It = 8 \times 7 \times (5 \times 60) = \mathbf{16800\ J.}$$

37. (*i*) Total emf the three cells in series

= P.D. corresponding to zero current = 6.0 V

∴ EMF of each cell = $6.0 / 3 =$ **2.0 V**

(*ii*) When $i = 1.0$ A, $V = 3.0 / 3 = 1.0$ V

∴ $r = \frac{\mathcal{E} - V}{i} = \frac{2.0 - 1.0}{1.0} = 1.0\,\Omega$

The output power is maximum, when

external resistance = internal resistance = $3r$

∴ $i_{max} = \frac{\text{Total emf}}{\text{Total resistance}} = \frac{3\mathcal{E}}{3r + 3r} = \frac{\mathcal{E}}{2r}$

$$= \frac{2.0}{2 \times 1.0} = \mathbf{1.0\ A}.$$

3.31 KIRCHHOFF'S LAWS

Introductory concepts. In 1942, a German physicist *Kirchhoff* extended Ohm's law to complicated circuits and gave two laws, which enable us to determine current in any part of such a circuit. Before understanding these laws, we first define a few terms.

1. **Electric network.** *The term electric network is used for a complicated system of electrical conductors.*
2. **Junction.** *Any point in an electric circuit where two or more conductors are joined together is a junction.*
3. **Loop or Mesh.** *Any closed conducting path in an electric network is called a loop or mesh.*
4. **Branch.** *A branch is any part of the network that lies between two junctions.*

56. *State the two Kirchhoff's laws for electrical circuits and explain them giving suitable illustrations. Also state the sign conventions used.*

Kirchhoff's first law or junction rule. *In an electric circuit, the algebraic sum of currents at any junction is zero. Or, the sum of currents entering a junction is equal to the sum of currents leaving that junction.*

Mathematically, this law may be expressed as

$$\Sigma I = 0$$

Sign convention for applying junction rule :

1. The currents flowing towards the junction are taken as positive.
2. The currents flowing away from the junction are taken as negative.

Figure 3.141 represents a junction J in a circuit where four currents meet. The currents I_1 and I_2 flowing towards the junction are positive, while the currents I_3 and I_4 flowing away from the junction are negative, therefore, by *junction rule* :

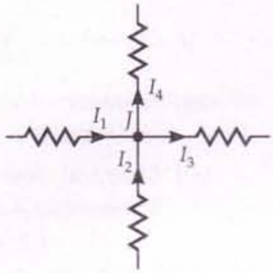


Fig. 3.141 Junction rule : $I_1 + I_2 = I_3 + I_4$.

$$\Sigma I = 0$$

or $\quad I_1 + I_2 - I_3 - I_4 = 0$

or $\quad I_1 + I_2 = I_3 + I_4$

i.e., Incoming current = Outgoing current

First law is also called **Kirchhoff's current law (KCL).**

Justification. This law is based on the law of conservation of charge. When currents in a circuit are steady, charges cannot accumulate or originate at any point of the circuit. So whatever charge flows towards the junction in any time interval, an equal charge must flow away from that junction in the same time interval.

Kirchhoff's second law or loop rule. *Around any closed loop of a network, the algebraic sum of changes in potential must be zero. Or, the algebraic sum of the emfs in any loop of a circuit is equal to the sum of the products of currents and resistances in it.*

Mathematically, the loop rule may be expressed as

$$\Sigma \Delta V = 0 \quad \text{or} \quad \Sigma \varepsilon = \Sigma\ IR$$

Sign convention for applying loop rule :

1. We can take any direction (clockwise or anti-clockwise) as the direction of traversal.
2. The emf of cell is taken as positive if the direction of traversal is from its negative to the positive terminal (through the electrolyte).

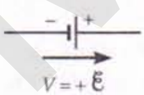


Fig. 3.142 Positive emf.

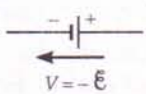


Fig. 3.143 Negative emf.

3. The emf of a cell is taken as negative if the direction of traversal is from its positive to the negative terminal.
4. The current-resistance (IR) product is taken as positive if the resistor is traversed in the same direction of assumed current.

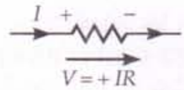


Fig. 3.144 Positive potential drop across a resistor.

5. The IR product is taken as negative if the resistor is traversed in the opposite direction of assumed current.

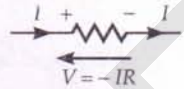
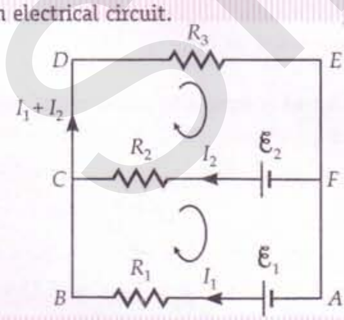


Fig. 3.145 Negative potential drop across a resistor.

Illustration. Let us consider the circuit shown in Fig. 3.146.

Fig. 3.146 An electrical circuit.



In Fig. 3.146, traversing in the clockwise direction around the loop $ABCFA$, we find that :

Algebraic sum of current resistance products

$$= I_1R_1 - I_2R_2$$

Algebraic sum of emfs $= \varepsilon_1 - \varepsilon_2$

Applying Kirchhoff's loop rule to closed path $ABCFA$, we get $\varepsilon_1 - \varepsilon_2 = I_1R_1 - I_2R_2$

Similarly, applying Kirchhoff's second rule to mesh $CDEFC$, we get

$$\varepsilon_2 = I_2R_2 + (I_1 + I_2)\ R_3$$

Second law is also called **Kirchhoff's voltage law (KVL).**

Justification. This law is based on the law of conservation of energy. As the electrostatic force is a conservative force, so the work done by it along any closed path must be zero.

Examples based on Kirchhoff's Laws

Formulae Used

1. $\Sigma I = 0$ *(Junction rule)*

   or Total incoming current = Total outgoing current

2. $\Sigma \varepsilon = \Sigma\ IR$ *(Loop rule)*

Units Used

Current I is in ampere, resistance R in ohm and emf ε in volt.

Example 137. *Network PQRS (Fig. 3.147) is made as under : PQ has a battery of 4 V and negligible resistance with positive terminal connected to P, QR has a resistance of 60 Ω. PS has a battery of 5 V and negligible resistance with positive terminal connected to P, RS has a resistance of 200 Ω. If a milliammeter, of 20 Ω resistance is connected between P and R, calculate the reading of the milliammeter.* [NCERT]

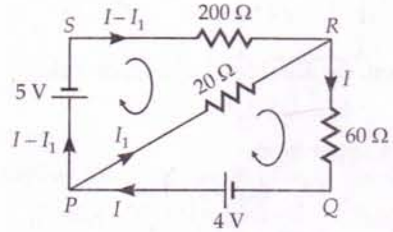


Fig. 3.147

Solution. Applying Kirchhoff's second law to the loop *PRQP*, we get

$$20 I_1 + 60 I = 4 \quad ...(i)$$

Similarly, from the loop *PSRP*, we get

$$200(I - I_1) - 20 I_1 = -5$$

or $$40 I - 44 I_1 = -1 \quad ...(ii)$$

Multiplying (*i*) by 2 and (*ii*) by 3, we get

$$120 I + 40 I_1 = 8 \quad ...(iii)$$

and $$120 I - 132 I_1 = -3 \quad ...(iv)$$

Subtracting (*iv*) from (*iii*), we get

$$172 I_1 = 11$$

or $$I_1 = \frac{11}{172} = \mathbf{0.064\ A}$$

Thus the milliammeter of 20 Ω will read 0.064 A.

Example 138. *Using Kirchhoff's laws in the electrical network shown in Fig. 3.148, calculate the values of I_1, I_2 and I_3.* [CBSE D 2000C]

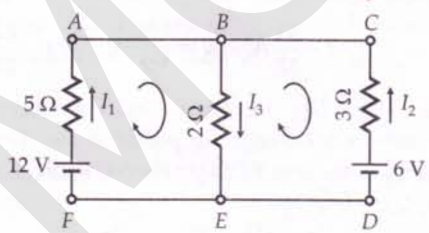


Fig. 3.148

Solution. Applying Kirchhoff's first law at junction *B*,

$$I_1 + I_2 = I_3 \quad ...(1)$$

Applying Kirchhoff's second law to loops *ABEFA* and *BCDEB*, we get

$$2I_3 + 5I_1 = 12 \quad ...(2)$$

$$-2I_3 - 3I_2 = -6 \quad ...(3)$$

Solving equations (1), (2) and (3), we get

$$I_1 = \frac{48}{31}\ \mathbf{A}, \quad I_2 = \frac{18}{31}\ \mathbf{A}, \quad I_3 = \frac{66}{31}\ \mathbf{A}$$

Example 139. *Find the potential difference across each cell and the rate of energy dissipation in R. [Fig. 3.149(a)].* [CBSE Sample Paper 11]

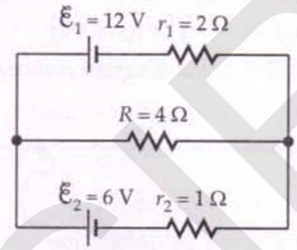


Fig. 3.149(*a*)

Solution. Applying Kirchhoff's laws,

For closed loop *ADCBA*

$$12 = 4(I_1 + I_2) + 2 I_1 = 6 I_1 + 4 I_2 \quad ...(i)$$

For closed loop *ADEFA*,

$$6 = 4(I_1 + I_2) + I_1 = 4 I_1 + 5 I_2 \quad ...(ii)$$

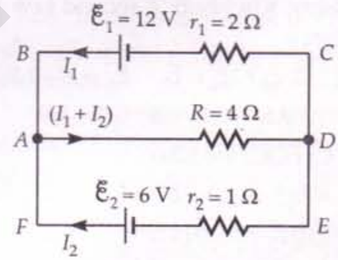


Fig. 3.149(*b*)

Solving (*i*) and (*ii*), we get

$$I_1 = \frac{18}{7}\,\text{A} \quad \text{and} \quad I_2 = -\frac{6}{7}\,\text{A}$$

P.D. across $R = V$

$$= (I_1 + I_2) R$$

$$= \left(\frac{18 - 6}{7}\right) \times 4 \text{ volt} = \frac{48}{7} \text{ volt}$$

P.D. across each cell = P.D. across $R = \frac{48}{7}$ V

Energy dissipated in $R = 4\ \Omega$ resistor

$$= (I_1 + I_2)^2 R = \left(\frac{12}{7}\right)^2 \times 4\ \text{J}$$

$$= \frac{576}{49}\ \text{J} = \mathbf{11.75\ J.}$$

Example 140. *Two cells of emfs 1.5 V and 2.0 V and internal resistances 1 Ω and 2 Ω respectively are connected in parallel so as to send current in the same direction through an external resistance of 5 Ω.* [CBSE OD 05]

(*i*) *Draw the circuit diagram.*

(*ii*) *Using Kirchhoff 's laws, calculate*

(*a*) *current through each branch of the circuit.*

(*b*) *p.d. across the 5 Ω resistance.*

Solution. (*i*) The circuit diagram is shown in Fig. 3.150.

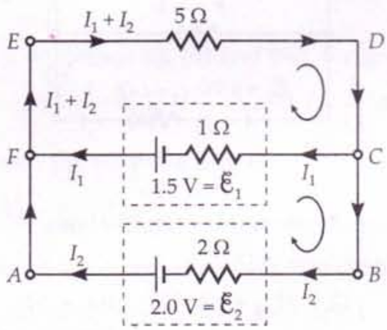


Fig. 3.150

(*ii*) (*a*) Let I_1 and I_2 be the currents as shown in Fig. 3.150. Using Kirchhoff 's second law for the loop *AFCBA*, we get

$$2I_2 - 1I_1 = \mathcal{E}_2 - \mathcal{E}_1 = 2 - 1.5$$

or $$2I_2 - I_1 = 0.5 \quad ...(1)$$

For loop *CFEDC*, we have

$$1I_1 + 5(I_1 + I_2) = \mathcal{E}_1 = 1.5$$

or $$5I_2 + 6I_1 = 1.5 \quad ...(2)$$

Solving equations (1) and (2), we get

$$I_1 = \frac{1}{34}\text{ A}, \quad I_2 = \frac{9}{34}\text{ A}$$

∴ Current through branch *BA*,

$$I_1 = \mathbf{\frac{1}{34}\ A}$$

Current through branch *CF*,

$$I_2 = \mathbf{\frac{9}{34}\ A}$$

Current through branch *DE*,

$$I_1 + I_2 = \mathbf{\frac{10}{34}\ A.}$$

(*b*) P.D. across the 5 Ω resistance

$$= (I_1 + I_2) \times 5 = \frac{10}{34} \times 5 \text{ V} = \mathbf{1.47\ V.}$$

Example 141. *Use Kirchhoff's rules to write the expressions for the currents I_1, I_2 and I_3 in the circuit diagram shown in Fig. 3.151.* [CBSE OD 10]

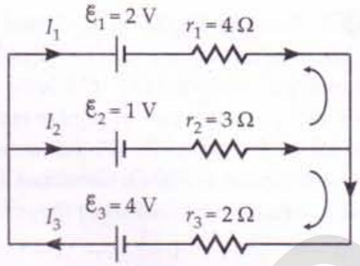


Fig. 3.151

Solution. By Kirchhoff's junction rule,

$$I_3 = I_1 + I_2 \quad ...(i)$$

From upper loop,

$$3I_2 - 4I_1 = 2 - 1 = 1 \quad ...(ii)$$

From lower loop,

$$3I_2 + 2I_3 = 4 - 1 = 3 \quad ...(iii)$$

On solving equations (*i*), (*ii*) and (*iii*), we get

$$I_1 = \mathbf{\frac{2}{13}\ A}, \quad I_2 = \mathbf{\frac{7}{13}\ A}, \quad I_3 = \mathbf{\frac{9}{13}\ A.}$$

Example 142. *Apply Kirchhoff's rules to the loops ACBPA and ACBQA to write the expression for the currents I_1, I_2 and I_3 in the network shown in Fig. 3.152.* [CBSE OD 10]

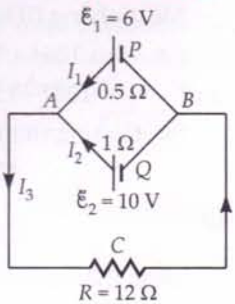


Fig. 3.152

Solution. By Kirchhoff's junction rule,

$$I_3 = I_1 + I_2 \quad ...(i)$$

From loop *AQBPA*,

$$0.5\,I_1 - I_2 = 6 - 10 = -4 \quad ...(ii)$$

From loop *ACBPA*,

$$12\,I_3 + 0.5\,I_1 = 6 \quad ...(iii)$$

On solving equations (*i*), (*ii*) and (*iii*), we get

$$I_1 = -\mathbf{\frac{84}{37}\ A}, \quad I_2 = \mathbf{\frac{106}{37}\ A}, \quad I_3 = -\mathbf{\frac{22}{37}\ A.}$$

Example 143. *Use Kirchhoff's rules to detemine the potential difference between the points A and D when no current flows in the arm BE of the electric network shown in Fig. 3.153(a).* [CBSE OD 15]

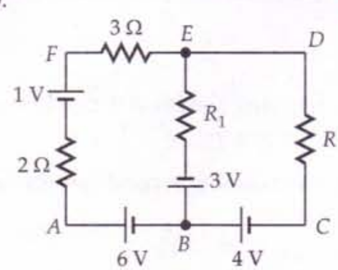


Fig. 3.153 (*a*)

Solution. No current flows through the arm BE.

Let I be the current along the outer loop as shown in Fig. 3.153(b).

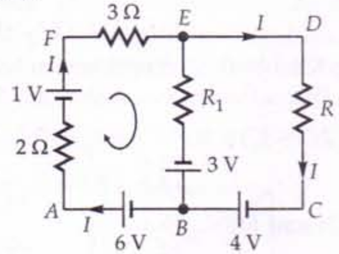


Fig. 3.153 (b)

Applying Kirchhoff' loop rule to the loop $AFEBA$,

$$(2+3)I + R_1 \times 0 = 1+3+6$$

$$\therefore \qquad I = 2\text{ A}$$

From A to D along AFD,

$$V_{AD} = 2\times 2 - 1 + 3\times 2 = \mathbf{9\ V}.$$

Example 144. *In the circuit Fig. 154, assuming point A to be at zero potential, use Kirchhoff's rules to determine the potential at point B.*

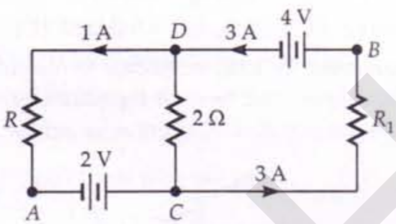


Fig. 3.154

Solution. From the loop $BDCR_1B$, we get

$$2\times 2 + 3R_1 = 4 \quad \text{or} \quad R_1 = 0$$

$$\therefore \qquad V_B = V_C = 2 - V_A = 2 - 0 = \mathbf{2\ V}.$$

Example 145. *Determine the current in each branch of the network shown in Fig. 3.155.* [NCERT]

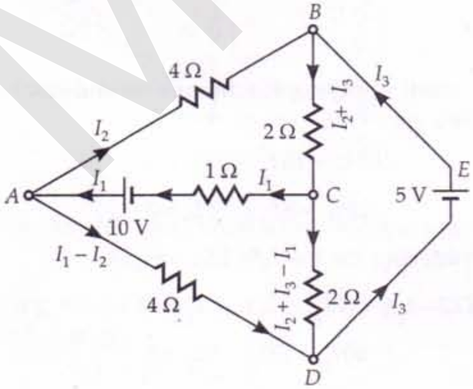


Fig. 3.155

Solution. Let I_1, I_2 and I_3 be the currents as shown in Fig. 3.155. Kirchhoff 's second rule for the closed loop $ADCA$ gives

$$10 - 4(I_1 - I_2) + 2(I_2 + I_3 - I_1) - I_1 = 0$$

or $$7I_1 - 6I_2 - 2I_3 = 10 \qquad ...(1)$$

For the closed loop $ABCA$, we get

$$10 - 4I_2 - 2(I_2 + I_3) - I_1 = 0$$

or $$I_1 + 6I_2 + 2I_3 = 10 \qquad ...(2)$$

For the closed loop $BCDEB$, we get

$$5 - 2(I_2 + I_3) - 2(I_2 + I_3 - I_1) = 0$$

or $$2I_1 - 4I_2 - 4I_3 = -5 \qquad ...(3)$$

On solving equations (1), (2) and (3), we get

$$I_1 = 2.5\ A, \quad I_2 = \frac{5}{8}\ A, \quad I_3 = 1\frac{7}{8}\ A$$

The currents in the various branches of the network are :

$$I_{AB} = \mathbf{\frac{5}{8}\ A}; \quad I_{CA} = \mathbf{2\frac{1}{2}\ A}; \quad I_{DEB} = \mathbf{1\frac{7}{8}\ A}$$

$$I_{AD} = \mathbf{1\frac{7}{8}\ A}; \quad I_{CD} = \mathbf{0}; \quad I_{BC} = \mathbf{2\frac{1}{2}\ A}.$$

Example 146. *In the circuit shown in Fig. 3.156(a), E, F, G and H are cells of emf 2 V, 1 V, 3 V and 1 V, and their internal resistances are 2 Ω, 1 Ω, 3 Ω and 1 Ω, respectively. Calculate (i) the potential difference between B and D and (ii) the potential difference across the terminals of each of the cells G and H.* [CBSE D 04C ; CBSE Sample Paper 08]

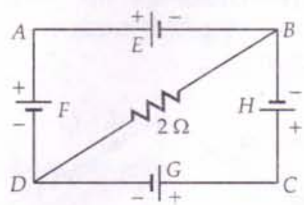


Fig. 3.156(a)

Solution. In Fig. 3.156(b), the network has been redrawn showing the emfs and internal resistances of the cells explicitly.

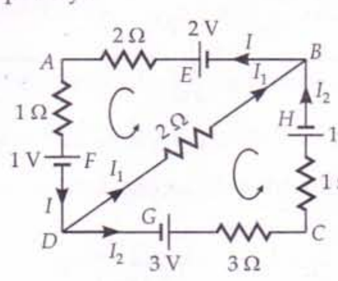


Fig. 3.156(b)

(*i*) Applying Kirchhoff's first law at junction *D*, we get

$$I = I_1 + I_2 \quad ...(i)$$

Applying Kirchhoff's second law to loop *ADBA*, we get

$$2I + I + 2I_1 = 2 - 1$$

or $$3I + 2I_1 = 1 \quad ...(ii)$$

Applying Kirchhoff's second law to loop *DCBD*

$$3I_2 + I_2 - 2I_1 = 3 - 1$$

or $$4I_2 - 2I_1 = 2 \quad ...(iii)$$

On solving equations (*i*), (*ii*) and (*iii*), we get

$$I_1 = -\frac{1}{13}\ A,\ I_2 = \frac{6}{13}\ A \text{ and } I = \frac{5}{13}\ A$$

P.D. between the points *B* and *D*,

$$V_1 = I_1 \times 2 = \frac{\mathbf{2}}{\mathbf{13}}\ \mathbf{V}.$$

(*ii*) P.D. between the terminals of *G* (giving current),

$$V_2 = \mathcal{E} - I_2 r = 3 - \frac{6}{13} \times 3 = \mathbf{1.615\ V}$$

P.D. between the terminals of *H* (taking current),

$$V_3 = \mathcal{E}' + I_2 r' = 1 + \frac{6}{13} \times 1 = \mathbf{1.46\ V.}$$

Example 147. *In a Wheatstone bridge, $P = 1\,\Omega$, $Q = 2\,\Omega$, $R = 2\,\Omega$, $S = 3\,\Omega$ and $R_g = 4\,\Omega$. Find the current through the galvanometer in the unbalanced position of the bridge, when a battery of 2 V and internal resistance 2 Ω is used.*

Solution. The circuit for the given Wheatstone bridge is shown in Fig. 3.157. Let I, I_1 and I_g be the currents as shown.

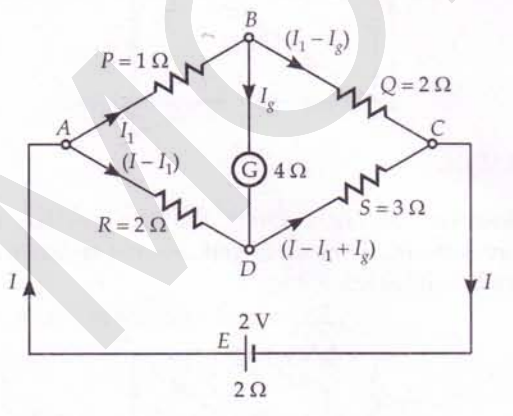


Fig. 3.157

Applying Kirchhoff's second law to loop *ABDA*, we get,

$$I_1 \times 1 + I_g \times 4 - (I - I_1) \times 2 = 0$$

or $$3I_1 - 2I + 4I_g = 0 \quad ...(1)$$

Applying Kirchhoff's second law to loop *BCDB*, we get

$$(I_1 - I_g) \times 2 - (I - I_1 + I_g) \times 3 - I_g \times 4 = 0$$

$$5I_1 - 3I - 9I_g = 0 \quad ...(2)$$

Applying Kirchhoff's second law to loop *ADCEA*, we get

$$2(I - I_1) + 3(I - I_1 + I_g) + 2I = 2$$

or $$-5I_1 + 7I + 3I_g = 2 \quad ...(3)$$

Adding (2) and (3),

$$4I - 6I_g = 2 \quad ...(4)$$

Multiplying (1) by 5 and (2) by 3 and subtracting, we get

$$-I + 47I_g = 0 \quad \text{or} \quad I = 47\,I_g$$

From (4),

$$4 \times 47I_g - 6I_g = 2 \quad \text{or} \quad 182\,I_g = 2$$

$$\therefore \quad I_g = \frac{2}{182} = \frac{\mathbf{1}}{\mathbf{91}}\ \mathbf{A.}$$

Example 148. *The four arms of a Whetstone bridge (Fig. 3.158) have the following resistances :*

$AB = 100\,\Omega$, $BC = 10\,\Omega$, $CD = 5\,\Omega$ and $DA = 60\,\Omega$.

A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC. [NCERT]

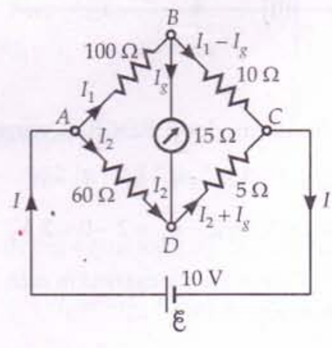


Fig. 3.158

Solution. Applying Kirchhoff's second law to loop *BADB*, we get

$$100I_1 + 15I_g - 60I_2 = 0$$

or $$20I_1 + 3I_g - 12I_2 = 0 \quad ...(1)$$

Considering the loop *BCDB*, we get

$$10(I_1 - I_g) - 15I_g - 5(I_2 + I_g) = 0$$

$$10I_1 - 30I_g - 5I_2 = 0$$

$$2I_1 - 6I_g - I_2 = 0 \quad ...(2)$$

Considering the loop *ADCEA*, we get

$$60I_2 + 5(I_2 + I_g) = 10$$

$$65I_2 + 5I_g = 10$$

$$13I_2 + I_g = 2 \quad \text{...(3)}$$

Multiplying Eq. (2) by 10, we get

$$20I_1 - 60I_g - 10I_2 = 0 \quad \text{...(4)}$$

From equations (1) and (4), we get

$$63I_g - 2I_2 = 0$$

$$I_2 = \frac{63}{2} I_g = 31.5 I_g$$

Substituting the value of I_2 in Eq. (3), we get

$$13(31.5I_g) + I_g = 2$$

or $$410.5 I_g = 2$$

or $$I_g = \frac{2}{410.5}\ \text{A} = \mathbf{4.87\ mA.}$$

Example 149. *Two cells of emfs 1.5 V and 2 V and internal resistances 2 Ω and 1 Ω respectively have their negative terminals joined by a wire of 6 Ω and positive terminals by a wire of 4 Ω resistance. A third resistance wire of 8 Ω connects middle points of these wires. Draw the circuit diagram. Using Kirchhoff laws, find the potential difference at the end of this third wire.* [CBSE D 2000C]

Solution. As shown in Fig. 3.159, the positive terminals of cells $\mathcal{E}_1$ and $\mathcal{E}_2$ are connected to the wire *AE* of resistance 4 Ω and negative terminals to the wire *BD* of resistance 6 Ω. The 8 Ω wire is connected between the middle points *F* and *C* of the wires *AE* and *BD* respectively.

$$\therefore \quad R_1 = R_2 = \frac{4}{2} = 2\ \Omega$$

and $$R_3 = R_4 = \frac{6}{2} = 3\ \Omega$$

The distribution of current in various branches is shown in Fig. 3.159.

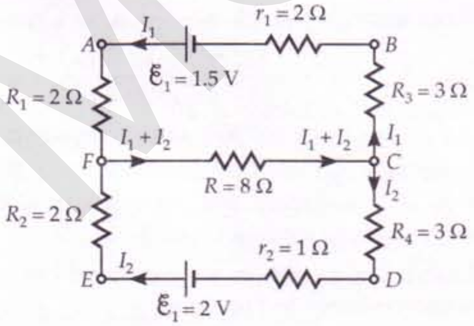


Fig. 3.159

Applying Kirchhoff's second law to the loop *ABCFA*, we get

$$I_1 \times r_1 + I_1 \times R_1 + (I_1 + I_2)R + I_1 \times R_3 = \mathcal{E}_1$$

$$I_1 \times 2 + I_1 \times 2 + (I_1 + I_2) \times 8 + I_1 \times 3 = 1.5$$

$$15I_1 + 8I_2 = 1.5 \quad \text{...(i)}$$

Applying Kirchhoff's second law to the loop *CDEFC*, we get

$$I_2 \times r_2 + I_2 \times R_2 + (I_1 + I_2) \times R + I_2 \times R_4 = \mathcal{E}_2$$

$$I_2 \times 1 + I_2 \times 2 + (I_1 + I_2) \times 8 + I_2 \times 3 = 2$$

$$8I_1 + 14I_2 = 2$$

or $$4I_1 + 7I_2 = 1 \quad \text{...(ii)}$$

On solving equations (*i*) and (*ii*), we get

$$I_1 = \frac{5}{146}\ \text{A} \quad \text{and} \quad I_2 = \frac{18}{146}\ \text{A}$$

Current through the 8 Ω resistance wire is

$$I_1 + I_2 = \frac{5}{146} + \frac{18}{146} = \frac{23}{146}\ \text{A}$$

P.D. across the ends of 8 Ω resistance wire

$$= \frac{23}{146} \times 8 = \mathbf{1.26\ V.}$$

Example 150. *AB, BC, CD and DA are resistors of 1, 1, 2 and 2 Ω respectively connected in series. Between A and C is a 1 volt cell of resistance 2 Ω, A being positive. Between B and D is a 2 V cell of 1 Ω resistance, B being positive. Find the current in each branch of the circuit.*

Solution. The circuit arrangement and current distribution is shown in Fig. 3.160.

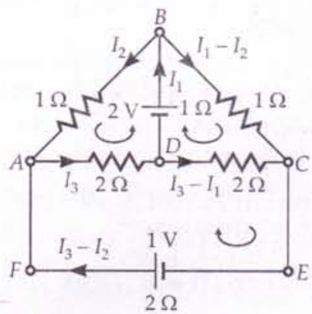


Fig. 3.160

Applying Kirchhoff's second law to loops *BADB*, *BCDB* and *ADCEFA*, we get

$$1 . I_2 + 2 . I_3 + 1 . I_1 = 2$$

or $$I_1 + I_2 + 2I_3 = 2 \quad \text{...(1)}$$

or $$1(I_1 - I_2) - 2(I_3 - I_1) + I_1 = 2$$

or $$4I_1 - I_2 - 2I_3 = 2 \quad \text{...(2)}$$

and $$2I_3 + 2(I_3 - I_1) + 2(I_3 - I_2) = 1$$

or $$-2I_1 - 2I_2 + 6I_3 = 1 \quad \text{...(3)}$$

Solving equations (1), (2) and (3), we get

$$I_1 = 0.8\ \text{A}, \quad I_2 = 0.2\ \text{A} \quad \text{and} \quad I_3 = 0.5\ \text{A}$$

Currents in different branches are

$$I_{AB} = I_2 = \mathbf{0.2\ A}\ ;$$
$$I_{BC} = I_1 - I_2 = \mathbf{0.6\ A}\ ;$$
$$I_{CD} = I_1 - I_3 = \mathbf{0.3\ A}\ ;$$
$$I_{AD} = I_3 = \mathbf{0.5\ A}\ ;$$
$$I_{EF} = I_3 - I_2 = \mathbf{0.3\ A.}$$

Example 151. *Find the equivalent resistance between the terminals A and B in the network shown in Fig. 3.161. Given each resistor R is of* $10\ \Omega$.

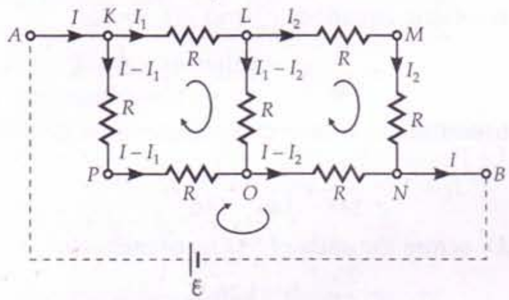


Fig. 3.161

Solution. Imagine a battery of emf ℰ, having no internal resistance, connected between the points A and B. The distribution of current through various branches is as shown in Fig. 3.161.

Applying Kirchhoff 's second law to loop $KLOPK$, we get

$$I_1 R + (I_1 - I_2) R - 2(I - I_1) R = 0$$

or $$4 I_1 - I_2 = 2 I \quad ...(1)$$

Similarly, from the loop $LMNOL$, we have

$$2 I_2 R - (I - I_2) R - (I_1 - I_2) R = 0$$

or $$- I_1 + 4 I_2 = I \quad ...(2)$$

From the loop $AKPONBEA$, we have

$$2(I - I_1) R + (I - I_2) R = \mathcal{E} \quad ...(3)$$

Solving equations (1) and (2), we get

$$I_1 = \frac{3}{5} I \quad \text{and} \quad I_2 = \frac{2}{5} I$$

Substituting these values in equation (3), we get

$$2\left(I - \frac{3}{5} I\right) R + \left(I - \frac{2}{5} I\right) R = \mathcal{E}$$

or $$\frac{7}{5} IR = \mathcal{E} \quad ...(4)$$

If R' is the equivalent resistance between A and B, then

$$I R' = \mathcal{E} \quad ...(5)$$

From (4) and (5), $$I R' = \frac{7}{5} IR$$

or $$R' = \frac{7}{5} R = \frac{7}{5} \times 10 = \mathbf{14\ \Omega}.$$

Example 152. *Two squares ABCD and BEFC have the side BC in common. The sides are of conducting wires with resistances as follows : AB, BE, FC and CD each* $2\ \Omega$ *; AD, BC, EF each* 1Ω. *A cell of emf 2 V and internal resistance* $2\ \Omega$ *is joined across AD. Find the currents in various branches of the circuit.*

Solution. The current distribution in various branches of the circuit is shown in Fig. 3.162.

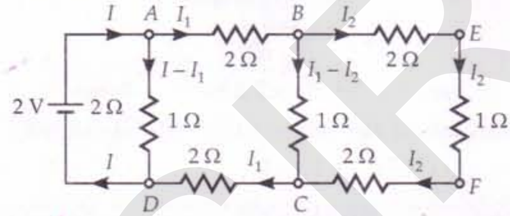


Fig. 3.162

Applying Kirchhoff 's second law to the loop containing the cell and AD, we get

$$2 \times I + 1 \times (I - I_1) = 2$$

or $$3 I - I_1 = 2 \quad ...(1)$$

From the loop $ABCDA$, we get

$$2 \times I_1 + 1 \times (I_1 - I_2) + 2 \times I_1 - 1 \times (I - I_1) = 0$$

or $$- I + 6 I_1 - I_2 = 0 \quad ...(2)$$

Similarly, from the loop $BEFCB$, we get

$$2 \times I_2 + 1 \times I_2 + 2 \times I_2 - 1 \times (I_1 - I_2) = 0$$

or $$- I_1 + 6 I_2 = 0 \quad ...(3)$$

Solving equations (1), (2) and (3), we get

$$I = \frac{70}{99}\ \text{A}, \quad I_1 = \frac{12}{99}\ \text{A}, \quad I_2 = \frac{2}{99}\ \text{A}$$

Currents in different branches are

$$I_{AB} = I_{CD} = I_1 = \mathbf{\frac{12}{99}\ A}, \quad I_{BE} = I_{EF} = I_{CF} = I_2 = \mathbf{\frac{2}{99}\ A}$$

$$I_{AD} = I - I_1 = \mathbf{\frac{58}{99}\ A}, \quad I_{BC} = I_1 - I_2 = \mathbf{\frac{10}{99}\ A}$$

Current through the cell $= I = \mathbf{\frac{70}{99}\ A}$.

Example 153. *Two points A and B are maintained at a constant potential difference of 110 V. A third point is connected to A by two resistances of 100 and* $200\ \Omega$ *in parallel, and to B by a single resistance of* $300\ \Omega$. *Find the current in each resistance and the potential difference between A and C and between C and B.*

Solution. The circuit arrangement and the current distribution is shown in Fig. 3.163.

Applying Kirchhoff's second law to the loop $DEFGHID$, we get

$$I_1 \times 100 - (I - I_1) \times 200 = 0$$

or $$300 I_1 - 200 I = 0 \quad ...(1)$$

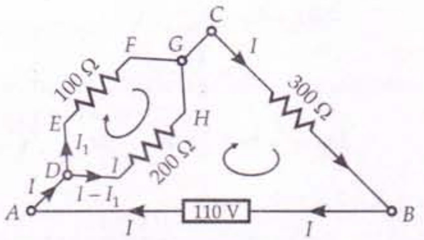


Fig. 3.163

Similarly, from loop *ADIHGCBA*, we get

$$(I - I_1)200 + I \times 300 = 110$$

or $$500\ I - 200\ I_1 = 110 \qquad ...(2)$$

Solving equations (1) and (2), we get

$$I = \frac{3}{10}\ \text{A} \quad \text{and} \quad I_1 = \frac{1}{5}\ \text{A}$$

∴ Current through 100 Ω resistance

$$= I_1 = \frac{\mathbf{1}}{\mathbf{5}}\ \mathbf{A}$$

Current through 200 Ω resistance

$$= I - I_1 = \frac{\mathbf{1}}{\mathbf{10}}\ \mathbf{A}$$

Current through 300 Ω resistance

$$= I = \frac{\mathbf{3}}{\mathbf{10}}\ \mathbf{A}$$

P.D. between *A* and *C* = P.D. across 100 Ω resistor

$$= I_1 \times 100 = \frac{1}{5} \times 100 = \mathbf{20\ V}$$

P.D. between *C* and *B* = P.D. across 300 Ω resistor

$$= I \times 300 = \frac{3}{10} \times 300 = \mathbf{90\ V}.$$

Example 154. *A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of resistance 1 Ω. Determine the equivalent resistance of the network and the current along each edge of the cube.* [NCERT]

Solution. Let 6*I* be the current through the cell. Since the paths *AA'*, *AD* and *AB* are symmetrically placed, current through each of them is same, *i.e.*, 2*I*. At the junctions *A'*, *B* and *D* the incoming current 2*I* splits equally into the two outgoing branches, the current through each branch is *I*, as shown in Fig 3.164. At the junctions *B'*, *C* and *D'*, these currents reunite and the currents along *B'C'*, *D'C'* and *CC'* are 2*I* each. The total current at junction *C'* is 6*I* again.

Applying Kirchhoff's second law to the loop *ABCC'EA*, we get

$$-2\ IR - IR - 2\ IR + \mathcal{E} = 0 \qquad \text{or} \qquad \mathcal{E} = 5\ IR$$

where *R* is the resistance of each edge and $\mathcal{E}$ is the emf of the battery.

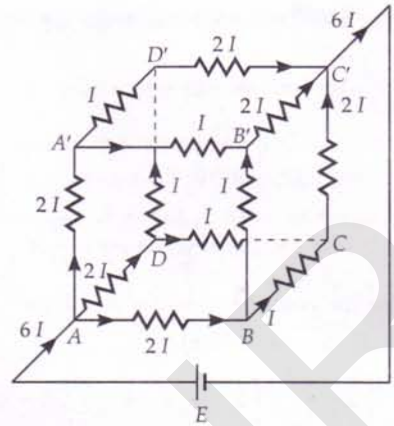


Fig. 3.164

∴ The equivalent resistance of the network is

$$R' = \frac{\text{Total emf}}{\text{Total current}} = \frac{\mathcal{E}}{6\ I} = \frac{5\ IR}{6\ I} = \frac{5}{6}\ R$$

But $R = 1\ \Omega$

$$\therefore \qquad R' = \frac{5}{6}\ \Omega$$

Total current in the network is

$$6\ I = \frac{\mathcal{E}}{R'} = \frac{10}{\frac{5}{6}} = 12\ \text{A or } I = \mathbf{2\ A}$$

The current flowing in each branch can be read off easily.

Example 155. *Twelve wires each having a resistance of r Ω are connected to form a skeleton cube ; find the resistance of the cube between the two corners of the same edge.*

Solution. Let a current $x + 2y$ enter the junction *A* of the cube *ABCDEFGH*. From the symmetry of the parallel paths, current distribution will be as shown in Fig. 3.165.

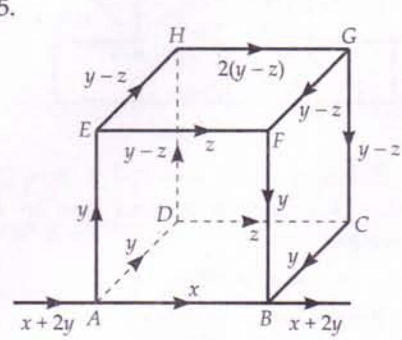


Fig. 3.165

Applying Kirchhoff's second law to the loop *DHGCD*, we get

$$(y - z)r + 2(y - z)r + (y - z)r - z\,r = 0$$

or $$4y\,r - 5z\,r = 0 \quad \text{or} \quad 5z = 4y \quad \text{or} \quad z = \frac{4}{5}\,y$$

Applying Kirchhoff 's second law to the loop *ABCDA*, we get

$$xr - yr - zr - yr = 0$$

or $$x - 2y - z = 0$$

or $$x - 2y - \frac{4}{5}y = 0 \qquad \left(\because \; z = \frac{4}{5}y\right)$$

or $$x = \frac{14}{5}y; \quad y = \frac{5}{14}x$$

Let R be the resistance across AB. Then

P.D. across $AB = xr$

or $$\left(x + \frac{10}{14}x\right)R = xr \quad \text{or} \quad \frac{12}{7}R = r$$

Hence $$R = \frac{7}{12}r\,\Omega.$$

Example 156. *Eleven equal wires each of resistance r form the edges of an incomplete cube. Find the total resistance from one end of the vacant edge of the cube to the other.*

Solution. Let A and B be the vacant edges of the cube. Let an emf $\mathcal{E}$ applied across AB send a current $2x$ in the circuit. Since the paths AD and AE are symmetrical, the current $2x$ at A is divided into two equal parts x and x. At other points, the current is divided as shown in Fig. 3.166, so that again the currents combine at B to give current $2x$. Let R be the total resistance of the cube between A and B.

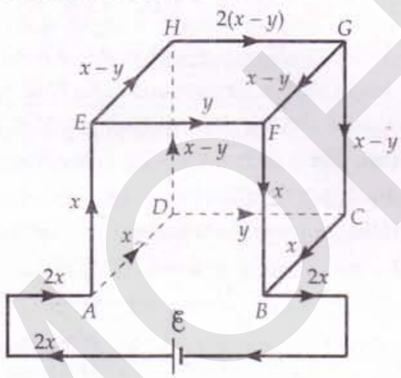


Fig. 3.166

Applying Kirchhoff 's second law to the loop, *ABCDA*, we get

$$xr + yr + xr = \mathcal{E}$$

From Ohm's law,

$$\mathcal{E} = 2x.\,R$$

$$\therefore \qquad 2xr + yr = 2xR$$

Applying Kirchhoff's second law to the loop *EFGHE*, we get

$$yr - (x - y)r - 2(x - y)r - (x - y)r = 0$$

or $$y - x + y - 2x + 2y - x + y = 0$$

or $$-4x + 5y = 0$$

or $$y = \frac{4}{5}x$$

$$\therefore \qquad 2xr + \frac{4}{5}xr = 2x\,R \quad \text{or} \quad \frac{14r}{5} = 2R$$

Hence $R = \mathbf{1.4}\,\boldsymbol{r}\,\Omega$.

Example 157. *Twelve wires each having a resistance of 1 Ω are connected to form a cube. Find the resistance of the cube between two corners of a diagonal of one face of a cube.*

Solution. Imagine a battery connected between points A and C so that a current of 1 A enters junction A. This current is divided equally along AB and AD. The distribution of current in various branches is shown in Fig. 3.167. These currents finally add so that a current of 1 A flows out of junction C.

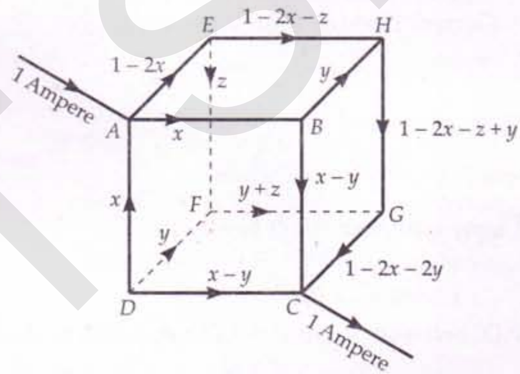


Fig. 3.167

Applying Kirchhoff 's second law to the loop *AEFDA*, we get

$$-(1 - 2x) - z + y + x = 0 \qquad ...(1)$$

Similarly, from the loop *BHGCB*, we have

$$-y - (1 - 2x - z + y) - (1 - 2x + 2y) + (x - y) = 0 \qquad ...(2)$$

Again, from the loop *FGCDF*, we have

$$-(y + z) - (1 - 2x + 2y) + (x - y) - y = 0 \qquad ...(3)$$

On solving equations (1), (2) and (3), we get

$$x = \frac{3}{3}\text{ A}, \quad y = 0, \quad z = \frac{1}{8}\text{ A}$$

Now $$V_{AC} = V_{AB} + V_{BC}$$

$$= 1 \times \frac{3}{8} + 1 \times \frac{3}{8} = \frac{6}{8} = \frac{3}{4}\text{ V}$$

$\therefore$ Equivalent resistance between A and C,

$$R = \frac{V_{AC}}{I} = \frac{3/4}{1} = \frac{\mathbf{3}}{\mathbf{4}}\,\Omega.$$

Example 158. *In the network as shown in Fig. 3.168, each resistance r is of 2 Ω. Find the effective resistance between points A and B.*

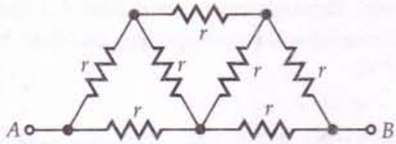


Fig. 3.168

Solution. The distribution of current is shown in Fig. 3.169. By symmetry, current in arm AE = current in arm EB. As the current in arm CE is equal to the current

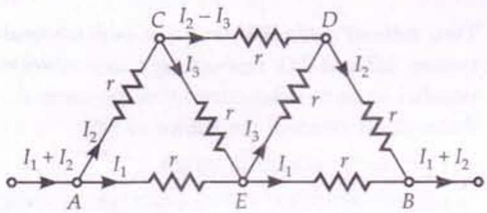


Fig. 3.169

in arm ED, so the resistance of the network will not be affected if the wire CED is disconnected from the wire AEB at the point E, as shown in Fig. 3.170.

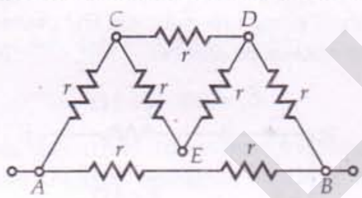


Fig. 3.170

Resistance of wire $ABD \quad = r + r = 2r$

Resistance of wire $ACDEB = r + \dfrac{2r \times r}{2r + r} + r = \dfrac{8r}{3}$

As these two resistances are in parallel, so the equivalent resistance R between points A and B is given by

$$\frac{1}{R} = \frac{1}{2r} + \frac{3}{8r} = \frac{7}{8r} \quad \text{or} \quad R = \frac{8r}{7}$$

Given $r = 2\ \Omega$, therefore, $R = \dfrac{8 \times 2}{7} = \dfrac{\mathbf{16}}{\mathbf{7}}\ \Omega$.

Example 159. *Calculate the equivalent resistance between the points A and B in the network shown in Fig. 3.171.*

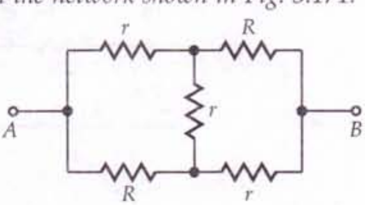


Fig. 3.171

Solution. Suppose a cell of emf $\mathcal{E}$ is connected between A and B. Then the given circuit can be represented by an unbalanced Wheatstone bridge as shown in Fig. 3.172. The distribution of current is also shown.

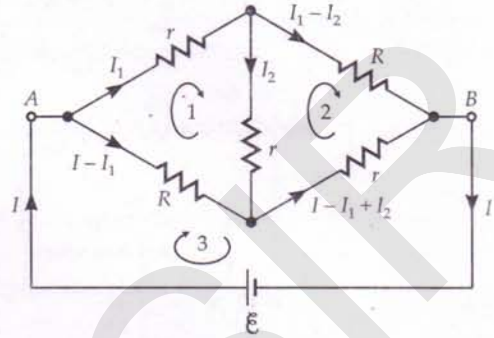


Fig. 3.172

Applying Kirchhoff 's second law to the loop 1, we get

$$I_1 r + I_2 r - (I - I_1) R = 0$$

or $\quad I_1(r + R) + I_2 r - I R = 0 \qquad ...(1)$

From the loop 2, we have

$$(I_1 - I_2) R - (I - I_1 + I_2) r - I_2 r = 0$$

or $\quad I_1 (R + r) - I_2 (R + 2r) - I r = 0 \qquad ...(2)$

Solving equations (1) and (2), we get

$$I_1 = \frac{R + r}{R + 3r} I$$

and $\quad I_2 = \dfrac{R - r}{R + 3r} I \qquad ...(3)$

Similarly, from the loop 3, we have

$$(I - I_1) R + (I - I_1 + I_2) r = \mathcal{E}$$

or $\quad - I_1 (R + r) + I_2 r + I (R + r) = \mathcal{E}$

Substituting the values of I_1 and I_2 from equation (3), we get

$$-\frac{(R + r)^2}{R + 3r} I + \frac{(R - r)}{R + 3r} I + (R + r) I = \mathcal{E}$$

or $\quad \dfrac{3rR + r^2}{R + 3r} I = \mathcal{E}$

Equivalent resistance between A and B,

$$R' = \frac{\mathcal{E}}{I} = \frac{3r R + r^2}{R + 3r} = \frac{r(3R + r)}{(R + 3r)}.$$

Problems For Practice

1. Apply Kirchhoff's rules to the loops *PRSP* and *PRQP* to write the expressions for the currents I_1, I_2 and I_3 in the circuit shown in Fig. 3.173.
[CBSE OD 10]

$$\left(\text{Ans. } \frac{39}{860}\text{ A}, \frac{4}{215}\text{ A}, \frac{11}{172}\text{ A}\right)$$

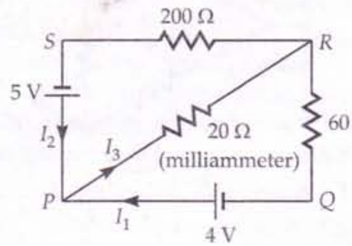


Fig. 3.173

2. Use Kirchhoff's rules to determine the value of the current I_1 flowing in the circuit shown in Fig. 3.174.
[CBSE D 13C]
(Ans. $I_1 = -0.75$ A)

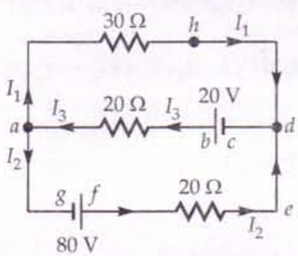


Fig. 3.174

3. Using Kirchhoff's laws, determine the currents I_1, I_2 and I_3 for the network shown in Fig. 3.175.
[CBSE D 99C]
(Ans. 3 A, – 1.5 A, 4.5 A)

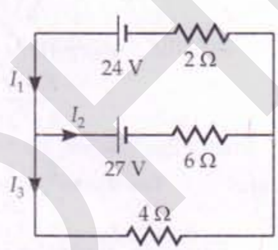


Fig. 3.175

4. The circuit diagram shown in Fig. 3.176 has two cells $\mathcal{E}_1$ and $\mathcal{E}_2$ with emfs 4 V and 2 V respectively, each one having an internal resistance of 2 Ω. The external resistance *R* is of 8 Ω. Find the magnitude and direction of currents flowing through the two cells.
[ISCE 98]

$$\left(\text{Ans. } I_1 = \frac{2}{3}\text{ A}, I_2 = -\frac{1}{3}\text{ A}\right)$$

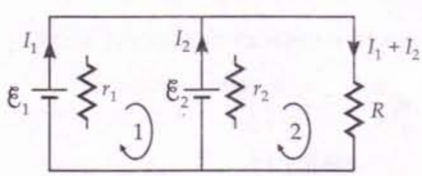


Fig. 3.176

5. Fig. 3.177 shows *n* cells connected to form a series circuit. Their internal resistances are related to their emfs as $r_i = \alpha\, \mathcal{E}_i$, where α is a constant. Find (*i*) the current through the circuit and (*ii*) the potential difference between the terminals of *i*th battery.

[Ans. (*i*) $\frac{1}{\alpha}$ (*ii*) 0]

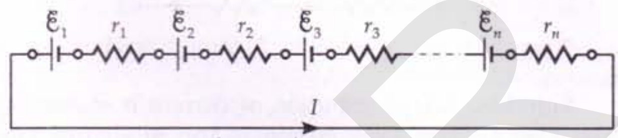


Fig. 3.177

6. Two cells of emfs 3 V and 4 V and internal resistances 1 Ω and 2 Ω respectively are connected in parallel so as to send current in the same direction through an external resistance of 5 Ω.
   (*i*) Draw the circuit diagram.
   (*ii*) Using Kirchhoff's laws, calculate (*a*) the current through each branch of the circuit. (*b*) p.d. across the 5 Ω resistance. **[CBSE OD 95, 96 C]**

$$\left(\text{Ans. } (a)\ \frac{1}{17}\text{ A}, \frac{9}{17}\text{ A}, \frac{8}{17}\text{ A } (b)\ 2.35\text{ V}\right)$$

7. In the electric network shown in Fig. 3.178, use Kirchhoff's rules to calculate the power consumed by the resistance $R = 4\ \Omega$. **[CBSE D 14C]**
(Ans. 9 W)

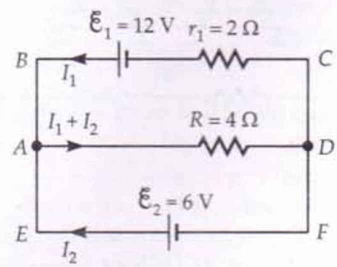


Fig. 3.178

8. A network of resistors is connected to a battery of negligible internal resistance, as shown in Fig. 3.179. Calculate the equivalent resistance between the points *A* and *D*, and the value of the current I_3.
(Ans. 1.25 Ω, 0.5 A)

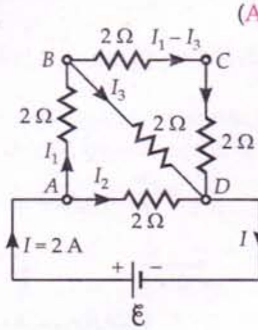


Fig. 3.179

9. Using Kirchhoff's rules, determine the value of unknown resistance R in the circuit shown in Fig. 3.180 so that no current flows through $4\,\Omega$ resistance. Also find the potential difference between A and D. **[CBSE D 12]**

(Ans. 3 V)

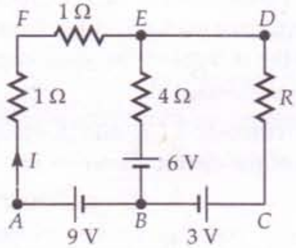


Fig. 3.180

10. Find the current flowing through each cell in the circuit shown in Fig. 3.181. Also calculate the potential difference across the terminals of each cell. (Ans. 0, – 3 A, 3 A, 3 V)

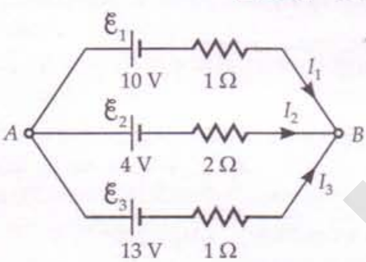
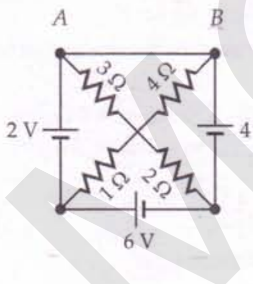


Fig. 3.181

11. In the network shown in Fig. 3.182, (*i*) calculate the current of the 6 V battery and (*ii*) determine the potential difference between the points A and B.

[Ans. (*i*) 2 A (*ii*) 4 V]



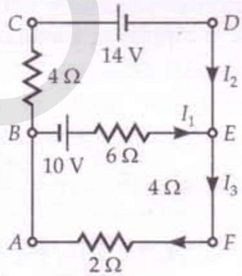


Fig. 3.182 Fig. 3.183

12. In the network shown in Fig. 3.183, find (*i*) the currents I_1, I_2 and I_3 and (*ii*) the potential difference between the points B and E.

[Ans. (*i*) $I_1 = 2$ A, $I_2 = -3$ A, $I_3 = -1$ A (*ii*) – 2 V]

13. Calculate the potential difference between the junctions B and D in the Wheatstone's bridge shown in Fig. 3.184. **[Roorkee 89]** (Ans. 0.2 V)

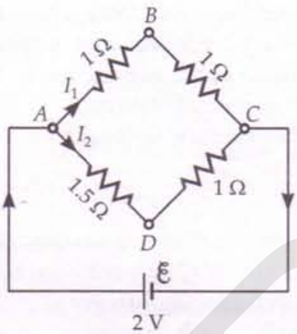


Fig. 3.184

14. In the given electrical networks shown in Figs. 3.185(*a*) and (*b*), identical cells each of emf $\mathcal{E}$, are giving same current I. Find the values of the resistors R_1 and R_2 in the network (*b*).

$\left(\text{Ans. } 9.9\,\Omega, \dfrac{11}{9}\,\Omega\right)$

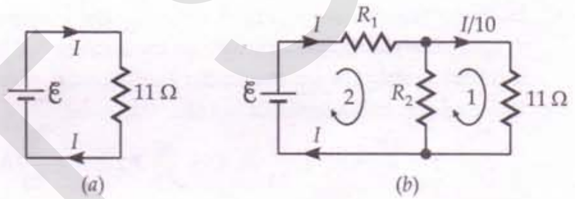


Fig. 3.185

15. What does the ammeter A read in the circuit shown in Fig. 3.186 ? What if the positions of the cell and the ammeter are interchanged ?

$\left(\text{Ans. } \dfrac{5}{11}\text{ A}, \dfrac{5}{11}\text{ A}\right)$

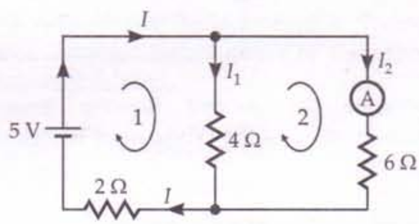


Fig. 3.186

16. In the circuit shown in Fig. 3.187, determine the current in the resistance CD and equivalent resistance between the points A and B. The internal resistance of cell is negligible.

(Ans. 7 Ω, 0.4 A)

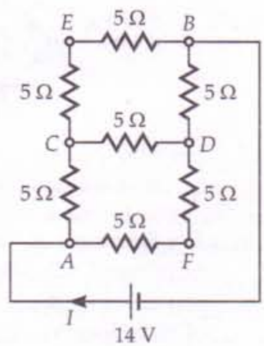


Fig. 3.187

17. A certain length of a uniform wire of resistance 12 Ω is bent into a circle and two points, a quarter of circumference apart, are connected to a battery of emf 4 V and internal resistance 1 Ω. Find the current in the different parts of the circuit.

$\left(\text{Ans. } \frac{12}{13}\text{ A}, \frac{4}{13}\text{ A}\right)$

18. In Fig. 3.188, *ABCDA* is a uniform circular wire of resistance 2 Ω. *AOC* and *BOD* are two wires along two perpendicular diameters of the circle, each having same resistance 1 Ω. A battery of emf ℰ and internal resistance *r* is connected between the points *A* and *D*. Calculate the equivalent resistance of the network.

(Ans. $\frac{15}{14}$ Ω)

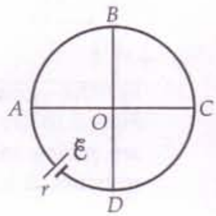


Fig. 3.188

19. In the circuit shown in Fig. 3.189, find the currents I, I_1, I_2 and I_3. Given that emf of the battery = 2 V, internal resistance of the battery = 2 Ω and resistance of the galvanometer = 4 Ω.

$\left(\text{Ans. } I = \frac{47}{91}\text{ A}, I_1 = \frac{17}{91}\text{ A}, I_2 = \frac{30}{91}\text{ A}, I_3 = -\frac{1}{91}\text{ A}\right)$

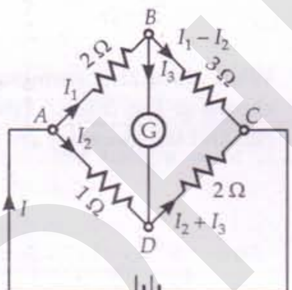


Fig. 3.189

20. Determine the current flowing through the galvanometer *G* of the Wheatstone bridge shown in Fig. 3.190. (Ans. 0.0454 A)

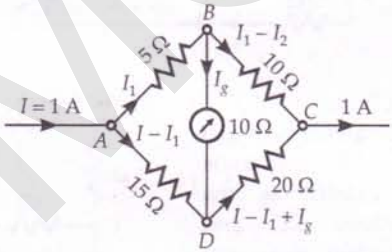


Fig. 3.190

21. The terminals of a battery of emf 3 V and internal resistance 2.5 Ω are joined to the diagonally opposite corners of a cubical skeleton frame of 12 wires, each of resistance 3 Ω. Find the current in the battery.

(Ans. 0.6 A)

22. Twelve identical wires each of resistance 6 Ω are arranged to form a skeleton cube. A current of 40 mA is led into cube at one corner and out at the diagonally opposite corner. Calculate the potential difference developed across these corners and the effective resistance of the network. (Ans. 0.2 V, 5 Ω)

23. Twelve identical wires each of resistance 6 Ω are joined to form a skeleton cube. Find the resistance between the corners of the same edge of the cube.

(Ans. 3.5 Ω)

24. Find the currents I_1, I_2 and I_3 through the three resistors of the circuit shown in Fig. 3.191.

(Ans. Zero in each resistor)

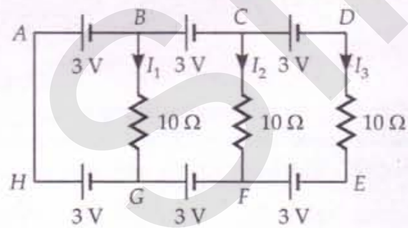


Fig. 3.191

HINTS

1. By Kirchhoff's junction rule,

$$I_3 = I_1 + I_2 \quad \text{...}(i)$$

From loop *PRSP*, $20I_3 + 200I_2 = 5$ ...(*ii*)

From loop *PRQP*, $20I_3 + 60I_1 = 4$ ...(*iii*)

On solving equations (*i*), (*ii*) and (*iii*), we get

$$I_1 = \frac{39}{860}\text{ A}, \quad I_2 = \frac{4}{215}\text{ A}, \quad I_3 = \frac{11}{172}\text{ A}$$

2. Applying Kirchhoff's junction rule at '*a*', we get

$$I_3 = I_1 + I_2$$

Applying Kirchhoff's loop rule to the loop *ahdcba*, we get

$$30I_1 + 20I_3 = 20$$

or $30I_1 + 20(I_1 + I_2) = 20$

or $50I_1 + 20I_2 = 20$

$5I_1 + 2I_2 = 2$ ...(*i*)

Again, from the loop *agfedcba*, we get

$$20I_2 + 20I_3 = 80 + 20$$

or $20I_2 + 20(I_1 + I_2) = 100$

or $20I_1 + 40I_2 = 100$

or $I_1 + 2I_2 = 5$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$$4I_1 = -3 \quad \text{or} \quad I_1 = -0.75\text{ A}$$

The negative sign shows that the actual direction of current I_1 is opposite to that shown in the given circuit diagram.

3. Traversing the upper and lower loops anticlockwise, we get

$$2I_1 + 6I_2 = 24 - 27$$

or $\quad 2I_1 + 6I_2 = -3 \quad$ ...(1)

and $\quad 4I_3 - 6I_2 = 27$

or $\quad 4(I_1 - I_2) - 6I_2 = 27$

or $\quad 4I_1 - 10\,I_2 = 27 \quad$ ...(2)

On solving (1) and (2), we get

$$I_1 = 3\text{ A},\ I_2 = -\mathbf{1.5\ A}$$

$$I_3 = I_1 - I_2 = 3 + 1.5 = \mathbf{4.5\ A}$$

4. Applying Kirchhoff's second law to loop 1, we get

$$I_1 r_1 - I_2 r_2 = \varepsilon_1 - \varepsilon_2$$

or $\quad 2I_1 - 2I_2 = 4 - 2$

or $\quad I_1 - I_2 = 1 \quad$ ...(1)

Similarly, from loop 2, we get

$$I_2 r_2 + (I_1 + I_2)\,R = \varepsilon_2$$

or $\quad 2I_2 + 8(I_1 + I_2) = 2$

or $\quad 4I_1 + 5I_2 = 1 \quad$ ...(2)

On solving equations (1) and (2), we get

$$I_1 = \frac{2}{3}\text{ A}$$

(From –ve to +ve terminal inside ε_1)

and $\quad I_2 = -\frac{1}{3}\text{ A}$

(From +ve to –ve terminal inside ε_2)

5. Suppose a current I flows in the circuit in the indicated direction. Applying Kirchhoff's loop law,

$$Ir_1 + Ir_2 + Ir_3 + \ldots + Ir_n = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots + \varepsilon_n$$

or $\quad I = \dfrac{\varepsilon_1 + \varepsilon_2 + \varepsilon_2 + \ldots + \varepsilon_n}{r_1 + r_2 + r_3 + \ldots + r_n}$

$$= \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots + \varepsilon_n}{\alpha\,(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots + \varepsilon_n)} = \frac{1}{\alpha}.$$

(*ii*) P.D. between the terminals of ith battery

$$= \varepsilon_i - I\,r_i = \varepsilon_i - \frac{1}{\alpha}\,.\,\alpha\,\varepsilon_i = 0.$$

8. Here $R_{BCD} = 2 + 2 = 4\,\Omega$. It is in parallel with $2\,\Omega$ resistance in BD. Their equivalent resistance $= \frac{4 \times 2}{4 + 2} = \frac{4}{3}\,\Omega$. This resistance is in series with 2Ω resistance in AB. Their equivalent resistance $= 2 + 4/3 = 10/3\,\Omega$. This resistance is in parallel with $2\,\Omega$ resistance in AD. The equivalent resistance between A and D,

$$R_{AD} = \frac{\frac{10}{3} \times 2}{\frac{10}{3} + 2} = \frac{5}{4} = \mathbf{1.25\,\Omega}$$

$\therefore \quad \varepsilon = IR = 2 \times 1.25 = 2.5\text{ V}$

Applying Kirchhoff's second law to the lower rectangular loop,

$$2I_2 = \varepsilon = 2.5\text{ V}$$

or $\quad I_2 = 1.25\text{ A}$

Now $\quad I_1 + I_2 = I$

$\therefore \quad I_1 = I - I_2 = 2 - 1.25 = 0.75\text{ A}$

From loop $BCDB$, we get

$$2(I_1 - I_3) + 2(I_1 - I_3) - 2I_3 = 0$$

or $\quad 4I_1 - 6I_3 = 0$

or $\quad I_3 = \frac{4}{6}\,I_1 = \frac{4}{6} \times 0.75 = \mathbf{0.50\ A}$

9. Applying Kirchhoff's loop rule to the loop $AFEBA$,

$$(1+1)\,I + 4 \times 0 = -6 + 9$$

$\therefore \quad I = 1.5\text{ A}$

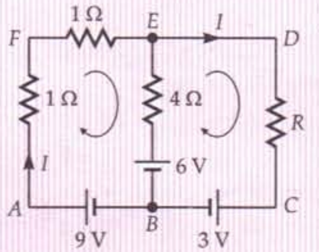


Fig. 3.192

From the loop $BEDCB$, we get

$$1.5\,R + 4 \times 0 = -3 + 6$$

$\therefore \quad R = 2\Omega$

$$V_{AD} = (1+1) \times 1.5 = \mathbf{3\ V}.$$

10. Applying Kirchhoff's first law at the junction B, we get

$$I_1 + I_2 + I_3 = 0 \quad \text{...(1)}$$

Applying Kirchhoff's second law to the loop $AE_1\,BE_2\,A$, we have

$$I_1 \times 1 - I_2 \times 2 = (10 - 4)$$

$$I_1 - 2\,I_2 = 6. \quad \text{...(2)}$$

Similarly, from the closed loop $AE_2BE_3\ A$, we have

$$I_2 \times 2 - I_3 \times 1 = 4 - 13 \text{ or } 2\,I_2 - I_3 = -9 \quad \text{...(3)}$$

Solving equations (1), (2) and (3), we get

$$I_1 = \mathbf{0}, \quad I_2 = -\mathbf{3\ A}, \quad I_3 = \mathbf{3\ A}$$

Thus, the current in the 10 V cell is zero. The current given by the 13 V cell to the circuit is 3 A, and the current taken by the 4 V cell from the circuit is 3 A.

As there is no current in the 10 V cell, so the potential difference across its ends is equal to its e.m.f. *i.e.*, 10 V. Since all the three cells are in parallel, the potential difference across the terminals of each is 10 V.

11. (*i*) The distribution of current in various branches of the circuit is shown in Fig. 3.193.

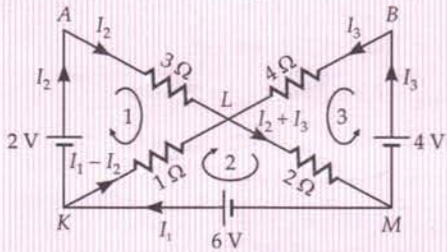


Fig. 3.193

Applying Kirchhoff's second law to loop 1,

$$3I_2 + (I_1 - I_2) = 2$$

or $I_1 + 2I_2 = 2$ ...(*i*)

From loop (2), we get

$$(I_1 - I_2) + 2(I_1 + I_3) = 6$$

or $3I_1 - I_2 + 2I_3 = 6$ ...(*ii*)

From loop (3), we get

$$4I_3 + 2(I_1 + I_3) = 4$$

or $2I_1 + 6I_3 = 4$

or $I_1 + 3I_3 = 2$ ...(*iii*)

On solving equations (*i*), (*ii*) and (*iii*), we get

$$I_1 = \mathbf{2\ A}$$

(*ii*) $V_A - V_B = \varepsilon_2 + \varepsilon_1 - \varepsilon_3 = 2 + 6 - 4 = \mathbf{4\ V}.$

12. (*i*) Applying Kirchhoff's first law at the junction E,

$$I_3 = I_1 + I_2$$ ...(*i*)

From the loop *BCDEB*, we get

$$-6I_1 + 4I_2 = -14 - 10$$

or $-3I_1 + 2I_2 = -12$ ...(*ii*)

From the loop *ABEFA*, we get

$$6I_1 + 2I_3 = 10$$

or $3I_1 + I_3 = 5$ ...(*iii*)

On solving equations (*i*), (*ii*) and (*iii*), we get

$$I_1 = \mathbf{2\ A},\ I_2 = \mathbf{-3A},\ \ I_3 = \mathbf{-1\ A}$$

(*iii*) P.D. between points B and E

$$= 10 - 6I_1 = 10 - 6 \times 2 = \mathbf{-2\ V}.$$

13. Applying Kirchhoff's second law to the loop *ABCEA*, we get

$$I_1 \times 1 + I_1 \times 1 = 2 \quad \therefore \quad I_1 = 1.0\ \text{A}$$

Similarly, from the loop *ADCEA*, we have

$$I_2 \times 1.5 + I_2 \times 1 = 2$$

$$\therefore \qquad I_2 = \frac{2}{2.5} = 0.8\ \text{A}$$

Potential difference between the points *A* and *B* is

$$V_A - V_B = 1.0\ \text{A} \times 1\Omega = 1.0\ \text{V}$$

Potential difference between the points *A* and *D* is

$$V_A - V_D = 0.8\ \text{A} \times 1.5\Omega = 1.2\ \text{V}$$

∴ Potential difference between the points *B* and *D* is

$$V_B - V_D = (V_A - V_D) - (V_A - V_B)$$
$$= 1.2 - 1.0 = \mathbf{0.2\ V}.$$

14. From the network of Fig. 3.194(*a*), $\varepsilon = 11\,I$

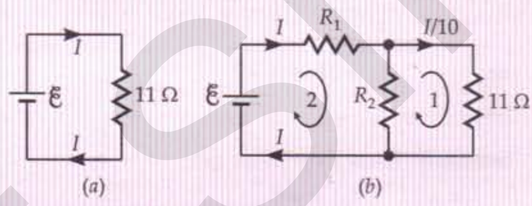


Fig. 3.194

In the network Fig. 3.194(*b*), the main current *I* passes through R_1, a part $\frac{I}{10}$ through the 11Ω resistor and the remaining current, $I - \frac{I}{10} = \frac{9I}{10}$ through the resistor R_2.

Applying Kirchhoff's law to the loop 1, we get

$$\frac{I}{10} \times 11 - \frac{9I}{10} \times R_2 = 0 \quad \text{or} \quad R_2 = \mathbf{\frac{11}{9}}\,\Omega$$

Similarly, from the loop 2, we get

$$I R_1 + \frac{9I}{10} \times R_2 = \varepsilon \quad \text{or} \quad I R_1 + \frac{9I}{10} \times \frac{11}{9} = 11\,I$$

$$\therefore \qquad R_1 = \mathbf{9.9\,\Omega}.$$

15. From Kirchhoff's first law, $\quad I = I_1 + I_2$

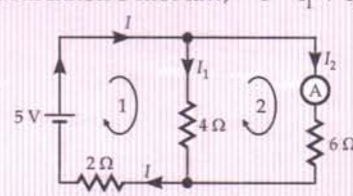


Fig. 3.195

Applying Kirchhoff's second law for the loop 1 of Fig. 3.195, we get

$$I_1 \times 4 + I \times 2 = 5$$

or $I_1 \times 4 + (I_1 + I_2) \times 2 = 5$

or $6\,I_1 + 2\,I_2 = 5$ ...(1)

Similarly, from the loop 2, we get

$$I_2 \times 6 - I_1 \times 4 = 0$$

or $4\,I_1 = 6\,I_2.$ ...(2)

Solving equations (1) and (2), $I_2 = \frac{5}{11}$ **A**

This will be the reading of the ammeter.

On interchanging the cell and the ammeter, the circuit takes the form as shown in Fig. 3.196. Again, we can show that

$$I_2 = \frac{5}{11} \text{ A.}$$

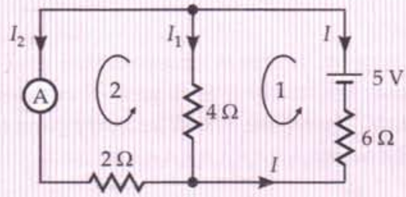


Fig. 3.196

16. Proceeding as in Example 151, we obtain equivalent resistance between points A and B as

$$R' = \frac{7}{5} R = \frac{7}{5} \times 5 = 7\Omega \qquad [\because R = 5\,\Omega]$$

Main current,

$$I = \frac{\mathcal{E}}{R} = \frac{14}{7} = 2 \text{ A}$$

Current through 5 Ω resistance in arm CD

$$= I_1 - I_2 = \frac{3}{5} I - \frac{2}{5} I = \frac{1}{5} I = \frac{1}{5} \times 2 = \mathbf{0.4 \text{ A.}}$$

18. The current distribution is shown in Fig. 3.197.

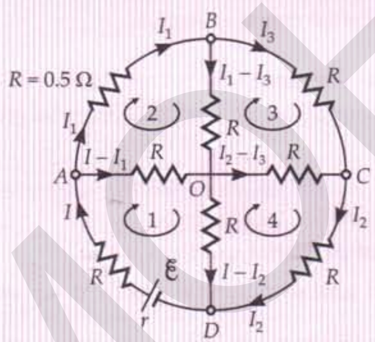


Fig. 3.197

Applying Kirchhoff's law to different loops, we get

$$R(I - I_1) + R(I - I_2) + (R + r)\, I = \mathcal{E} \qquad ...(1)$$

$$RI_1 + R(I_1 - I_3) - R(I - I_1) = 0 \qquad ...(2)$$

$$RI_3 - R(I_2 - I_3) - R(I_1 - I_3) = 0 \qquad ...(3)$$

$$RI_2 - R(I - I_2) + R(I_2 - I_3) = 0 \qquad ...(4)$$

On simplifying and solving these equations,

$$I_1 = I_2, \quad I_3 = \frac{2}{3} I_2, \quad I = \frac{7}{3} I_2$$

and $$\frac{7}{3} I_2 r + 5 I_2 R = \mathcal{E}$$

If R' is the equivalent resistance of the network, then

$$I(r + R') = \mathcal{E}$$

$$\therefore \frac{7}{3} I_2 r + 5 I_2 R = I(r + R') = \frac{7}{3} I_2 (r + R')$$

or $$R' = \frac{15}{7} R = \frac{15}{7} \times 0.5 = \mathbf{\frac{15}{14}\,\Omega.}$$

19. Applying Kirchhoff's first law at the junction A,

$$I = I_1 + I_2 \qquad ...(i)$$

Applying Kirchhoff's second law to the loop $ABDA$, we get

$$2I_1 + 4I_3 - I_2 = 0 \qquad ...(ii)$$

From the loop $BCDB$, we get

$$3(I_1 - I_3) - 2(I_2 + I_3) - 4I_3 = 0$$

or $$3I_1 - 2I_2 - 9I_3 = 0 \qquad ...(iii)$$

From the loop $ABCEA$, we get

$$2I_1 + 3(I_1 - I_3) + 2(I_1 + I_2) = 2$$

or $$7I_1 + 2I_2 - 3I_3 = 2 \qquad ...(iv)$$

On solving equations (*i*), (*ii*) and (*iii*), we get

$$I_1 = \mathbf{\frac{17}{91}\text{ A}},\; I_2 = \mathbf{\frac{30}{91}\text{ A}} \text{ and } I_3 = -\mathbf{\frac{1}{91}\text{ A}}$$

20. From the loop $ABDA$, we get

$$5I_1 + 10I_g - (1 - I_1)\,15 = 0 \qquad [I = 1\text{ A}]$$

or $$20I_1 + 10I_g = 15$$

or $$4I_1 + 2I_g = 3 \qquad ...(i)$$

From the loop $BCDB$, we get

$$10(I_1 - I_g) - 20(1 - I_1 + I_g) - 10I_g = 0$$

$$30I_1 - 40I_g = 20$$

or $$3I_1 - 4I_g = 2 \qquad ...(ii)$$

On solving equations (*i*) and (*ii*), we get

$$I_g = \frac{1}{22} \text{ A} = \mathbf{0.0454 \text{ A}}$$

21. Proceeding as in Example 154, we obtain the effective resistance,

$$R' = \frac{5}{6} R$$

But $R = 3\,\Omega$, therefore, $R = \frac{5 \times 3}{6} = 2.5\,\Omega$

Total resistance of the circuit = 2.5 + 2.5 = 5.0 Ω

Current,

$$I = \frac{\text{emf}}{\text{total resistance}} = \frac{3}{5.0} = \mathbf{0.6 \text{ A.}}$$

22. Proceeding as in Example 154, we obtain the effective resistance, $R' = \frac{5}{6} R$

But $R = 6\,\Omega$, therefore, $R' = \frac{5 \times 6}{6} = 5\,\Omega$

P.D. developed = Resistance × Current

$$= 5 \times (40 \times 10^{-3}) = \mathbf{0.2 \text{ V.}}$$

23. Proceeding as in Example 155, we obtain effective resistance, $R = \frac{7}{12} r$

But $r = 6\,\Omega$, therefore, $R = \frac{7 \times 6}{12} = \mathbf{3.5\,\Omega}$.

24. From the loop *ABGHA*, we get

$$10I_1 = 3 - 3 \quad \text{or} \quad I_1 = 0.$$

From the loop *BCFGB*, we get

$$10I_2 - 10I_1 = 3 - 3 \quad \text{or} \quad I_2 = 0.$$

From the loop *CDEFC*, we get

$$10I_3 - 10I_2 = 3 - 3 \quad \text{or} \quad I_3 = 0.$$

3.32 POTENTIOMETER

57. *What is a potentiometer ? Give its construction and principle.*

Potentiometer. An ideal voltmeter which does not change the original potential difference, needs to have infinite resistance. But a voltmeter cannot be designed to have an infinite resistance. Potentiometer is one such device which does not draw any current from the circuit and still measures the potential difference. So it acts as an ideal voltmeter.

A potentiometer is a device used to measure an unknown emf or potential difference accurately.

Construction. As shown in Fig. 3.198, a potentiometer consists of a long wire *AB* of uniform cross-section, usually 4 to 10 m long, of material having high resistivity and low temperature coefficient such as constantan or manganin. Usually, 1 m long separate pieces of wire are fixed on a wooden board parallel to each other. The wires are joined in series by thick copper strips. A metre scale is fixed parallel to the wires. The ends *A* and *B* are connected to a strong battery, a plug key *K* and a rheostat *Rh*. This circuit, called *driving* or *auxiliary circuit*, sends a constant current *I* through the wire *AB*. Thus, the potential gradually falls from *A* to *B*. A jockey can slide along the length of the wire.

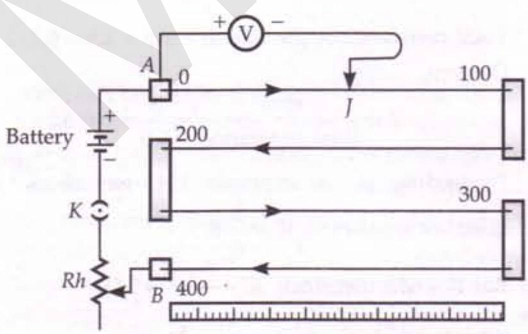


Fig. 3.198 Principle of a potentiometer.

Principle. *The basic principle of a potentiometer is that when a constant current flows through a wire of uniform cross-sectional area and composition, the potential drop across any length of the wire is directly proportional to that length.*

In Fig. 3.198, if we connect a voltmeter between the end *A* and the jockey *J*, it reads the potential difference *V* across the length *l* of the wire *AJ*. By Ohm's law,

$$V = IR = I \cdot \frac{\rho l}{A} \qquad \left[\because R = \rho \frac{l}{A}\right]$$

For a wire of uniform cross-section and uniform composition, resistivity ρ and area of cross-section *A* are constants. Therefore, when a steady current *I* flows through the wire,

$$\frac{I\rho}{A} = \text{a constant, } k$$

Hence $\quad V = k\,l \quad \text{or} \quad V \propto l$

This is the principle of a potentiometer. A graph drawn between *V* and *l* will be a straight line passing through the origin *O*, as shown in Fig. 3.199.

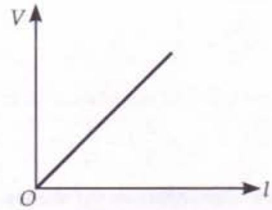


Fig. 3.199 Potential drop $V \propto$ length l

Potential gradient. *The potential drop per unit length of the potentiometer wire is known as potential gradient*. It is given by

$$k = \frac{V}{l}$$

SI unit of potential gradient = Vm^{-1}

Practical unit of potential gradient = V cm^{-1}.

3.33 APPLICATIONS OF A POTENTIOMETER

58. *With the help of a circuit diagram, explain how can a potentiometer be used to compare the emfs of two primary cells.*

Comparison of emfs of two primary cells. Fig. 3.200 shows the circuit diagram for comparing the emfs of two cells. A constant current is maintained in the potentiometer wire *AB* by means of a battery of emf $\mathcal{E}$ through a key *K* and rheostat *Rh*. Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be the emfs of the two primary cells which are to be compared. The positive terminals of these cells are connected to the end *A* of the potentiometer wire and their negative terminals are connected to a high resistance box R.B., a galvanometer G and a jockey J through a two way key. A high resistance R is inserted in the circuit from resistance box R.B. to prevent excessive currents flowing through the galvanometer.

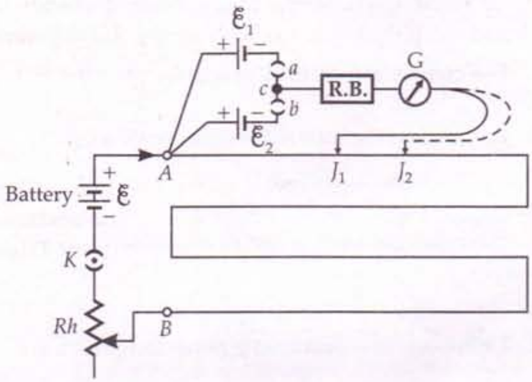


Fig. 3.200 Comparing emfs of two cells by a potentiometer.

As the plug is inserted between a and c, the cell ε_1 gets introduced in the circuit. The jockey J is moved along the wire AB till the galvanometer shows no deflection. Let the position of the jockey be J_1 and length of wire $AJ_1 = l_1$. If k is the potential gradient along the wire AB, then at null point,

$$\varepsilon_1 = kl_1$$

By inserting the plug between b and c, the null point is again obtained for cell ε_2. Let the balancing length be $AJ_2 = l_2$. Then

$$\varepsilon_2 = kl_2$$

Hence, $$\frac{\varepsilon_2}{\varepsilon_1} = \frac{l_2}{l_1}$$

If one of the two cells is a standard cell of known emf, then emf of the other cell can be determined.

$$\varepsilon_2 = \frac{l_2}{l_1} \cdot \varepsilon_1$$

In order to get the null point on the potentiometer wire, it is necessary that the emf, ε of the auxiliary battery must be greater than both ε_1 and ε_2.

59. *With the help of a circuit diagram, explain how can a potentiometer be used to measure the internal resistance of a primary cell.*

Internal resistance of a primary cell by a potentiometer. As shown in the Fig. 3.201, the +ve terminal of the cell of emf ε whose internal resistance r is to be measured is connected to the end A of the potentiometer wire and its negative terminal to a galvanometer G and jockey J. A resistance box R.B. is connected across the cell through a key K_2.

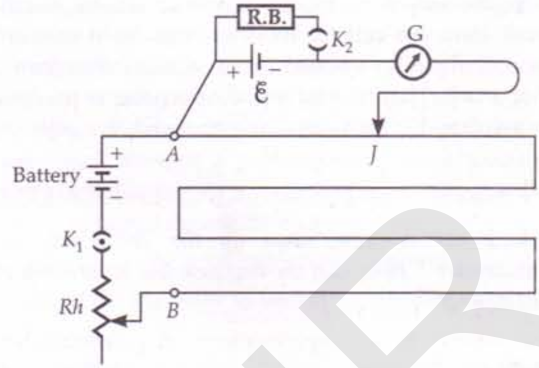


Fig. 3.201 To determine the internal resistance of a cell by a potentiometer.

Close the key K_1. A constant current flows through the potentiometer wire. With key K_2 kept open, move the jockey along AB till it balances the emf ε of the cell. Let l_1 be the balancing length of the wire. If k is the potential gradient, then emf of the cell will be

$$\varepsilon = kl_1$$

With the help of resistance box R.B., introduce a resistance R and close key K_2. Find the balance point for the terminal potential difference V of the cell. If l_2 is the balancing length, then

$$V = kl_2$$

$$\therefore \quad \frac{\varepsilon}{V} = \frac{l_1}{l_2}$$

Let r be the internal resistance of the cell. If current I flows through cell when it is shunted with resistance R, then from Ohm's law we get

$$\varepsilon = I(R + r) \quad \text{and} \quad V = IR$$

$$\therefore \quad \frac{\varepsilon}{V} = \frac{R + r}{R} = \frac{l_1}{l_2}$$

$$1 + \frac{r}{R} = \frac{l_1}{l_2}$$

or $$\frac{r}{R} = \frac{l_1 - l_2}{l_2}$$

$\therefore$ Internal resistance, $$r = R\left[\frac{l_1 - l_2}{l_2}\right].$$

60. *Why is a potentiometer preferred over a voltmeter for measuring the emf of a cell ?*

Superiority of a potentiometer to a voltmeter. Potentiometer is a null method device. At null point, it does not draw any current from the cell and thus there is no potential drop due to the internal resistance of the cell. It measures the p.d. in an open circuit which is equal to the actual emf of the cell.

On the other hand, a voltmeter draws a small current from the cell for its operation. So it measures the terminal p.d. in a closed circuit which is less than the emf of a cell. That is why a potentiometer is preferred over a voltmeter for measuring the emf of a cell.

3.34 SENSITIVENESS OF A POTENTIOMETER

61. *What do you mean by the sensitivity of a potentiometer ? How can we increase the sensitivity of a potentiometer ?*

Sensitivity of a potentiometer. *A potentiometer is sensitive if*

(*i*) *it is capable of measuring very small potential differences, and*

(*ii*) *it shows a significant change in balancing length for a small change in the potential difference being measured.*

The sensitivity of a potentiometer depends on the potential gradient along its wire. *Smaller the potential gradient, greater will be the sensitivity of the potentiometer.*

The sensitivity of a potentiometer can be increased by reducing the potential gradient. This can be done in two ways :

(*i*) For a given potential difference, the sensitivity can be increased by increasing the length of the potentiometer wire.

(*ii*) For a potentiometer wire of fixed length, the potential gradient can be decreased by reducing the current in the circuit with the help of a rheostat.

For Your Knowledge

- A potentiometer can be regarded as an *ideal voltmeter with infinite resistance* because it does not draw any current from the source of emf at the null point.
- The principle of potentiometer requires that (*i*) the potentiometer wire should be of uniform cross-section and (*ii*) the current through the wire should remain constant.
- The emf of the auxiliary battery must be greater than the emf of the cell to be measured.
- The balance point cannot be obtained on the potentiometer if the fall of potential along the potentiometer wire due to the auxiliary battery is less than the emf of the cell to be measured.
- The positive terminals of the auxiliary battery and the cell whose emf is to be determined must be connected to the zero end of the potentiometer.
- **Other uses of a potentiometer.** Any physical quantity that can produce or control a potential difference can be measured using a potentiometer. Thus, a potentiometer can be used to measure and control stress, temperature, radiation, pH, frequency, etc.

Examples based on

(i) Comparison of EMFs of two Cells (ii) Measurement of Internal Resistance of a Cell by a Potentiometer

Formulae Used

1. For comparing emfs of two cells, $\frac{\varepsilon_2}{\varepsilon_1} = \frac{l_2}{l_1}$
2. For measuring internal resistance of a cell, $$r = \frac{l_1 - l_2}{l_2} \times R$$
3. Potential gradient of the potentiometer wire, $$k = \frac{V}{l}$$
4. Unknown emf balanced against length l, $\varepsilon = k\,l$

Units Used

The emfs ε_1 and ε_2 are in volt, lengths l_1 and l_2 of potentiometer wire in metre.

Example 160. *A potentiometer wire is 10 m long and has a resistance of 18 Ω. It is connected to a battery of emf 5 V and internal resistance 2 Ω. Calculate the potential gradient along the wire.*

Solution. Here $l = 10$ m, $R = 18\,\Omega$, $\varepsilon = 5$ V, $r = 2\,\Omega$

Current through the potentiometer wire,

$$I = \frac{\varepsilon}{R + r} = \frac{5}{18 + 2} = \frac{5}{20} = \frac{1}{4}\text{ A}$$

$$\therefore \text{ Potential gradient} = \frac{IR}{l} = \frac{1}{4} \times \frac{18}{10} = \mathbf{0.45\ Vm^{-1}}.$$

Example 161. *A potentiometer wire is supplied a constant voltage of 3 V. A cell of emf 1.08 V is balanced by the voltage drop across 216 cm of the wire. Find the total length of the potentiometer wire.*

Solution. Here $\varepsilon = 3$ V, $\varepsilon_1 = 1.08$ V, $l_1 = 216$ cm, $l = ?$

$$\text{As } \frac{\varepsilon}{\varepsilon_1} = \frac{l}{l_1} \quad \therefore\ l = \frac{\varepsilon}{\varepsilon_1} \times l_1 = \frac{3 \times 216}{1.08} = \mathbf{600\ cm.}$$

Example 162. *Two cells of emfs* ε_1 and ε_2 ($\varepsilon_1 > \varepsilon_2$) *are connected as shown in Fig. 3.202.*

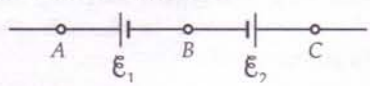


Fig. 3.202

When a potentiometer is connected between A and B, the balancing length of the potentiometer wire is 300 cm. On connecting the same potentiometer between A and C, the balancing length is 100 cm. Calculate the ratio of ε_1 *and* ε_2.

[CBSE D 94]

Solution. As emf ∝ balancing length of the potentiometer wire

∴ When the potentiometer is connected between A and B, $\varepsilon_1 \propto 300$

When potentiometer is connected between A and C,

$$\varepsilon_1 - \varepsilon_2 \propto 100$$

Hence $\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1} = \frac{100}{300}$ or $1 - \frac{\varepsilon_2}{\varepsilon_1} = \frac{100}{300}$

or $\frac{\varepsilon_2}{\varepsilon_1} = 1 - \frac{1}{3} = \frac{2}{3}$ or $\frac{\varepsilon_1}{\varepsilon_2} = \frac{3}{2} = \mathbf{3:2.}$

Example 163. *In Fig. 3.203, a long uniform potentiometer wire AB is having a constant potential gradient along its length. The null points for the two primary cells of emfs ε_1 and ε_2 connected in the manner shown are obtained at a distance of 120 cm and 300 cm from the end A. Find (i) $\varepsilon_1 / \varepsilon_2$ and (ii) position of null point for the cell ε_1.*

How is the sensitivity of a potentiometer increased ? [CBSE D 12]

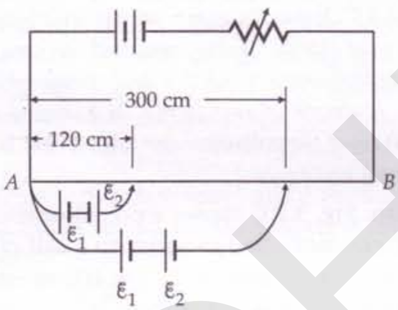


Fig. 3.203

Solution. (*i*) Let k be the potential gradient in volt/cm. Then

$$\varepsilon_1 + \varepsilon_2 = 300k \quad \text{and} \quad \varepsilon_1 - \varepsilon_2 = 120k$$

∴ $\varepsilon_1 = 210k$ and $\varepsilon_2 = 90k$

Hence, $\frac{\varepsilon_1}{\varepsilon_2} = \frac{7}{3}$

(*ii*) As $\varepsilon_1 = 210k$

∴ Balancing length for cell ε_1 is

$$l_1 = \frac{\varepsilon_1}{k} = \mathbf{210\ cm}$$

The sensitivity of a potentiometer wire can be increased by decreasing potential gradient either through increasing length of the potentiometer wire or through increasing resistance put in series with the main cell.

Example 164. *In a potentiometer, a standard cell of emf 5 V and of negligible resistance maintains a steady current through the potentiometer wire of length 5 m. Two primary cells of emfs ε_1 and ε_2 are joined in series with (i) same polarity, and (ii) opposite polarity. The combination is connected through a galvanometer and a jockey to the potentiometer. The balancing lengths in the two cases are found to be 350 cm and 50 cm respectively.*

(*i*) *Draw the necessary circuit diagram.*

(*ii*) *Find the value of the emfs of the two cells.* [CBSE D 04C]

Solution. (*i*) The circuit diagram is shown in Fig. 3.204.

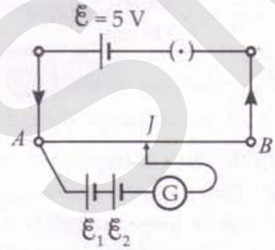


Fig. 3.204

(*ii*) Here $k = \frac{5\ \text{V}}{5\ \text{m}} = \frac{5\ \text{V}}{500\ \text{cm}} = \frac{1}{100}\ \text{V cm}^{-1}$

In first case,

$$\varepsilon_1 + \varepsilon_2 = kl_1 = \frac{1}{100} \times 350$$

or $\varepsilon_1 + \varepsilon_2 = 3.50$ V ...(*i*)

In second case,

$$\varepsilon_1 - \varepsilon_2 = kl_2 = \frac{1}{100} \times 50 = 0.50\ \text{V} \quad ...(ii)$$

On solving (*i*) and (*ii*), we get

$$\varepsilon_1 = \mathbf{2.0\ V} \quad \text{and} \quad \varepsilon_2 = \mathbf{1.50\ V}.$$

Example 165. *A 10 metre long wire of uniform cross-section of 20 Ω resistance is used as a potentiometer wire. This wire is connected in series with a battery of 5 V, along with an external resistance of 480 Ω. If an unknown emf ε is balanced at 600 cm of this wire, calculate (i) the potential gradient of the potentiometer wire and (ii) the value of the unknown emf ε.* [CBSE D 06]

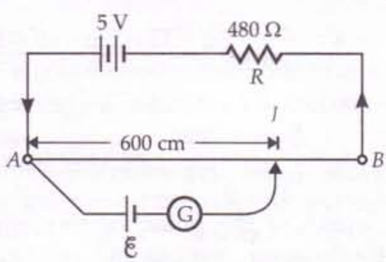


Fig. 3.205

Solution. Current in the circuit or through the potentiometer wire is

$$I = \frac{V}{R_{AB} + R} = \frac{5\text{ V}}{(20 + 480)\,\Omega} = 0.01\text{ A}$$

Resistance of potentiometer wire,

$$R_{AB} = 20\,\Omega$$

∴ P.D. across the wire,

$$V = IR_{AB} = 0.01 \times 20 = 0.2\text{ V}$$

Length of potentiometer wire,

$$l = 10\text{ m} = 1{,}000\text{ cm}$$

∴ Potential gradient,

$$k = \frac{V}{l} = \frac{0.2\text{ V}}{1{,}000\text{ cm}} = 0.0002\text{ V cm}^{-1}$$

Unknown emf balanced against 600 cm length is

$$\mathcal{E} = kl' = 0.0002 \times 600 = \mathbf{0.12\text{ V}}.$$

Example 166. *In the circuit diagram given below, AB is a uniform wire of resistance 15 ohm and length one metre. It is connected to a series arrangement of cell* $\mathcal{E}_1$ *of emf 2.0 V and negligible internal resistance and a resistor R. Terminal A is also connected to an electrochemical cell* $\mathcal{E}_2$ *of emf 75 mV and a galvanometer G. In this set-up, a balancing point is obtained at 30 cm mark from A. Calculate the resistance of R. If* $\mathcal{E}_2$ *were to have an emf of 300 mV, where will you expect the balancing point to be ?* [CBSE D 99C]

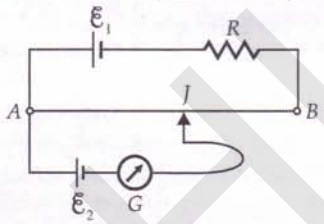


Fig. 3.206

Solution. Current through the potentiometer wire,

$$I = \frac{\mathcal{E}_1}{R + R_{AB}} = \frac{2}{R + 15}$$

Resistance of the 30 cm length of wire, which balances the emf $\mathcal{E}_2$, is

$$R' = \frac{15}{100} \times 30 = 4.5\,\Omega$$

Now, $\mathcal{E}_2$ = Potential drop across R'

$$\therefore\ 75 \times 10^{-3} = \frac{2}{R + 15} \times 4.5$$

or $$R = \frac{2 \times 4.5}{75 \times 10^{-3}} - 15 = 120 - 15 = \mathbf{105\,\Omega}.$$

For $\mathcal{E}_2 = 300$ mV, the balancing length is given by

$$l_2 = \frac{\mathcal{E}_2}{\mathcal{E}_1} . l_1 = \frac{300}{75} \times 30 = 120\text{ cm}$$

As the length of the potentiometer wire is just 100 cm, so this balance point cannot be obtained on the wire.

Example 167. *The length of a potentiometer wire is 5 m. It is connected to a battery of constant emf. For a given Leclanche cell, the position of zero galvanometer deflection is obtained at 100 cm. If the length of the potentiometer wire be made 8 m instead of 5 m, calculate the length of wire for zero deflection in the galvanometer for the same cell.* [CBSE F 97]

Solution. Here $l = 5$ m, $l_1 = 100$ cm $= 1$ m, $l' = 8$ m, $l_1' = ?$

Let $\mathcal{E}$ be the emf of the Leclanche cell.

In first case, $$\mathcal{E} = \frac{IR\,l_1}{l} \quad ...(1)$$

In second case, $$\mathcal{E} = \frac{IRl_1'}{l'} \quad ...(2)$$

Comparing equations (1) and (2),

$$\frac{l_1'}{l'} = \frac{l_1}{l}$$

or $$l_1' = \frac{l_1}{l} \times l' = \frac{1}{5} \times 8 = \mathbf{1.6\text{ m}}.$$

Example 168. *A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a battery of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. What is the value of the external resistance ?* [IIT]

Solution. Fig. 3.207 shows a potentiometer wire of length 100 cm connected in series to a cell of emf 2 V and an unknown resistance *R*. The cell of emf 10 mV balances length $AJ = 40$ cm of the wire.

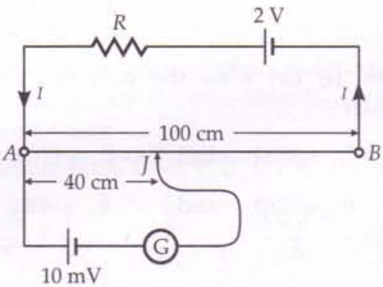


Fig. 3.207

Resistance of wire $AJ = \frac{10}{100} \times 40 = 4\,\Omega$

Current through wire *AJ*,

$$I = \frac{10\text{ mV}}{4\,\Omega} = \frac{10 \times 10^{-3}\text{ V}}{4\,\Omega} = 2.5 \times 10^{-3}\text{ A}$$

The same current flows through the potentiometer wire and through the external resistance *R*.

Total resistance $= (R + 10)\,\Omega$

$$\therefore \quad 2.5\times10^{-3}\,\text{A} = \frac{2\ \text{V}}{(R+10)\,\Omega}$$

or $$R+10 = \frac{2}{2.5\times10^{-3}} = 800$$

$$R = 800 - 10 = \mathbf{790\ \Omega}.$$

Example 169. *AB is 1 metre long uniform wire of* 10 Ω *resistance. Other data are as shown in Fig. 3.208. Calculate (i) potential-gradient along AB and (ii) length AO, when galvanometer shows no deflection.* [CBSE D 2000C]

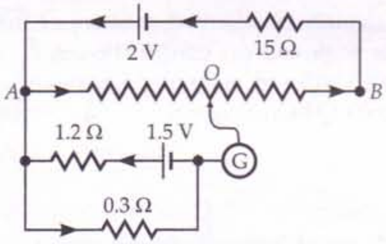


Fig. 3.208

Solution. (*i*) Total resistance of the primary circuit

$= 15 + 10 = 25\,\Omega$, emf = 2 V

∴ Current in the wire *AB*,

$$I = \frac{2}{25} = 0.08\ \text{A}$$

P.D. across the wire *AB*

= Current × resistance of wire *AB*

$= 0.08 \times 10 = 0.8$ V

Potential gradient

$$= \frac{\text{P.D.}}{\text{length}} = \frac{0.8}{100} = \mathbf{0.008\ V\ cm^{-1}}.$$

(*ii*) Resistance of secondary circuit

$= 1.2 + 0.3 = 1.5\,\Omega$

emf = 1.5 V

Current in the secondary circuit $= \frac{1.5}{1.5} = 1.0$ A

The same is the current in 0.3 Ω resistor.

P.D. between points *A* and *O*

= P.D. across 0.3 Ω resistor in the zero-deflection condition

= Current × resistance

$= 1.0 \times 0.3 = 0.3$ V

Length *AO*

$$= \frac{\text{Potential difference}}{\text{Potential gradient}}$$

$$= \frac{0.3\ \text{V}}{0.008\ \text{V cm}^{-1}} = \mathbf{37.5\ cm.}$$

Example 170. *A cell gives a balance with 85 cm of a potentiometer wire. When the terminals of the cell are shorted through a resistance of 7.5 Ω, the balance is obtained at 75 cm. Find the internal resistance of the cell.* **[ISCE 95]**

Solution. Here $l_1 = 85$ cm, $l_2 = 75$ cm, $R = 7.5\,\Omega$

Internal resistance,

$$r = R\,\frac{(l_1 - l_2)}{l_2} = 7.5\left(\frac{85-75}{75}\right) = \mathbf{1\ \Omega}.$$

Example 171. *When a resistor of 5 Ω is connected across cell, its terminal p.d. is balanced by 150 cm of potentiometer wire and when a resistor of 10 Ω resistance is connected across the cell, the terminal p.d. is balanced by 175 cm of the potentiometer wire. Find the internal resistance of the cell.*

Solution. In the first case, $r = R_1\left(\frac{l-l_1}{l_1}\right)$

$$\therefore \quad r\frac{l_1}{R_1} = l - l_1 \qquad \text{...(1)}$$

In the second case,

$$r = R_2\left(\frac{l-l_2}{l_2}\right)$$

$$\therefore \quad r\frac{l_2}{R_2} = l - l_2 \qquad \text{...(2)}$$

Subtracting (2) from (1),

$$r\left[\frac{l_1}{R_1} - \frac{l_2}{R_2}\right] = l - l_1 - l + l_2$$

$$\therefore \quad r = \frac{l_2 - l_1}{\frac{l_1}{R_1} - \frac{l_2}{R_2}} = \frac{175-150}{\frac{150}{5} - \frac{175}{10}} = \frac{25}{12.5} = \mathbf{2\ \Omega.}$$

Problems For Practice

1. A potentiometer wire is 10 m long and a potential difference of 6 V is maintained between its ends. Find the emf of a cell which balances against a length of 180 cm of the potentiometer wire.

(Ans. 1.08 V)

2. The resistance of a potentiometer wire of length 10 m is 20 Ω. A resistance box and a 2 volt accumulator are connected in series with it. What resistance should be introduced in the box to have a potential drop of one microvolt per millimetre of the potentiometer wire ? **[Kerala 94]**

(Ans. 3980 Ω)

3. In a potentiometer arrangement, a cell of emf 1.20 volt gives a balance point at 30 cm length of the wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emfs of the two

cells is 1.5, calculate the difference in the balancing length of the potentiometer wire in the two cases.

[CBSE D 06C] (Ans. 10 cm)

4. Two cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are connected together in two ways shown here. The 'balance points' in a given potentiometer experiment for these two com-

binations of cells are found to be at 351.0 cm and 70.2 cm respectively. Calculate the ratio of the emfs of the two cells. **[CBSE Sample Paper 08]** (Ans. 3 : 2)

5. A potentiometer has 400 cm long wire which is connected to an auxiliary of steady voltage 4 V. A Leclanche cell gives null point at 140 cm and Daniel cell at 100 cm. (*i*) Compare emfs of the two cells. (*ii*) If the length of wire is increased by 100 cm, find the position of the null point with the first cell.

[Ans. (*i*) 7 : 5, (*ii*) 175 cm]

6. With a certain cell, the balance point is obtained at 60 cm from the zero end of the potentiometer wire. With another cell whose emf differs from that of the first cell by 0.1 V, the balance point is obtained at 55 cm mark. Calculate the emf of the two cells.

(Ans. 1.2 V, 1.1 V)

7. A potentiometer wire has a potential gradient of 0.0025 volt/cm along its length. Calculate the length of the wire at which null-point is obtained for a 1.025 volt standard cell. Also, find the emf of another cell for which the null-point is obtained at 860 cm length. (Ans. 410 cm, 2.15 V)

8. *AB* is a potentiometer wire of length 100 cm. When a cell $\mathcal{E}_2$ is connected across *AC*, where *AC* = 75 cm, no current flows from $\mathcal{E}_2$. Find (*i*) the potential gradient along *AB* and (*ii*) emf of the cell $\mathcal{E}_2$. The internal resistance of the cell $\mathcal{E}_1$ is negligible.

[Ans. (*i*) 0.02 volt/cm (*ii*) 1.5 V]

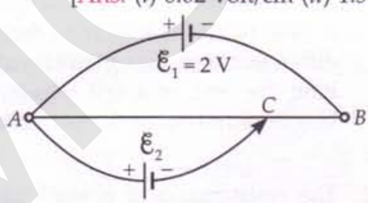


Fig. 3.209

9. A cell can be balanced against 110 cm and 100 cm of potentiometer wire respectively when in open circuit and in circuit shorted through a resistance of 10 Ω. Find the internal resistance of the cell. (Ans. 1Ω)

10. A potentiometer wire of length 1 m has a resistance of 10 Ω. It is connected to a 6 V battery in series with a resistance of 5Ω. Determine the emf of the primary cell which gives a balance point at 40 cm.

[CBSE D 14]

(Ans. 1.6 V)

11. A standard cell of emf 1.08 V is balanced by the potential difference across 91 cm of a metre long wire supplied by a cell of emf 2 V through a series resistor of resistance 2 Ω. The internal resistance of the cell is zero. Find the resistance per unit length of the potentiometer wire. (Ans. 0.03 Ω cm^{-1})

12. Potentiometer wire *PQ* of 1 m length is connected to a standard cell $\mathcal{E}_1$. Another cell, $\mathcal{E}_2$, of emf 1.02 V is connected as shown in the circuit diagram with a resistance '*r*' and a switch, *S*. With switch *S* open, null position is obtained at a distance of 51 cm from *P*. Calculate (*i*) potential gradient of the potentiometer wire and (*ii*) emf of the cell $\mathcal{E}_1$. (*iii*) When switch *S* is closed, will null point move towards *P* or towards *Q*? Give reason. **[CBSE OD 04]**

(Ans. 0.02 V cm^{-1}, 2 V, no effect)

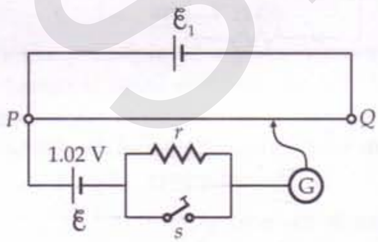


Fig. 3.210

13. A battery $\mathcal{E}_1$ of 4 V and a variable resistance *Rh* are connected in series with the wire *AB* of the potentiometer. The length of the wire of the potentiometer is 1 metre. When a cell $\mathcal{E}_2$ of emf 1.5 volt is connected between points *A* and *C*, no current flows through $\mathcal{E}_2$. Length of *AC* = 60 cm.
   (*i*) Find the potential difference between the ends *A* and *B* of the potentiometer.
   (*ii*) Would the method work, if the battery $\mathcal{E}_1$ is replaced by a cell of emf of 1 V?

[CBSE D 03]

[Ans. (*i*) 2.5 V, (*ii*) No]

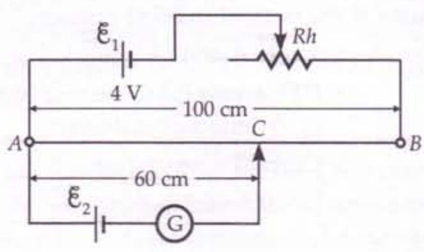


Fig. 3.211

14. The potentiometer wire of length 200 cm has a resistance of 20 Ω. It is connected in series with a resistance 10 Ω and an accumulator of emf 6 V

having negligible resistance. A source of 2.4 V is balanced against a length 'L' of the potentiometer wire. Find the value of L. **[CBSE F 03]**

(Ans. 120 cm)

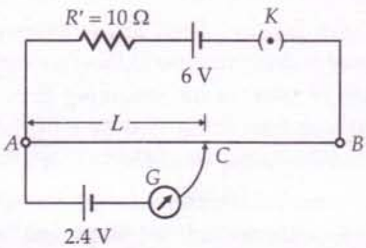


Fig. 3.212

15. A potentiometer wire carries a steady current. The potential difference across 70 cm length of it balances the potential difference across a 2 Ω coil supplied by a cell of emf 2.0 V and an unknown internal resistance r. When a 1Ω coil is placed in parallel with the 2Ω coil, a length equal to 50 cm of the potentiometer wire is required to balance the potential difference across the parallel combination. Find the value of r. (Ans. 0.5 Ω)

HINTS

2. Resistance of the potentiometer wire, $R = 20\,\Omega$

Length of the potentiometer wire $= 10\text{ m} = 10^4$ mm

Required potential gradient, $k = 1\,\mu\text{V mm}^{-1}$

Potential drop along the potentiometer wire,

$V = kl = 10\,\mu\text{V mm}^{-1} \times 10^4\text{mm} = 10^4\mu\text{V} = 10^{-2}\text{V}$

Current through the potentiometer wire,

$$I = \frac{V}{R} = \frac{10^{-2}}{20} = 5 \times 10^{-4}\text{ A}$$

If R' is the required resistance to be introduced in the resistance box, then

$$I = \frac{\varepsilon}{R + R'} \quad \text{or} \quad 5 \times 10^{-4} = \frac{2}{20 + R'}$$

or $R' = \mathbf{3980\,\Omega}$.

3. Here $\varepsilon_1 = 1.20\text{ V}, l_1 = 30\text{ cm}$

Also $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} = 1.5$

$\therefore \quad l_2 = \frac{l_1}{1.5} = \frac{30}{1.5} = 20\text{ cm}$

Difference in the balancing lengths,

$l_1 - l_2 = 30 - 20 = \mathbf{10\text{ cm.}}$

4. Proceed as in Example 162 on page 3.98.

5. (*i*) Here $l_1 = 140\text{ cm}, l_2 = 100\text{ cm}$

$\therefore \quad \frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} = \frac{140}{100} = \frac{7}{5} = \mathbf{7:5.}$

(*ii*) Let ε be the emf of the auxiliary battery and l be the length of potentiometer wire. Then $\varepsilon = 4$ V and $l = 400$ cm.

$\therefore \quad \frac{\varepsilon_1}{\varepsilon} = \frac{l_1}{l} \quad \text{or} \quad \frac{\varepsilon_1}{4} = \frac{140}{400} = \frac{7}{20}$

$\therefore \quad \varepsilon_1 = 1.4\text{ V}$

When length is increased by 100 cm, new length,

$l' = 400 + 100 = 500\text{ cm}$

Now $\frac{\varepsilon_1}{\varepsilon} = \frac{l_1'}{l'} \quad \text{or} \quad \frac{1.4}{4} = \frac{l_1'}{500}$

$\therefore$ New balancing length, $l_1' = \frac{1.4 \times 500}{4} = \mathbf{175\text{ cm.}}$

6. Let the emf of the two cells be ε and $\varepsilon - 0.1$. Then

$$\frac{\varepsilon}{\varepsilon - 0.1} = \frac{60}{55} \quad \therefore \quad \varepsilon = \mathbf{1.2\text{ V.}}$$

emf of the other cell $= 1.2 - 0.1 = \mathbf{1.1\text{ V.}}$

7. (*i*) $l = \frac{\varepsilon}{k} = \frac{1.025}{0.0025} = \mathbf{410\text{ cm.}}$

(*ii*) $\varepsilon' = kl' = 0.0025 \times 860 = \mathbf{2.15\text{ V.}}$

8. (*i*) $k = \frac{\varepsilon_1}{l_1} = \frac{2\text{ V}}{100\text{ cm}} = \mathbf{0.02\text{ V cm}^{-1}}$.

(*ii*) $\varepsilon_2 = kl_2 = 0.02 \times 75 = \mathbf{1.5\text{ V.}}$

9. $r = R\left(\frac{l_1 - l_2}{l_2}\right) = 10\left(\frac{110 - 100}{100}\right) = \mathbf{1\,\Omega.}$

10. $I = \frac{V}{R_{AB} + R} = \frac{6\text{V}}{(10 + 5)\,\Omega} = 0.4\text{ A}$

$V = IR_{AB} = 0.4 \times 10 = 4.0\text{ V}$

$k = \frac{V}{l} = \frac{4.0\text{ V}}{1\text{ m}} = \frac{4.0\text{ V}}{100\text{ cm}} = 0.04\text{ V cm}^{-1}$

Unknown emf balanced against 40 cm of the wire,

$\varepsilon = kl' = 0.04\text{ V cm}^{-1} \times 40\text{ cm} = \mathbf{1.6\text{ V.}}$

11. Let r ohm be the resistance per cm of the potentiometer wire. Then

$$k = \frac{IR_{AB}}{l} = \frac{\varepsilon\, R_{AB}}{l\,(R + R_{AB})} = \frac{2 \times 100\,r}{100\,(2 + 100\,r)}\text{ V cm}^{-1}$$

As the emf of 1.08 V balances against a length of 91 cm, so

$$k = \frac{1.08}{91}\text{ V cm}^{-1} \quad \therefore \quad \frac{2 \times 100\,r}{100\,(2 + 100\,r)} = \frac{1.08}{91}$$

On solving, $r = 0.029 \simeq \mathbf{0.03\,\Omega\text{ cm}^{-1}}$.

12. (*i*) $k = \frac{\varepsilon_2}{l_2} = \frac{1.02\text{ V}}{51\text{ cm}} = \mathbf{0.02\text{ V cm}^{-1}}$.

(*ii*) $\varepsilon_1 = kl_{PQ} = 0.02\text{ V cm}^{-1} \times 100\text{ cm} = \mathbf{2\text{ V.}}$

(*iii*) With switch S closed, the null point is not affected because no current flows through the cell ε_2 at the null point.

13. (i) As $\frac{V_{AB}}{\varepsilon_2} = \frac{l_1}{l_2}$

$\therefore$ P.D. between A and B,

$$V_{AB} = \frac{l_1}{l_2} . \varepsilon_2 = \frac{100 \text{ cm}}{60 \text{ cm}} \times 1.5 = \mathbf{2.5\ V}.$$

(ii) No, this method would not work when $\varepsilon_1 = 1$ V, because then $\varepsilon_1 < \varepsilon_2$ and null point cannot be obtained through the potentiometer wire.

14. $I_{AB} = \frac{6}{10+20} = \frac{6}{30} = 0.2$ A

$V_{AB} = I_{AB} R_{AB} = 0.2 \times 20 = 4$ V

Potential gradient,

$$k = \frac{V_{AB}}{l} = \frac{4 \text{ V}}{200 \text{ cm}} = 0.02 \text{ V cm}^{-1}$$

Balancing length, $L = \frac{\text{Potential difference}}{\text{Potential gradient}}$

$$= \frac{2.4 \text{ V}}{0.02 \text{ V cm}^{-1}} = \mathbf{120\ cm}.$$

15. *In first case.* Current sent by the 2.0 V cell through $2\,\Omega$ coil,

$$I_1 = \frac{\varepsilon}{\text{Total resistance}} = \frac{2}{2+r}$$

Potential drop across $2\,\Omega$ coil,

$$V_1 = RI_1 = 2 \times \frac{2}{2+r} = \frac{4}{2+r}$$

But $V_1 \propto 70$ cm

$$\therefore \quad \frac{4}{2+r} \propto 70 \quad ...(i)$$

In second case. The combined resistance of the parallel combination of $2\,\Omega$ and $1\,\Omega$ coil,

$$R_p = \frac{2 \times 1}{2+1} = \frac{2}{3}\,\Omega$$

Current sent by the cell through the parallel combination,

$$I_g = \frac{\varepsilon}{\text{Total resistance}} = \frac{2}{(2/3)+r} = \frac{6}{2+3r}$$

Potential drop across R_p,

$$V_2 = R_p\, I_2 = \frac{2}{3} \times \frac{6}{2+3r} = \frac{4}{2+3r}$$

But $V_2 \propto 50$ cm

$$\therefore \quad \frac{4}{2+3r} \propto 50 \quad ...(ii)$$

Dividing (i) by (ii), we get

$$\frac{4}{2+r} \times \frac{2+3r}{4} = \frac{70}{50} \quad \text{or} \quad r = \mathbf{0.5\,\Omega}.$$

3.35 WHEATSTONE BRIDGE

62. *What is a Wheatstone bridge ? When is the bridge said to be balanced ? Apply Kirchhoff's laws to derive the balance condition of the Wheatstone bridge.*

Wheatstone bridge. *It is an arrangement of four resistances used to determine one of these resistances quickly and accurately in terms of the remaining three resistances.* This method was first suggested by a British physicist *Sir Charles F. Wheatstone* in 1843.

A Wheatstone bridge consists of four resistances P, Q, R and S ; connected to form the arms of a quadrilateral ABCD. A battery of emf ε is connected between points A and C and a sensitive galvanometer between B and D, as shown in Fig. 3.213.

Let S be the resistance to be measured. The resistance R is so adjusted that there is no deflection in the galvanometer. The *bridge is said to balanced* when the potential difference across the galvanometer is zero so that there is no current through the galvanometer. In the balanced condition of the bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

Unknown resistance, $S = \frac{Q}{P} . R$

Knowing the ratio of resistances P and Q, and the resistance R, we can determine the unknown resistance S. That is why the arms containing the resistances P and Q are called *ratio arms*, the arm AD containing R *standard arm* and the arm CD containing S the *unknown arm*.

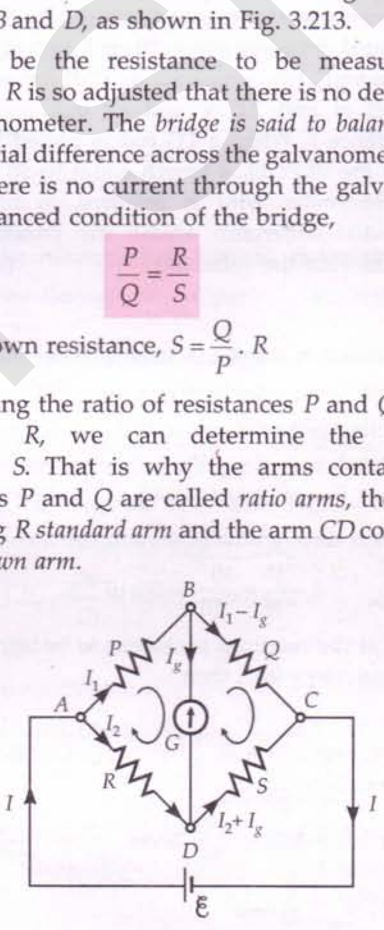


Fig. 3.213 Wheatstone bridge.

Derivation of balance condition from Kirchhoff's laws. In accordance with Kirchhoff's first law, the currents through various branches are as shown in Fig. 3.213.

Applying Kirchhoff's second law to the loop ABDA, we get

$$I_1 P + I_g G - I_2 R = 0$$

where G is the resistance of the galvanometer. Again applying Kirchhoff's second law to the loop $BCDB$, we get

$$(I_1 - I_g)\,Q - (I_2 + I_g)\,S - GI_g = 0$$

In the balanced condition of the bridge, $I_g = 0$. The above equations become

$$I_1 P - I_2 R = 0 \quad \text{or} \quad I_1 P = I_2 R \qquad ...(i)$$

and $$I_1 Q - I_2 S = 0 \quad \text{or} \quad I_1 Q = I_2 S \qquad ...(ii)$$

On dividing equation (*i*) by equation (*ii*), we get

$$\frac{P}{Q} = \frac{R}{S}$$

This proves the condition for the balanced Wheatstone bridge.

63. *What do you mean by sensitivity of a Wheatstone bridge ? On what factors does it depend ?*

Sensitivity of a Wheatstone bridge. *A Wheatstone bridge is said to be sensitive if it shows a large deflection in the galvanometer for a small change of resistance in the resistance arm.*

The sensitivity of the Wheatstone bridge depends on two factors :

(*i*) Relative magnitudes of the resistances in the four arms of the bridge. The bridge is most sensitive when all the four resistances are of the same order.

(*ii*) Relative positions of battery and galvanometer.

According to *Callender* for the greater sensitivity of the Wheatstone bridge, *the battery should be so connected that the resistance in series with the resistance to be measured is greater than the resistance in parallel with it.*

According to *Maxwell* for the greater sensitivity of the Wheatstone bridge, *out of the battery and the galvanometer, the one having the higher resistance should be connected between the junction of the two highest resistances and the junction of the two lowest resistances.*

64. *What are the advantages of measuring resistance by Wheatstone bridge method over other methods ?*

Advantages of Wheatstone bridge method. The bridge method has following advantages over other methods for measuring resistance :

(*i*) It is a null method. Hence the internal resistance of the cell and the resistance of the galvanometer do not affect the null point.

(*ii*) As the method does not involve any measurement of current and potential difference, so the resistances of ammeters and voltmeters do not affect the measurements.

(*iii*) The unknown resistance can be measured to a very high degree of accuracy by increasing the ratio of the resistances in arms P and Q.

For Your Knowledge

- When the Wheatstone bridge is balanced, the potential difference between the points B and D is zero.
- The Wheatstone bridge is most sensitive when the resistances in the four arms are of the same order.
- Wheatstone bridge method is not suitable for the measurement of very low and very high resistances.
- In the balanced Wheatstone bridge, the resistance in arm BD is ineffective. The equivalent resistance of the balanced Wheatstone bridge between the points A and C will be

$$R_{eq} = \frac{(P+Q)(R+S)}{P+Q+R+S}$$

- If the bridge is balanced, then on interchanging the positions of the galvanometer and the battery there is no effect on the balance of the bridge. That is why the arms BD and AC are called *conjugate arms* of the bridge.
- The Wheatstone bridge is the simplest example of an arrangement, the variants of which are used for a large number of electrical measurements. The important applications of Wheatstone bridge are metre bridge, Carey-Foster's bridge and post office box.

3.36 METRE BRIDGE OR SLIDE WIRE BRIDGE

65. *What is a metre bridge ? With the help of a circuit diagram, explain how it can be used to find an unknown resistance. Explain the principle of the experiment and give the formula used.*

Metre bridge or slide wire bridge. *It is the simplest practical application of the Wheatstone bridge that is used to measure an unknown resistance.*

Principle. Its working is based on the principle of Wheatstone bridge.

When the bridge is balanced,

$$\frac{P}{Q} = \frac{R}{S}$$

Construction. It consists of usually one metre long magnanin wire of uniform cross-section, stretched along a metre scale fixed over a wooden board and with its two ends soldered to two L-shaped thick copper strips A and C. Between these two copper strips, another copper strip is fixed so as to provide two gaps ab and a_1b_1. A resistance box R.B. is connected in the gap ab and the unknown resistance S is connected in the gap a_1b_1. A source of emf ε is connected across AC. A movable jockey and a galvanometer are connected across BD, as shown in Fig. 3.214.

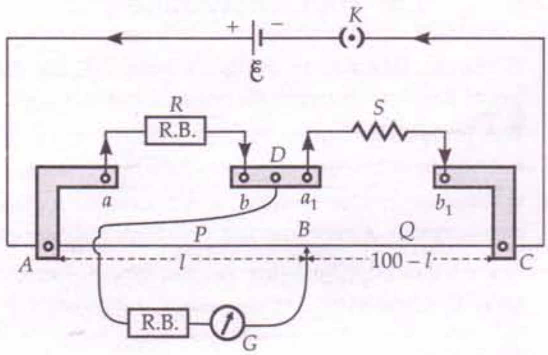


Fig. 3.214 Measurement of unknown resistance by a metre bridge.

Working. After taking out a suitable resistance R from the resistance box, the jockey is moved along the wire AC till there is no deflection in the galvanometer. This is the balanced condition of the Wheatstone bridge. If P and Q are the resistances of the parts AB and BC of the wire, then for the balanced condition of the bridge, we have

$$\frac{P}{Q}=\frac{R}{S}$$

Let total length of wire $AC = 100$ cm and $AB = l$ cm, then $BC = (100 - l)$ cm. Since the bridge wire is of uniform cross-section, therefore,

resistance of wire ∝ length of wire

or $$\frac{P}{Q}=\frac{\text{resistance of } AB}{\text{resistance of } BC}$$

$$=\frac{\sigma l}{\sigma(100-l)}=\frac{l}{100-l}$$

where σ is the resistance per unit length of the wire.

Hence
$$\frac{R}{S}=\frac{l}{100-l}$$

or $$S=\frac{R(100-l)}{l}$$

Knowing l and R, unknown resistance S can be determined.

Determination of resistivity. If r is the radius of the wire and l' its length, then resistivity of its material will be

$$\rho=\frac{SA}{l'}=\frac{S\times\pi r^2}{l'}.$$

Examples Based on
(i) Wheatstone Bridge
(ii) Slide Wire Bridge

Formulae Used

1. For a balanced Wheatstone bridge, $\frac{P}{Q}=\frac{R}{S}$

   If X is the unknown resistance
   $$\frac{P}{Q}=\frac{R}{X} \quad\text{or}\quad X=\frac{RQ}{P}$$

2. In a slide wire bridge, if balance point is obtained at l cm from the zero end, then
   $$\frac{P}{Q}=\frac{R}{X}=\frac{l}{100-l} \quad\text{or}\quad X=\left(\frac{100-l}{l}\right)R$$

Units Used

All resistances are in ohm and distances in cm.

Example 172. *Find out the magnitude of resistance X in the circuit shown in Fig. 3.215, when no current flows through the 5 Ω resistor.* [ISCE 98]

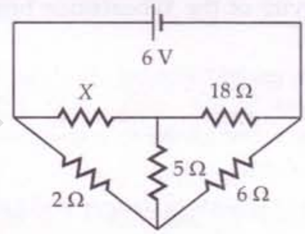


Fig. 3.215

Solution. As no current flows through the middle 5 Ω resistor, the circuit represents a balanced Wheatstone bridge.

$$\therefore \quad \frac{X}{18}=\frac{2}{6} \qquad\text{or}\qquad X=\frac{2}{6}\times 18=\mathbf{6\,\Omega}.$$

Example 173. *P, Q, R and S are four resistance wires of resistances 2, 2, 2 and 3 ohms respectively. Find out the resistance with which S must be shunted in order that bridge may be balanced.*

Solution. For a balanced Wheatstone bridge,
$$\frac{P}{Q}=\frac{R}{S}$$

But $P = 2\,\Omega,\ Q = 2\,\Omega,\ R = 2\,\Omega \quad \therefore \frac{2}{2}=\frac{2}{S}$

i.e., resistance S must have a total resistance of 2 Ω. In arm S, the resistance of 3Ω must be shunted with a resistance r so that the combined resistance is of 2Ω.

i.e., $$\frac{1}{r}+\frac{1}{3}=\frac{1}{2} \quad\text{or}\quad \frac{1}{r}=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$$

∴ Required shunt, $r = \mathbf{6\,\Omega}$.

Example 174. *In a Wheatstone bridge arrangement, the ratio arms P and Q are nearly equal. The bridge is balanced when R = 500 Ω. On interchanging P and Q, the value of R for balancing is 505 Ω. Find the value of X and the ratio P/Q.*

Solution. For balanced Wheatstone bridge,

$$\frac{P}{Q}=\frac{R}{X}$$

In the first case, $R = 500\,\Omega$

$$\therefore \quad \frac{P}{Q}=\frac{500}{X} \quad ...(1)$$

In the second case when P and Q are interchanged, $R = 505\,\Omega$

$$\therefore \quad \frac{Q}{P}=\frac{505}{X} \quad ...(2)$$

Multiplying equations (1) and (2),

$$1=\frac{500\times 505}{X^2}$$

or

$$X=\sqrt{500\times 505}$$
$$= \mathbf{502.5\,\Omega}$$

Substituting the value of X in (1), we get

$$\frac{P}{Q}=\frac{500}{502.5}$$
$$=\frac{1}{1.005}=\mathbf{1:1.005}$$

Example 175. *The galvanometer, in each of the two given circuits, does not show any deflection. Find the ratio of the resistors R_1 and R_2, used in these two circuits.* [CBSE OD 13]

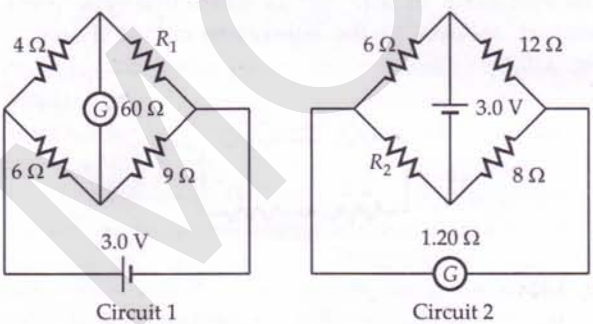


Fig. 3.216

Solution. In circuit 1, the Wheatstone bridge is in the balanced condition, so

$$\frac{4}{R_1}=\frac{6}{9} \Rightarrow R_1=\frac{4\times 9}{6}=6\Omega$$

In circuit 2, the interchange of the positions of the battery and the galvanometer does not affect the balance condition of the Wheatstone bridge, so

$$\frac{6}{12}=\frac{R_2}{8}$$

$$\Rightarrow \quad R_2=\frac{6\times 8}{12}=4\Omega$$

$$\therefore \quad \frac{R_1}{R_2}=\frac{6}{4}=\frac{3}{2}$$
$$= \mathbf{3:2}$$

Example 176. *Calculate the current drawn from the battery by the network of resistors shown in Fig. 3.217.* [CBSE OD 09, 15C]

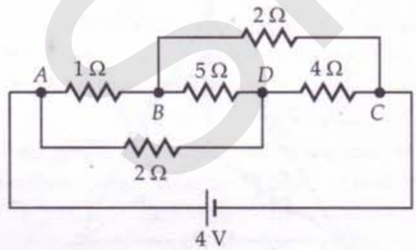


Fig. 3.217

Solution. The given network is equivalent to the circuit shown in Fig. 3.218.

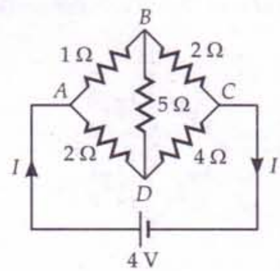


Fig. 3.218

Now $\quad \frac{1\,\Omega}{2\,\Omega}=\frac{2\,\Omega}{4\,\Omega} \quad$ *i.e.,* $\quad \frac{P}{Q}=\frac{R}{S}$

The given circuit is a balanced Wheatstone bridge. The resistance of 5 Ω in arm *BD* is ineffective. The equivalent circuit reduces to the circuit shown in Fig. 3.219.

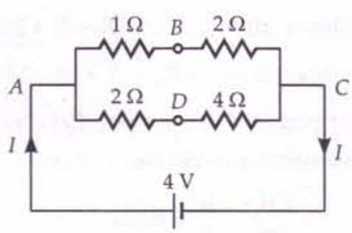


Fig. 3.219

Resistances in AB and BC are in series, their equivalent resistance $= 1 + 2 = 3\,\Omega$.

Resistances in AD and DC are in series, their equivalent resistance $= 2 + 4 = 6\,\Omega$

The resistances of $3\,\Omega$ and $6\,\Omega$ are in parallel.

The equivalent resistance R between A and C is

$$R = \frac{3 \times 6}{3 + 6} = 2\,\Omega$$

Current, $I = \frac{V}{R} = \frac{4}{2} = \mathbf{2\,A.}$

Example 177. *Each of the resistances in the network shown in Fig. 3.220 equals R. Find the resistance between two terminals A and C.*

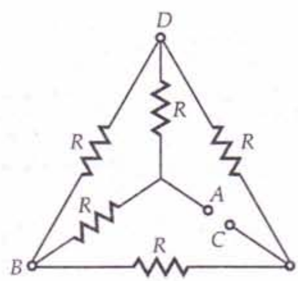


Fig. 3.220

Solution. The network shown in Fig. 3.221 is the equivalent network of the given network.

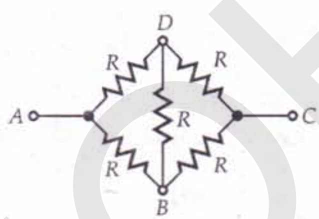


Fig. 3.221

It is a balanced Wheatstone bridge because

$$\frac{R}{R} = \frac{R}{R}$$

Hence the points B and D must be at the same potential. The resistance R in arm BD is ineffective.

Total resistance along $ADC = R + R = 2R\,\Omega$

Total resistance along $ABC = R + R = 2R\,\Omega$

These two resistances form a parallel combination.

$\therefore$ Effective resistance between A and C

$$= \frac{2R \times 2R}{2R + 2R} = R\,\Omega.$$

Example 178. *A potential difference of 2 V is applied between the points A and B shown in network drawn in Fig. 3.222. Calculate*

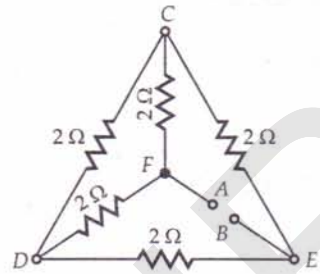


Fig. 3.222

(*i*) *the equivalent resistance of the network between the points A and B, and*

(*ii*) *the magnitudes of currents flowing in the arms AFCEB and AFDEB.* **[CBSE OD 98]**

Solution. (*i*) The equivalent network is shown in Fig. 3.223. It is a balanced Wheatstone bridge because

$$\frac{2\,\Omega}{2\,\Omega} = \frac{2\,\Omega}{2\,\Omega}$$

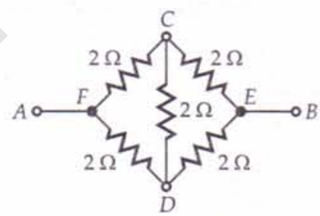


Fig. 3.223

Hence the points C and D are at the same potential. The resistance in arm CD is ineffective. The given network reduces to the equivalent circuit shown in Fig. 3.224.

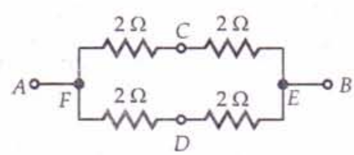


Fig. 3.224

Total resistance along $FCE = 2 + 2 = 4\,\Omega$

Total resistance along $FDE = 2 + 2 = 4\,\Omega$

These two resistances form a parallel combination.

$\therefore$ Equivalent resistance between points A and B

$$= \frac{4 \times 4}{4 + 4} = 2\,\Omega$$

(*ii*) Total current in the circuit $= \frac{V}{R} = \frac{2\text{ V}}{2\,\Omega} = 1\text{ A}$

Current through arm *AFCEB*

$= $ Current through arm *AFDEB*

$= \frac{1}{2}\text{ A} = \mathbf{0.5\ A.}$

Example 179. *Find the value of the unknown resistance X, in the following circuit, if no current flows through the section AO. Also calculate the current drawn by the circuit from the battery of emf 6 V and negligible internal resistance. (Fig. 3.225)* [CBSE OD 02]

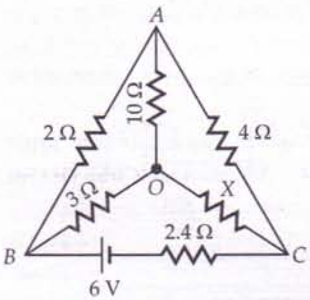


Fig. 3.225

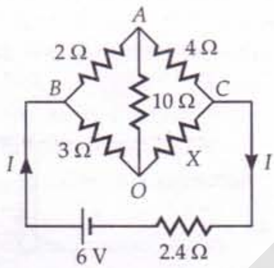


Fig. 3.226

Solution. The equivalent circuit for the given network is shown in Fig. 3.226.

As no current flows through the section *AO*, so the given circuit is a balanced Wheatstone bridge.

Hence

$$\frac{2}{4} = \frac{3}{X} \quad \text{or} \quad X = \frac{3\times 4}{2} = 6\,\Omega$$

The resistance of 10 Ω in section *AO* is not effective.

Total resistance along $BAC = 2 + 4 = 6\,\Omega$

Total resistance along $BOC = 3 + 6 = 9\,\Omega$

These two resistances form a parallel combination. The effective resistance between *B* and *C* is

$$R = \frac{6\times 9}{6+9} = \frac{18}{5} = 3.6\,\Omega.$$

Total resistance in the circuit $= 3.6 + 2.4 = 6\,\Omega$.

Current, $I = \frac{6\text{ V}}{6\,\Omega} = \mathbf{1\ A.}$

Example 180. *Six equal resistors, each of value R, are joined together as shown in Fig. 3.227. Calculate the equivalent resistance across AB. If a supply of emf ℰ is connected across AB, compute the current through the arms DE and AB.* [CBSE Sample Paper 03]

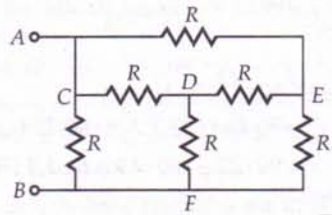


Fig. 3.227

Solution. The equivalent circuits are shown below. The resistance *R* in arm *DE* of the balanced Wheatstone bridge is ineffective.

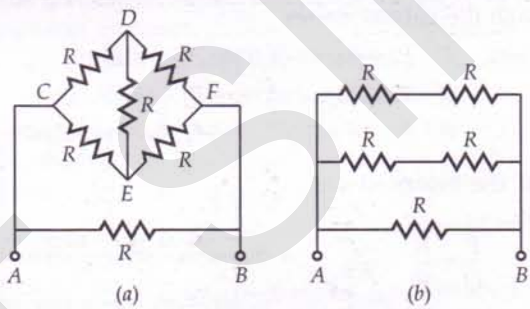


Fig. 3.228

The equivalent resistance *R'* across *AB* is given by

$$\frac{1}{R'} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} = \frac{4}{2R} = \frac{2}{R}$$

or $\quad R' = R/2$

Current through arm $AB = \frac{\mathcal{E}}{R'} = \frac{\mathcal{E}}{R/2} = \frac{\mathbf{2\mathcal{E}}}{\mathbf{R}}$.

Current through arm $DE = \mathbf{0}$.

Example 181. *Calculate the ratio of the heat produced in the four arms of the Wheatstone bridge shown in Fig. 3.229.*

Solution. As $\frac{40\,\Omega}{10\,\Omega} = \frac{60\,\Omega}{15\,\Omega}$

The bridge is balanced.

$\therefore$ P.D. across *AB*

= P.D. across *AD*

or $\quad 40\, I_1 = 60\, I_2$

or $\quad \frac{I_1}{I_2} = \frac{60}{40} = 1.5$

or $\quad I_1 = 1.5\, I_2$

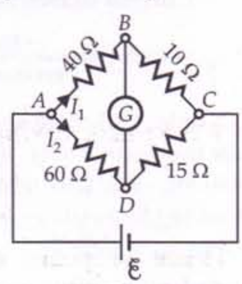


Fig. 3.229

Heats produced in time *t* in different arms of Wheatstone bridge are

$$H_{AB} = I_1^2\, Rt = (1.5\, I_2)^2 \times 40 \times t = 90\, I_2^2\, t$$

$$H_{BC} = I_1^2 \times 10 \times t = (1.5 I_2)^2 \times 10 \times t = 22.5\, I_2^2\, t$$

$$H_{AD} = I_2^2 \times 60 \times t = 60\, I_2^2\, t$$

$$H_{DC} = I_2^2 \times 15 \times t = 15\, I_2^2\, t$$

Hence the ratio of the heats produced in the four arms is

$$H_{AB} : H_{BC} : H_{AD} : H_{DC}$$
$$= 90\, I_2^2\, t : 22.5\, I_2^2\, t : 60\, I_2^2\, t : 15\, I_2^2\, t$$
$$= 90 : 22.5 : 60 : 15 = \mathbf{6 : 1.5 : 4 : 1.}$$

Example 182. *In the following circuit, a metre bridge is shown in its balanced state. The metre bridge wire has a resistance of 1 Ω cm. Calculate the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.*

Solution. In balanced condition, no current flows through the galvanometer.

Here P = Resistance of wire $AJ = 40\,\Omega$

Q = Resistance of wire $BJ = 60\,\Omega$

$R = X,\ S = 6\,\Omega$

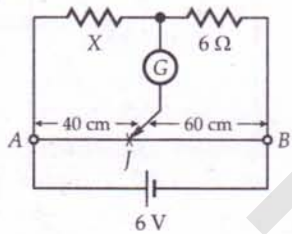


Fig. 3.230

In the balanced condition,

$$\frac{P}{Q} = \frac{R}{S}$$

or $$\frac{40}{60} = \frac{X}{6}$$

or $$X = 4\,\Omega$$

Total resistance of wire $AB = 100\,\Omega$

Total resistance of resistances X and $6\,\Omega$ connected in series $= 4 + 6 = 10\,\Omega$

This series combination is in parallel with wire AB.

$$\therefore \text{ Equivalent resistance} = \frac{10 \times 100}{10 + 100} = \frac{100}{11}\,\Omega$$

emf of the battery = 6 V

∴ Current drawn from the battery,

$$I = \frac{\text{emf}}{\text{resistance}} = \frac{6}{100/11} = \mathbf{0.66\ A.}$$

Example 183. *With a certain resistance in the left gap of a slide wire metre bridge, the balance point is obtained when a resistance of 10 Ω is taken out from the resistance box. On increasing the resistance from the resistance box by 12.5 Ω, the balance point shifts by 20 cm. Find the unknown resistance.*

Solution. With unknown resistance X in the left gap and known resistance of $10\,\Omega$ in the right gap, suppose the balance point is obtained at l cm from the zero end. Then

$$\frac{X}{10} = \frac{l}{100 - l} \qquad ...(1)$$

When the resistance in the right gap is increased by $12.5\,\Omega$, total resistance becomes $22.5\,\Omega$. The balance point shifts towards zero end by 20 cm.

$$\therefore \frac{X}{22.5} = \frac{l - 20}{100 - (l - 20)} = \frac{l - 20}{120 - l} \qquad ...(2)$$

Dividing (1) by (2),

$$\frac{22.5}{10} = \frac{l}{100 - l} \times \frac{120 - l}{l - 20}$$

On solving, we get $l^2 - 120l + 3600 = 0$

$$\therefore \quad l = 60 \text{ cm}$$

From (1),

$$\frac{X}{10} = \frac{60}{100 - 60} \quad \text{or} \quad X = \frac{60 \times 10}{40} = \mathbf{15\,\Omega.}$$

Example 184. *In metre bridge, the null point is found at a distance of 60.0 cm from A. If now a resistance of 5Ω is connected in series with S, the null point occurs at 50 cm. Determine the values of R and S. [Fig. 3.231].* [CBSE D 10]

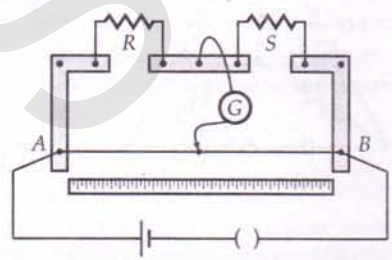


Fig. 3.231

Solution. In *first case*, $$\frac{R}{S} = \frac{60}{40} = \frac{3}{2} \qquad ...(i)$$

In *second case*, $$\frac{R}{S + 5} = \frac{50}{50} \qquad ...(ii)$$

On dividing (*i*) by (*ii*), $$\frac{S + 5}{S} = \frac{3}{2}$$

or $$2S + 10 = 3S$$

or $$S = \mathbf{10\,\Omega}$$

and $$R = \frac{3}{2} S = \frac{3}{2} \times 10 = \mathbf{15\,\Omega}$$

Example 185. *In a metre bridge, the null point is found at a distance of 40 cm from A. If a resistance of 12 Ω is connected in parallel with S, the null point occurs at 50.0 cm from A. Determine the values of R and S. [Fig. 3.232]*

[CBSE D 10]

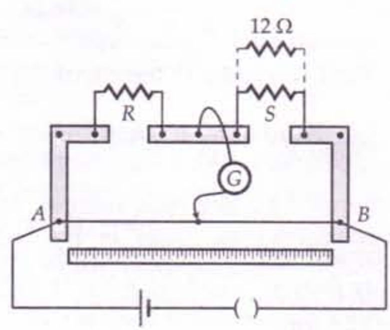


Fig. 3.232

Solution.

In *first case,* $\frac{R}{S}=\frac{40}{60}=\frac{2}{3}$

In *second case,* $\frac{R}{\frac{12\,S}{S+12}}=\frac{50}{50}$

or $\frac{R}{S}=\frac{12}{S+12}=\frac{2}{3}$ or $S=\mathbf{6\,\Omega}$

and $R=\frac{2}{3}\times S=\frac{2}{3}\times 6=\mathbf{4\,\Omega}$

Example 186. *A resistance $R=2\,\Omega$ is connected to one of the gaps in a metre bridge, which uses a wire of length 1 m. An unknown resistance $X>2\,\Omega$ is connected in the other gap as shown in the figure. The balance point is noticed at 'l' from the positive end of the battery. On interchanging R and X, it is found that the balance point further shifts by 20 cm (away from end A). Neglecting the end correction, calculate the value of unknown resistance X used.* [CBSE OD 08]

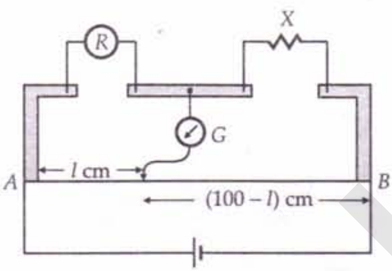


Fig. 3.233

Solution. In *first case,*

$$\frac{R}{X}=\frac{l}{100-l}$$

In *second case,*

$$\frac{X}{R}=\frac{l+20}{100-(l+20)}=\frac{l+20}{80-l}$$

On multiplying the two equations,

$$1=\frac{l}{100-l}\times\frac{l+20}{80-l}$$

or $8000-180l+l^2=l^2+20l$

or $200l=8000$

or $l=40$ cm

Now $X=\frac{l+20}{80-l}R=\frac{40+20}{80-40}\times 2=\mathbf{3\,\Omega.}$

Example 187. *The given figure shows the experimental setup of a metre bridge. The null point is found to be 60 cm away from the end A with X and Y in position as shown. When a resistance of $15\,\Omega$ is connected in series with Y, the null point is found to shift by 10 cm towards the end A of the wire. Find the position of null point if a resistance of $30\,\Omega$ were connected in parallel with Y.* [CBSE Sample Paper 08]

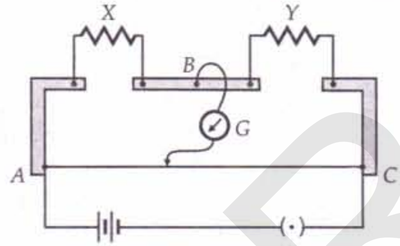


Fig. 3.234

Solution. In *first case,*

$$\frac{X}{Y}=\frac{60}{40}\quad\text{or}\quad\frac{X}{Y}=\frac{3}{2}$$

In *second case,*

$$\frac{X}{Y+15}=\frac{50}{50}=1$$

$\therefore$ $\frac{X}{Y}\times\frac{Y+15}{X}=\frac{3}{2}\times 1$

or $1+\frac{15}{Y}=\frac{3}{2}$

or $Y=30\,\Omega$

$$X=\frac{3}{2}Y=\frac{3}{2}\times 30=45\,\Omega$$

When a resistance of $30\,\Omega$ is connected in parallel with Y, the resistance in the right gap becomes

$$Y'=\frac{30Y}{30+Y}=\frac{30\times 30}{30+30}=15\,\Omega$$

Suppose the null point occurs at l cm from end A. Then

$$\frac{X}{15}=\frac{l}{100-l}\quad\text{or}\quad\frac{45}{15}=\frac{l}{100-l}$$

or $300-3l=l$

or $4l=300$ or $l=\mathbf{75\ cm.}$

Example 188. *When two known resistances, R and S, are connected in the left and right gaps of a metre bridge, the balance point is found at a distance l_1 from the 'zero end' of*

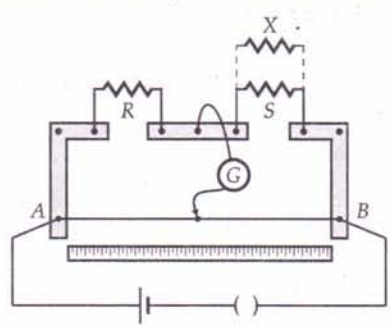


Fig. 3.235

the metre bridge wire. An unknown resistance X is now connected in parallel to the resistance S and the balance point is now found at a distance l_2 from the zero end of the metre bridge wire. Obtain a formula for X in terms of l_1, l_2 and S.
[CBSE D 04C, 10C ; OD 09]

Solution. In *first case,*

$$\frac{R}{S} = \frac{l_1}{100 - l_1} \quad ...(i)$$

In *second case,*

$$\frac{R}{XS/(X+S)} = \frac{l_2}{100 - l_2} \quad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{X+S}{X} = \frac{l_2}{l_1}\left(\frac{100-l_1}{100-l_2}\right) \quad \text{or} \quad 1 + \frac{S}{X} = \frac{l_2}{l_1}\left(\frac{100-l_1}{100-l_2}\right)$$

$$X = \frac{S}{\frac{l_2}{l_1}\left(\frac{100-l_1}{100-l_2}\right) - 1}.$$

Problems For Practice

1. Four resistances of 15 Ω, 12 Ω, 4 Ω and 10 Ω respectively are connected in cyclic order to form a Wheatstone bridge. Is the network balanced ? If not, calculate the resistance to be connected in parallel with the resistance of 10 Ω to balance the network. (Ans. Bridge is not balanced, 10 Ω)

2. The Wheatstone's bridge of Fig. 3.236 is showing no deflection in the galvanometer joined between the points B and D. Compute the value of R. (Ans. 25 Ω)

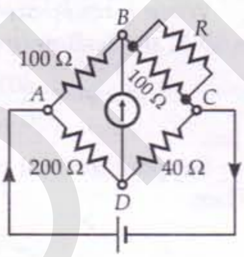


Fig. 3.236

3. (*i*) Calculate the equivalent resistance of the given electrical network between points A and B.
(*ii*) Also calculate the current through CD and ACB, if a 10 V d.c. source is connected between A and B, and the value of R is assumed as 2 Ω.
[CBSE OD 08]
[Ans. (*i*) $R_{AB} = R\,\Omega$
(*ii*) $I_{CD} = 0$,
$I_{ACB} = 2.5$ A]

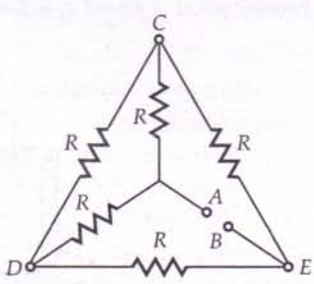


Fig. 3.237

4. Calculate the equivalent resistance between points A and B of the network shown in Fig. 3.238.
[CBSE D 99] (Ans. 2 Ω)

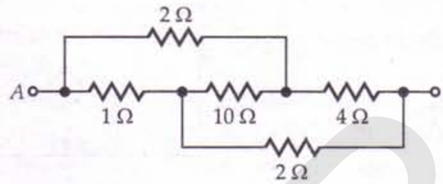


Fig. 3.238

5. Calculate the equivalent resistance between the points A and B of the network shown in Fig. 3.239. (Ans. R)

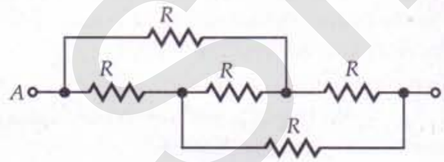


Fig. 3.239

6. Calculate the resistance between the points A and B of the network shown in Fig. 3.240. (Ans. 8 Ω)

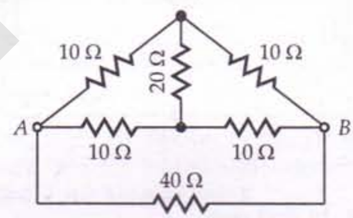


Fig. 3.240

7. For the network shown in Fig. 3.241, determine the value of R and the current through it, if the current through the branch AO is zero. (Ans. 6 Ω, 0.5 A)

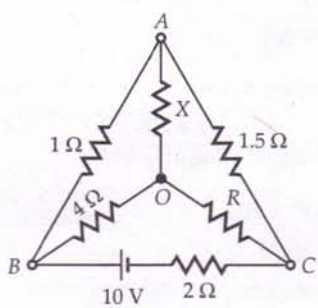


Fig. 3.241

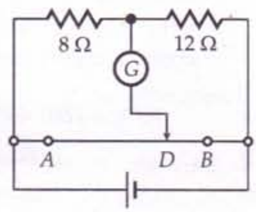


Fig. 3.242

8. The potentiometer wire AB shown in Fig. 3.242 is 40 cm long. Where should the free end of the galvanometer be connected on AB so that the galvanometer may show zero deflection ?
(Ans. 16 cm from A)

9. The potentiometer wire AB shown in Fig. 3.243 is 50 cm long. When $AD = 30$ cm, no deflection occurs in the galvanometer. Find R. (**Ans.** 4 Ω)

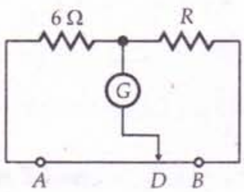


Fig. 3.243

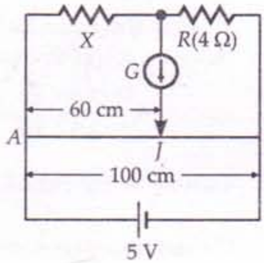


Fig. 3.244

10. Calculate the value of unknown resistance X and the current drawn by the circuit, assuming that no current flows through the galvanometer. Assume the resistance per unit length of the wire AB to be 0.01 Ω / cm. (Fig. 3.244) **[CBSE D 01]**

(**Ans.** 6 Ω, 5.5 A)

11. In Fig 3.245, $P = 3\,\Omega$, $Q = 2\,\Omega$, $R = 6\,\Omega$, $S = 4\,\Omega$ and $X = 5\,\Omega$. Calculate the current I. **[CBSE D 92]**

(**Ans.** 0.6 A)

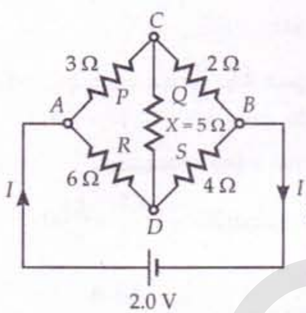


Fig. 3.245

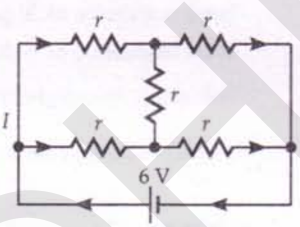


Fig. 3.246

12. Each resistor r shown in Fig. 3.246 has a resistance of 10 Ω and the battery has an emf of 6 V. Find the current supplied by battery. (**Ans.** 0.6 A)

13. Find the equivalent resistance between the points X and Y of the network shown in Fig. 3.247.

(**Ans.** 10 Ω)

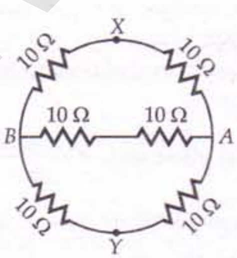


Fig. 3.247

14. In a metre bridge, the length of the wire is 100 cm. At what position will the balance point be obtained if the two resistances are in the ratio 2 : 3 ?

(**Ans.** 40 cm)

15. In the metre bridge experimental set up, shown in Fig. 3.248, the null point 'D' is obtained at a distance of 40 cm from end A of the metre bridge wire. If a resistance of 10 Ω is connected in series with X, null point is obtained at $AD = 60$ cm. Calculate the values of X and Y. **[CBSE D 13]**

(**Ans.** 8 Ω, 12 Ω)

16. In a metre-bridge experiment, two resistances P and Q are connected in series in the left gap. When the resistance in the right gap is 50 Ω, the balance point is at the centre of the slide wire. If P and Q are connected in parallel in the left gap, the resistance in the right gap has to be changed to 12 Ω so as to obtain the balance point at the same position. Find P and Q. (**Ans.** $P = 30\,\Omega$, $Q = 20\,\Omega$)

17. In a metre bridge when the resistance in the left gap is 2 Ω and an unknown resistance in the right gap, the balance point is obtained at 40 cm from the zero end. On shunting the unknown resistance with 2 Ω, find the shift of the balance point on the bridge wire. (**Ans.** 22.5 cm)

18. Fig. 3.248 shows experimental set up of a metre bridge. When the two unknown resistances X and Y are inserted, the null point D is obtained 40 cm from the end A. When a resistance of 10 Ω is connected in series with X, the null point shifts by 10 cm. Find the position of the null point when the 10 Ω resistance is instead connected in series with resistance 'Y'. Determine the values of the resistances X and Y. **[CBSE D 09]**

(**Ans.** $X = 20\,\Omega$, $Y = 30\,\Omega$, $l' = 33.3$ cm]

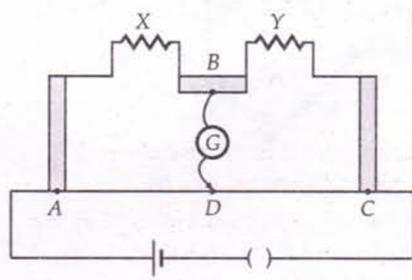


Fig. 3.248

HINTS

1. The four resistances are connected in a cyclic order as shown in Fig. 3.249.

As $\frac{15}{12} \neq \frac{10}{4}$

Thus Wheatstone bridge is not balanced. To balance the network, suppose

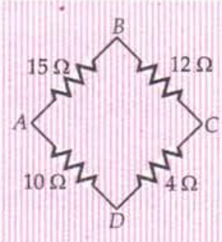


Fig. 3.249

resistance R is connected in parallel with $10\,\Omega$ resistance. Then

$$\frac{15}{12}=\frac{\frac{10\,R}{10+R}}{4} \quad \text{or} \quad \frac{10\,R}{10+R}=5$$

or $R = \mathbf{10\,\Omega}$.

2. $\dfrac{100}{\frac{100\,R}{100+R}}=\dfrac{200}{40}$ or $\dfrac{100\,R}{100+R}=20$ $\therefore$ $R=\mathbf{25\,\Omega}$.

3. Proceed as in Example 178 on page 3.108.

4. As $\dfrac{1}{2}=\dfrac{2}{4}$

$\therefore$ The given circuit is a balanced Wheatstone bridge as shown in Fig. 3.250. The resistance of $10\,\Omega$ is ineffective.

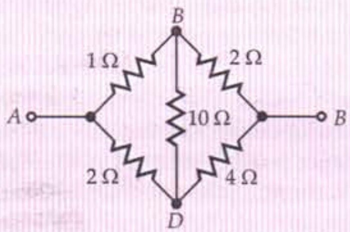


Fig. 3.250

We have $(1\,\Omega + 2\,\Omega)$ and $(2\,\Omega + 4\,\Omega)$ combinations in parallel.

$$\therefore \quad R=\frac{3\times 6}{3+6}=\mathbf{2\,\Omega}.$$

5. The given circuit is equivalent to the circuit shown in Fig. 3.251.

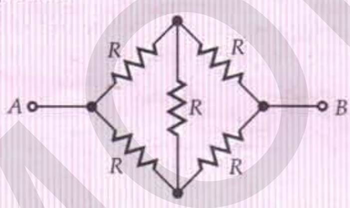


Fig. 3.251

Here $\dfrac{R}{R}=\dfrac{R}{R}$

So it is a balanced Wheatstone bridge. We have resistances $(R+R)$ and $(R+R)$ in parallel.

$$\therefore \quad \text{Equivalent resistance} = \frac{2R\times 2R}{2R+2R}=\mathbf{R}.$$

6. Here $\dfrac{10}{10}=\dfrac{10}{10}$ $\therefore$ Resistance of $20\,\Omega$ is ineffective.

We have resistances of $(10\,\Omega + 10\,\Omega)$, $(10\,\Omega + 10\,\Omega)$ and $40\,\Omega$ in parallel.

$$\therefore \quad \frac{1}{R}=\frac{1}{20}+\frac{1}{20}+\frac{1}{40}=\frac{5}{40} \quad \text{or} \quad R=\mathbf{8\,\Omega}.$$

7. As points A and O are at the same potential, therefore

$$\frac{1}{1.5}=\frac{4}{R} \quad \text{or} \quad R=4\times 1.5=\mathbf{6\,\Omega}$$

If R' is the equivalent resistance of the network between B and C, then

$$R'=\frac{2.5\times 10}{2.5+10}+2=4\,\Omega$$

Current in the circuit, $I=\dfrac{10}{4}=2.5$ A

Current through $R\,(=6\,\Omega)=\dfrac{2.5}{2.5+10}\times 2.5=\mathbf{0.5\ A.}$

8. $\dfrac{8}{12}=\dfrac{AD}{DB}=\dfrac{l}{40-l}$ $\therefore$ $l=\mathbf{16\ cm.}$

9. $\dfrac{6}{R}=\dfrac{AD}{DB}=\dfrac{30}{50-30}$ $\therefore$ $R=4\,\Omega$.

10. Resistance of wire $AJ = 60\times 0.01 = 0.60\,\Omega$

Resistance of wire $BJ = 40\times 0.01 = 0.40\,\Omega$

When no current flows through the galvanometer,

$$\frac{P}{Q}=\frac{R}{S} \quad \text{or} \quad \frac{0.60}{0.40}=\frac{X}{4}$$

$$\therefore \quad X=\frac{0.60\times 4}{0.40}=\mathbf{6\,\Omega}$$

Total resistance of X and R in series $= 6+4=10\,\Omega$

Total resistance of wire $AB = 0.60+0.40 = 1.0\,\Omega$

The above two resistances are in parallel.

$\therefore$ Total resistance of the circuit $=\dfrac{10\times 1}{10+1}=\dfrac{10}{11}\,\Omega$

Current, $I=\dfrac{EMF}{\text{Resistance}}=\dfrac{5}{10/11}=\mathbf{5.5\ A.}$

11. The circuit is a balanced Wheatstone bridge. Its effective resistance R is given by

$$\frac{1}{R}=\frac{1}{3+2}+\frac{1}{6+4}=\frac{3}{10} \quad \text{or} \quad R=\frac{10}{3}\,\Omega.$$

$$\therefore \text{ Current, } I=\frac{V}{R}=\frac{2}{10/3}=\mathbf{0.6\ A.}$$

12. As $\dfrac{r}{r}=\dfrac{r}{r}$, so the given circuit is a balanced Wheatstone bridge and the resistance r in the vertical arm is ineffective. The circuit is then equivalent to two resistances of $2r$ and $2r$ connected in parallel.

$$\therefore \text{ Equivalent resistance, } R=\frac{2r\times 2r}{2r+2r}=r=10\,\Omega$$

Current supplied by the battery of emf 6 V,

$$I=\frac{\mathcal{E}}{R}=\frac{6}{10}=\mathbf{0.6\ A.}$$

13. The equivalent circuit is shown in Fig. 3.252.

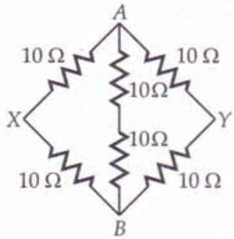


Fig. 3.252

The resistances in arm AB are ineffective.

$$\therefore \quad \frac{1}{R} = \frac{1}{10+10} + \frac{1}{10+10} = \frac{1}{10} \quad \text{or} \quad R = \mathbf{10\,\Omega}.$$

14. For a balanced metre bridge, $\dfrac{X}{R} = \dfrac{l}{100-l}$

But $\quad \dfrac{X}{R} = \dfrac{2}{3} \qquad \therefore \quad \dfrac{2}{3} = \dfrac{l}{100-l}$

or $\quad 200 - 2l = 3l \quad$ or $\quad l = \dfrac{200}{5} = \mathbf{40\ cm.}$

15. In *first* case : $\quad \dfrac{X}{Y} = \dfrac{40}{100-40} = \dfrac{2}{3}$

In *second* case : $\dfrac{X+10}{Y} = \dfrac{60}{100-60}$

or $\quad \dfrac{X}{Y} + \dfrac{10}{Y} = \dfrac{3}{2}$

or $\quad \dfrac{10}{Y} = \dfrac{3}{2} - \dfrac{X}{Y} = \dfrac{3}{2} - \dfrac{2}{3} = \dfrac{5}{6}$

$$\therefore \; Y = \frac{10 \times 6}{5} = \mathbf{12\,\Omega} \quad \text{and} \quad X = \frac{2}{3}Y = \frac{2}{3} \times 12 = \mathbf{8\,\Omega}.$$

16. When P and Q are connected in series in the left gap,

$$\frac{P+Q}{50} = \frac{50}{100-50}$$

$$\therefore \qquad P + Q = 50\,\Omega \qquad \text{...(1)}$$

When P and Q are connected in parallel in the left gap,

$$\frac{\frac{PQ}{P+Q}}{12} = \frac{50}{50} = 1$$

$$\therefore \qquad PQ = 12\,(P+Q) = 12 \times 50 = 600$$

$$(P-Q)^2 = (P+Q)^2 - 4\,PQ = 50^2 - 4 \times 600 = 100$$

$$\therefore \qquad P - Q = 10 \qquad \text{...(2)}$$

Solving (1) and (2), $P = \mathbf{30\,\Omega}$ and $Q = \mathbf{20\,\Omega}$.

17. If X is the unknown resistance, then

$$\frac{2}{X} = \frac{40}{100-40} \quad \text{or} \quad X = \frac{2 \times 60}{40} = 3\,\Omega$$

When resistance X is shunted with $2\,\Omega$ resistor, the effective resistance becomes

$$X' = \frac{X \times 2}{X+2} = \frac{3 \times 2}{3+2} = 1.2\,\Omega$$

Now if the balance point is obtained at distance l' from the left end, then

$$\frac{2}{X'} = \frac{l'}{100-l'} \quad \text{or} \quad \frac{2}{1.2} = \frac{l'}{100-l'}$$

$$\therefore \qquad l' = 62.5\ \text{cm}$$

Shift in the balance point

$$= l' - l = 62.5 - 40 = \mathbf{22.5\ cm.}$$

18. With the unknown resistances X and Y, the balance point is 40 cm from the end A.

$$\therefore \qquad \frac{X}{Y} = \frac{40}{100-40} = \frac{2}{3} \quad \text{or} \quad Y = \frac{3}{2}X$$

With $10\,\Omega$ resistance in series with X, the balance point is at $40 + 10 = 50$ cm from the end A.

$$\therefore \qquad \frac{X+10}{Y} = \frac{50}{100-50} = 1$$

or $\qquad Y = X + 10$

or $\qquad \dfrac{3}{2}X = X + 10$

or $\qquad X = \mathbf{20\,\Omega}$. and $\quad Y = 20 + 10 = \mathbf{30\,\Omega}$.

When $10\,\Omega$ resistance is connected in series with Y, let the balancing length be l'.

Then

$$\frac{X}{Y+10} = \frac{l'}{100-l'}$$

or $\qquad \dfrac{20}{30+10} = \dfrac{l'}{100-l'}$

or $\qquad l' = \mathbf{33.3\ cm.}$

3.1. *The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery ?*

Ans. Here $\mathcal{E} = 12$ V, $r = 0.4\,\Omega$

The current drawn from the battery will be maximum when the external resistance in the circuit is zero *i.e.*, $R = 0$.

$$\therefore \quad I_{max} = \frac{\mathcal{E}}{r} = \frac{12}{0.4}$$

$$= 30 \text{ A.}$$

3.2. *A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor ? What is the terminal voltage of the battery when the circuit is closed ?*

Ans. As

$$I = \frac{\mathcal{E}}{R + r}$$

or $$R + r = \frac{\mathcal{E}}{I}$$

$$\therefore \quad R = \frac{\mathcal{E}}{I} - r = \frac{10}{0.5} - 3 = 17\,\Omega$$

Terminal voltage,

$$V = IR = 0.5 \times 17 = 8.5 \text{ V}.$$

3.3. *(i) Three resistors of 1 Ω, 2 Ω and 3 Ω are combined in series. What is the total resistance of the combination ? (ii) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.*

Ans. (*i*) $R_s = R_1 + R_2 + R_3 = 6\,\Omega.$

(*ii*) Current in the circuit, $I = \frac{\mathcal{E}}{R} = \frac{12}{6} = 2$ A

$\therefore$ Potential drops across different resistors are

$$V_1 = I\,R_1 = 2 \times 1 = \mathbf{2\ V},$$
$$V_2 = I\,R_2 = 2 \times 2 = \mathbf{4\ V},$$
$$V_3 = I\,R_3 = 2 \times 3 = \mathbf{6\ V}.$$

3.4. *(i) Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination ? (ii) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.*

Ans. (*i*) $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{19}{20}$

$\therefore \quad R_p = \frac{\mathbf{20}}{\mathbf{19}}\,\Omega.$

(*ii*) Currents drawn through different resistors are

$$I_1 = \frac{\mathcal{E}}{R_1} = \frac{20}{2} = \mathbf{10\ A}, \quad I_2 = \frac{\mathcal{E}}{R_2} = \frac{20}{4} = \mathbf{5\ A},$$

$$I_3 = \frac{\mathcal{E}}{R_3} = \frac{20}{5} = \mathbf{4\ A}$$

Total current drawn from the battery,

$$I = I_1 + I_2 + I_3 = 10 + 5 + 4 = \mathbf{19\ A}.$$

3.5. *At room temperature (27° C), the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that temperature coefficient of the resistor material is $1.70 \times 10^{-4}\,°C^{-1}$.*

Ans. Here $R_1 = 100\,\Omega$, $R_2 = 117\,\Omega$, $t_1 = 27°C$, $\alpha = 1.70 \times 10^{-4}\ °C^{-1}$

As $\quad \alpha = \frac{R_2 - R_1}{R_1\,(t_2 - t_1)}$

$\therefore \quad t_2 - t_1 = \frac{R_2 - R_1}{R_1\,\alpha} = \frac{117 - 100}{100 \times 1.70 \times 10^{-4}} = 1000$

$\therefore \quad t_2 = 1000 + t_1 = 1000 + 27 = \mathbf{1027°\ C}.$

3.6. *A negligibly small current is passed through a wire of length 15 m and uniform cross-section 6.0×10^{-7} m^2 and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment ?*

Ans. Here $l = 15$ m, $A = 6.0 \times 10^{-7}\,\text{m}^2$, $R = 5.0\,\Omega$

Resistivity, $\rho = \frac{RA}{l} = \frac{5.0 \times 6.0 \times 10^{-7}}{15}$

$$= \mathbf{2.0 \times 10^{-7}\,\Omega\ m}.$$

3.7. *A silver wire has a resistance of 2.1 Ω at 27.5° C, and a resistance of 2.7 Ω at 100° C. Determine the temperature coefficient of resistivity of silver.*

Ans. Here $R_1 = 2.1\,\Omega, t_1 = 27.5°C, R_2 = 2.7\,\Omega, t_2 = 100°C$

Temperature coefficient of resistivity of silver,

$$\alpha = \frac{R_2 - R_1}{R_1\,(t_2 - t_1)}$$
$$= \frac{2.7 - 2.1}{2.1(100 - 27.5)} = \frac{0.6}{2.1 \times 72.5}$$
$$= \mathbf{0.00394°\ C^{-1}}.$$

3.8. *A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27° C ? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1.70 \times 10^{-4}\,°C^{-1}$.*

Ans. Here $V = 230$ V, $I_1 = 3.2$ A,

$I_2 = 2.8$ A, $\alpha = 1.70 \times 10^{-4}\ °C^{-1}$

Resistance at room temperature,

$$R_1 = \frac{V}{I_1} = \frac{230}{3.2} = 71.875\,\Omega$$

Resistance at steady temperature,

$$R_2 = \frac{V}{I_2} = \frac{230}{2.8} = 82.143\,\Omega$$

Now $\quad \alpha = \frac{R_2 - R_1}{R_1\,(t_2 - t_1)}$

$\therefore \quad t_2 - t_1 = \frac{R_2 - R_1}{R_1\,\alpha}$

$$= \frac{82.143 - 71.875}{71.875 \times 1.70 \times 10^{-4}}$$
$$= \frac{10.268 \times 10^4}{71.875 \times 1.7} = 840.35°C$$

$\therefore$ Steady temperature of element,

$$t_2 = 840.35 + 27 = \mathbf{867.35°\ C}.$$

3.9. *Determine the current in each branch of the network shown in Fig. 3.313.*

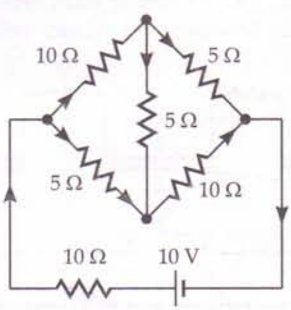


Fig. 3.313

Ans. Let I, I_1, I_2, I_3 be the currents as shown in Fig. 3.314. We apply Kirchhoff 's second rule to different loops.

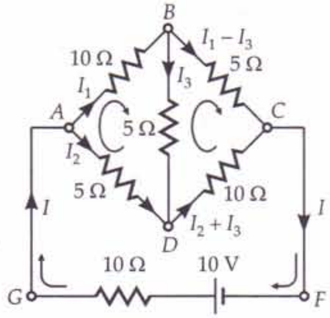


Fig. 3.314

For loop *ABDA*,

$$10I_1 + 5I_3 - 5I_2 = 0$$

For loop *BCDB*,

$$5(I_1 - I_3) - 10(I_2 + I_3) - 5I_3 = 0$$

For loop *ADCFGA*,

$$5I_2 + 10(I_2 + I_3) + 10(I_1 + I_2) = 10 \qquad (\because\ I_1 + I_2 = I)$$

or

$$10I_1 - 5I_2 + 5I_3 = 0 \qquad ...(1)$$

$$5I_1 - 10I_2 - 20I_3 = 0 \qquad ...(2)$$

$$10I_1 + 25I_2 + 10I_3 = 10 \qquad ...(3)$$

Solving equations (1), (2) and (3), we get

$$I_1 = \frac{4}{17}\text{ A},\ I_2 = \frac{6}{17}\text{ A},\ I_3 = -\frac{2}{17}\text{ A}$$

Currents in different branches are

$$I_{AB} = I_1 = \mathbf{\frac{4}{17}\ A},\ I_{BC} = I_1 - I_3 = \mathbf{\frac{6}{17}\ A},$$

$$I_{DC} = I_2 + I_3 = \mathbf{\frac{4}{17}\ A}$$

$$I_{AD} = I_2 = \mathbf{\frac{6}{17}\ A},\ I_{BD} = I_3 = -\mathbf{\frac{2}{17}\ A}$$

Total Current,

$$I = I_1 + I_2 = \mathbf{\frac{10}{17}\ A.}$$

3.10. *(i) In a metre bridge (Fig. 3.315), the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5 Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or metre bridge made of thick copper strips ? (ii) Determine the balance point of the bridge above if X and Y are interchanged. (iii) What happens if the galvanometer and cell are interchanged at the balance point of the bridge ? Would the galvanometer show any current ?* [CBSE D 05]

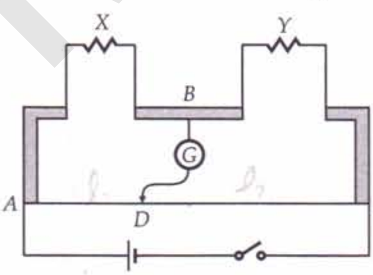


Fig. 3.315

Ans. Here $l = 35.9$ cm, $R = X = 7$, $S = Y = 12.5\,\Omega$

As $S = \frac{100 - l}{l} \times R \quad \therefore \quad 12.5 = \frac{100 - 39.5}{39.5} \times R$

or $R = \frac{12.5 \times 39.5}{60.5} = \mathbf{8.16\,\Omega}$

Connections are made by thick copper strips to minimise the resistances of connections which are not accounted for in the above formula.

(*ii*) When *X* and *Y* are interchanged,

$$R = Y = 12.5\,\Omega, \quad S = X = 8.16\,\Omega, l = ?$$

As $S = \frac{100 - l}{l} \times R \quad \therefore \quad 8.16 = \frac{100 - l}{l} \times 12.5$

or $8.16\,l = 1250 - 12.5\,l$

or $l = \frac{1250}{20.66} = \mathbf{60.5\,\Omega}$, from the end *A*.

(*iii*) When the galvanometer and cell are interchanged at the balance point, the conditions of the balanced bridge are still satisfied and so again the galvanometer will not show any current.

3.11. *A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging ? What is the purpose of having a series resistor in the charging circuit ?*

Ans. When the storage battery of 8.0 volt is charged with a dc supply of 120 V, the net emf in the circuit will be

$$\mathcal{E}' = 120 - 8.0 = 112\text{ V}$$

Current in the circuit during charging

$$I = \frac{\mathcal{E}'}{R + r} = \frac{112}{15.5 + 0.5} = 7\text{ A}$$

The terminal voltage of the battery during charging,

$$V = \mathcal{E} + Ir = 8.0 + 7 \times 0.5 = \mathbf{11.5\ V}$$

The series resistor limits the current drawn from the external source. In its absence, the current will be dangerously high.

3.12. *In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell ?*

Ans. Here $\mathcal{E}_1 = 1.25$ V, $l_1 = 35.0$ cm, $l_2 = 63.0$ cm, $\mathcal{E}_2 = ?$

As $\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1}$

$\therefore \quad \mathcal{E}_2 = \frac{l_2}{l_1} \times \mathcal{E}_1 = \frac{63 \times 1.25}{35} = \mathbf{2.25\ V.}$

3.13. *The number density of free electrons in a copper conductor is $8.5 \times 10^{28}\ m^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end ? The area of cross-section of the wire is $2.0 \times 10^{-6}\ m^2$ and it is carrying a current of 3.0 A.*

Ans. Here $n = 8.5 \times 10^{28} \text{m}^{-3}$, $l = 3$ m,
$A = 2.0 \times 10^{-6}\ \text{m}^2$, $e = 1.6 \times 10^{-19}$ C, $I = 3.0$ A

Drift speed,

$$v_d = \frac{I}{enA}$$
$$= \frac{3}{1.6 \times 10^{-19} \times 8.5 \times 10^{28} \times 2 \times 10^{-6}} \text{ ms}^{-1}$$
$$= \frac{3}{16 \times 85 \times 2 \times 10} \text{ ms}^{-1} = 1.1 \times 10^{-4} \text{ ms}^{-1}$$

Required time,

$$t = \frac{l}{v_d} = \frac{3}{1.1 \times 10^{-4}} \text{ s} = 2.73 \times 10^4 \text{ s} \simeq \mathbf{7.57\ h.}$$

3.14. *The earth's surface has a negative surface charge density of $10^{-9}\ Cm^{-2}$. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface ? (Radius of the earth = 6.37×10^6 m).*

Ans. Surface charge density,

$$\sigma = 10^{-9} \text{Cm}^{-2}$$

Radius of the earth,

$$R = 6.37 \times 10^6 \text{ m}$$

Current, $I = 1800$ A

Total charge of the globe,

$$q = \text{surface area} \times \sigma = 4\pi R^2 \sigma$$
$$= 4 \times 3.14 \times (6.37 \times 10^6)^2 \times 10^{-9}$$
$$= 509.65 \times 10^3 \text{ C}$$

Required time,

$$t = \frac{q}{I} = \frac{509.65 \times 10^3}{1800} = 283.13 \text{ s} \simeq \mathbf{283\ s.}$$

3.15. *(a) Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage ?*

(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell ? Could the cell drive the starting motor of a car ?

Ans. (a) Here $\mathcal{E} = 2$ V, $r = 0.015\,\Omega$, $R = 8.5\,\Omega$, $n = 6$

When the cells are joined in series, the current is

$$I = \frac{n\mathcal{E}}{R + nr} = \frac{6 \times 2}{8.5 + 6 \times 0.015} = \frac{12}{8.59} \text{ A} \simeq \mathbf{1.4\ A}$$

Terminal voltage,

$$V = IR = 1.4 \times 8.5 = \mathbf{11.9\ V.}$$

(b) Here $\mathcal{E} = 1.9$ V, $r = 380\,\Omega$

$$I_{max} = \frac{\mathcal{E}}{r} = \frac{1.9}{380} \text{ A} = \mathbf{0.005\ A}$$

This secondary cell cannot drive the starting motor of a car because that requires a large current of about 100 A for a few seconds.

3.16. *Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter ? Hence explain why aluminium wires are preferred for overhead power cables.*

Given : $\rho_{Al} = 2.63 \times 10^{-8}\,\Omega m$, $\rho_{Cu} = 1.72 \times 10^{-8}\,\Omega m$, relative density of Al = 2.7 and that of Cu = 8.9.

Ans. Mass = volume × density = $Al\,d$

$$= \frac{\rho l}{R} . ld = \frac{\rho d\, l^2}{R} \qquad \left[\because R = \rho \frac{l}{A}\right]$$

As the two wires are of equal length and have the same resistance, their mass ratio will be

$$\frac{m_{Cu}}{m_{Al}} = \frac{\rho_{Cu}\, d_{Cu}}{\rho_{Al}\, d_{Al}} = \frac{1.72 \times 10^{-8} \times 8.9}{2.63 \times 10^{-8} \times 2.7} = 2.1558 \simeq \mathbf{2.2}$$

i.e., copper wire is 2.2 times heavier than aluminium wire. Since aluminium is lighter, it is preferred for long suspension of cables otherwise heavy cable may sag down due to its own weight.

3.17. *What conclusion can you draw from the following observations on a resistor made of alloy manganin :*

Current I(A)	Voltage V	Current I(A)	Voltage V
0.2	3.94	3.0	59.2
0.4	7.87	4.0	78.8
0.6	11.8	5.0	98.6
0.8	15.7	6.0	118.5
1.0	19.7	7.0	138.5
2.0	39.4	8.0	158.0

Ans. We plot a graph between current I (along y-axis) and voltage V (along x-axis) as shown in Fig. 3.316.

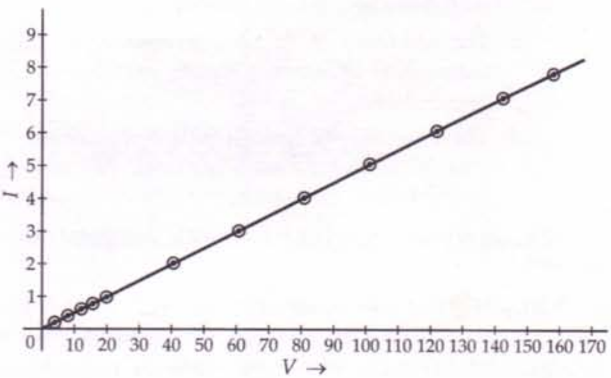


Fig. 3.316 *V-I* graph for manganin.

Since the *V-I* graph is almost a straight line, therefore, manganin resistor is an ohmic resistor for given ranges of votlage and current. As the current increases from 0 to 8 A, the temperature increases but the resistance of manganin does not change. This indicates that the temperature coefficient of resistivity of manganin alloy is negligibly small.

3.18. *Answer the following questions :*

(*a*) *A steady current flows in a metallic conductor of non-uniform cross-section. Say which of these quantities is constant along the conductor : current, current density, electric field, drift speed ?* [CBSE D 15C]

(*b*) *Is Ohm's law universally applicable for all conducting elements ? If not, give examples of elements which do not obey Ohm's law.*

(*c*) *A low voltage supply from which one needs high current must have very low internal resistance. Why ?*

(*d*) *Why a high tension (H.T.) supply of say 6 kV must have a very large internal resistance ?*

Ans. (*a*) Only current is constant because it is given to be steady. Other quantities : current density, electric field and drift speed vary inversely with area of cross-section.

(*b*) No, Ohm's law is not universally applicable for all conducting elements. Examples of non-ohmic elements are vacuum diode, semiconductor diode, thyristor, gas discharge tube, electrolytic solution, etc.

(*c*) The maximum current that can be drawn from a voltage supply is given by

$$I_{max} = \frac{\varepsilon}{r}$$

Clearly, I_{max} will be large if r is small.

(*d*) If the internal resistance is not very large, then the current will exceed the safety limits in case the circuit is short-circuited accidentally.

3.19. *Choose the correct alternative :*

(*a*) *Alloys of metals usually have (greater/ lesser) resistivity than that of their constituent metals.*

(*b*) *Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.*

(*c*) *The resistivity of the alloy manganin is nearly independent of/increases rapidly with increase of temperature.*

(*d*) *The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of ($10^{22}/10^{3}$).*

Ans. (*a*) greater (*b*) lower (*c*) is nearly independent of (*d*) 10^{22}.

3.20. (*a*) *Given n resistors each of resistance R, how will you combine them to get the (i) maximum, (ii) minimum effective resistance ? What is the ratio of the maximum to minimum resistance ?*

(*b*) *Given the resistance of* $1\,\Omega, 2\,\Omega, 3\,\Omega$, *how will you combine them to get an equivalent resistance of :*

(*i*) $\frac{11}{3}\,\Omega$ (*ii*) $\frac{11}{5}\,\Omega$ (*iii*) $6\,\Omega$ (*iv*) $\frac{6}{11}\,\Omega$?

[CBSE F 15]

(*c*) *Determine the equivalent resistance of the following networks :*

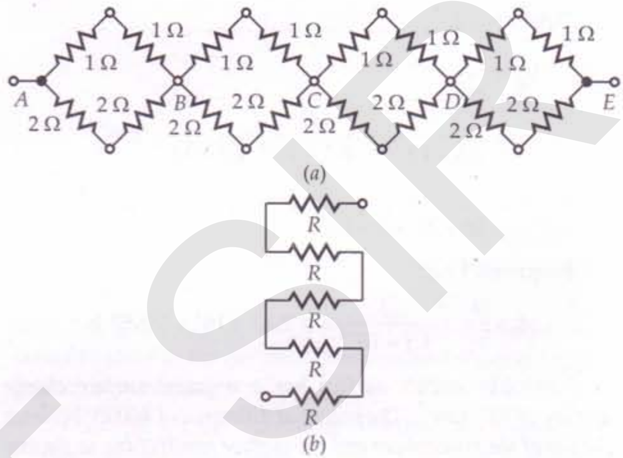


Fig. 3.317

Ans. (*a*) For maximum effective resistance, all the n resistors must be connected in series.

$\therefore$ Maximum effective resistance,

$$R_s = nR$$

For minimum effective resistance, all the n resistors must be connected in parallel. It is given by

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \ldots\ldots\ n \text{ terms} = \frac{n}{R}$$

$\therefore$ Minimum effective resistance,

$$R_p = \frac{R}{n}$$

Ratio of the maximum to minimum resistance is

$$\frac{R_s}{R_p} = \frac{nR}{R/n} = \frac{n^2}{1} = \mathbf{n^2 : 1.}$$

(*b*) Here $R_1 = 1\,\Omega,\ R_2 = 2\,\Omega,\ R_3 = 3\,\Omega$

(*i*) When parallel combination of $1\,\Omega$ and $2\,\Omega$ resistors is connected in series with $3\,\Omega$ resistor [Fig. 3.318(*a*)], the equivalent resistance is

$$R = R_p + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

$$= \frac{1 \times 2}{1 + 2} + 3 = \frac{2}{3} + 3 = \mathbf{\frac{11}{3}}\,\Omega.$$

(*ii*) When parallel combination of $2\,\Omega$ and $3\,\Omega$ resistors is connected in series with $1\,\Omega$ resistor [Fig. 3.318(*b*)], the equivalent resistance is

$$R = \frac{R_2 R_3}{R_2 + R_3} + R_1 = \frac{2 \times 3}{2 + 3} + 1 = \frac{6}{5} + 1 = \mathbf{\frac{11}{5}}\,\Omega.$$

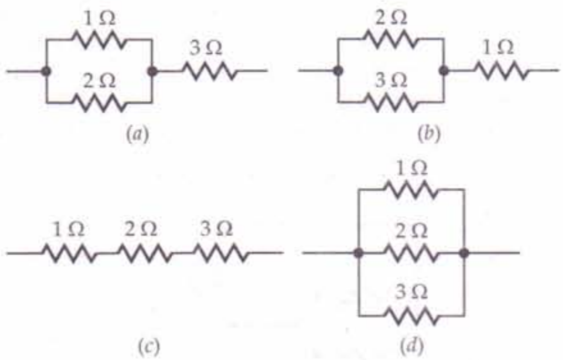


Fig. 3.318

(*iii*) When the three resistances are connected in series [Fig. 3.318(*c*)], the equivalent resistance is

$$R = R_1 + R_2 + R_3 = (1 + 2 + 3)\,\Omega = \mathbf{6\,\Omega}.$$

(*iv*) When all the resistances are connected in parallel [Fig. 3.318(*d*)],

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$\therefore$ Equivalent resistance, $R = \mathbf{\frac{6}{11}}\,\Omega.$

(*c*) The network shown in Fig. 3.317(*a*) is a series combination of four identical units. One such unit is shown in Fig. 3.319(*a*) and it is equivalent to a parallel combination of two resistances of 2 Ω and 4 Ω as shown in Fig. 3.319(*b*).

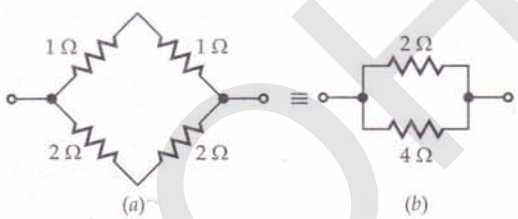


Fig. 3.319

Resistance R of one such unit is given by

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$$

or $$R = \frac{4}{3}\,\Omega$$

$\therefore$ Resistance of the total network (4 such units)

$$= 4 \times \frac{4}{3} = \mathbf{\frac{16}{3}}\,\Omega.$$

(*ii*) The network shown in Fig. 3.319(*b*) is a series combination of 5 resistors, each of resistance R.

$\therefore$ Equivalent resistance = **5 *R*.**

3.21. *Determine the current drawn from a 12 V supply with internal resistance 0.5 Ω by the following infinite network. Each resistor has 1 Ω resistance.*

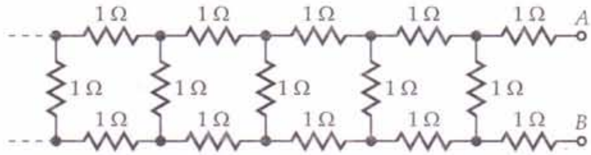


Fig. 3.320

Ans. Let the equivalent resistance of the infinite network be X. This network consists of infinite units of three resistors of 1 Ω, 1 Ω, 1 Ω. The addition of one more such unit across AB will not affect the total resistance. The network obtained by adding one more unit would appear as shown in Fig. 3.321.

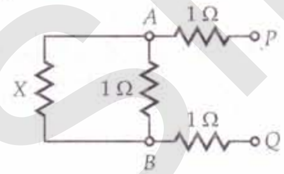


Fig. 3.321

Resistance between A and B

= Resistance equivalent to parallel combination of X and 1Ω

$$= \frac{X \times 1}{X+1} = \frac{X}{X+1}$$

Resistance between P and Q

$$= 1 + \frac{X}{X+1} + 1 = 2 + \frac{X}{X+1}$$

This must be equal to the original resistance X.

$\therefore$ $$X = 2 + \frac{X}{1+X}$$

or $$X^2 - 2X - 2 = 0$$

or $$X = 1 \pm \sqrt{3}$$

As the value of resistance cannot be negative, so

$$X = 1 + \sqrt{3} = 2.732\,\Omega$$

Current, $$I = \frac{\text{emf}}{\text{Total resistance}} = \frac{\varepsilon}{X + r} = \frac{12}{2.732 + 0.5}$$

$$= \mathbf{3.713\ A}$$

3.22. *Figure 3.322 shows a potentiometer with a cell of 2.0 V and internal resistance 0.40 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf of 1.02 V (for very moderate currents upto a few A) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf ε and the balance point found similarly turns out to be at 82.3 cm length of the wire.*

(*a*) *What is the value of ε ?*

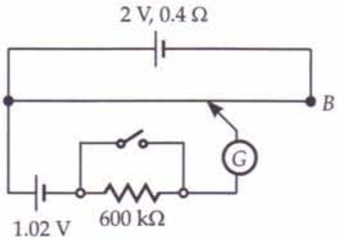


Fig. 3.322

(*b*) *What purpose does the high resistance of 600 kΩ have ?*

(*c*) *Is the balance point affected by this high resistance ?*

(*d*) *Is the balance point affected by the internal resistance of the driver cell ?*

(*e*) *Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V ?*

(*f*) *Would the circuit work well for determining extremely small emf, say of the order of a few mV (such as the typical emf of a thermocouple) ? If not, how will you modify the circuit ?*

Ans. (*a*) $\mathcal{E}_1 = 1.02$ V, $l_1 = 67.3$ cm, $\mathcal{E}_2 = \mathcal{E} = ?$, $l_2 = 82.3$ cm

Formula for the comparison of emfs by potentiometer is

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1} \qquad \therefore \qquad \frac{\mathcal{E}}{1.02} = \frac{82.3}{67.3}$$

or
$$\mathcal{E} = \frac{82.3}{67.2} \times 1.02 = \mathbf{1.25\ V.}$$

(*b*) High resistance of 600 kΩ protects the galvanometer for positions far away from the balance point, by decreasing current through it.

(*c*) No, balance point is not affected by high resistance because no current flows through the standard cell at the balance point.

(*d*) Yes, the balance point is affected by the internal resistance of the driver cell. The internal resistance affects the current through the potentiometer wire, so changes the potential gradient and hence affects the balance point.

(*e*) No, the arrangement will not work. If $\mathcal{E}$ is greater than the emf of the driver cell of the potentiometer, there will be no balance point on the wire *AB*.

(*f*) The circuit as it is would be unsuitable, because the balance point (for $\mathcal{E}$ of the order of a few mV) will be very close to the end *A* and the percentage error in measurement will be very large. The circuit is modified by putting a suitable resistor *R* in series with the wire *AB* so that potential drop across *AB* is only slightly greater than the emf to be measured. Then the balance point will be at larger length of the wire and the percentage error will be much smaller.

3.23. *Figure 3.323 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor $R = 10.0\,\Omega$ is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What might you do if you failed to find a balance point with the given cell $\mathcal{E}$?*

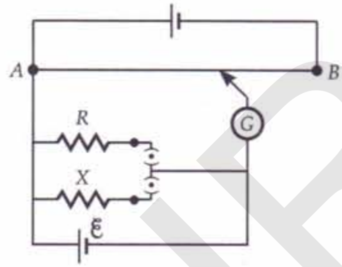


Fig. 3.323

Ans. Here $R = 10.0\,\Omega$, $l_1 = 58.3$ cm, $X = ?$, $l_2 = 68.5$ cm

Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be the potential drops across *R* and *X* respectively and *I* be the current in potentiometer wire.

Then
$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{IX}{IR} = \frac{X}{R}$$

But
$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1} \qquad \therefore \qquad \frac{X}{R} = \frac{l_2}{l_1}$$

or
$$X = \frac{l_2}{l_1} \cdot R = \frac{68.5}{58.3} \times 10 = \mathbf{11.75\,\Omega}$$

If there is no balance point, it means potential drops across *R* or *X* are greater than the potential drop across the potentiometer wire *AB*. We should reduce current in the outside circuit (and hence potential drops across *R* and *X*) suitably by putting a series resistor.

3.24. *Figure 3.324 shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.*

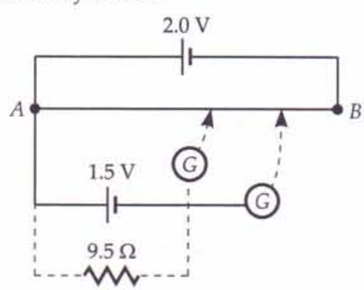


Fig. 3.324

Ans. Here $l_1 = 76.3$ cm, $l_2 = 64.8$ cm, $R = 9.5\,\Omega$

The formula for the internal resistance of a cell by potentiometer method is

$$r = R\left(\frac{l_1 - l_2}{l_2}\right) = 9.5\left(\frac{76.3 - 64.8}{64.8}\right) = \frac{9.5 \times 11.5}{64.8} \simeq \mathbf{1.7\,\Omega.}$$

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. Define electric current. What is the SI unit of electric current ?
2. Write the relation between a coulomb and an ampere. [ISCE 96]
3. What does the direction of electric current signify in an electric circuit ?
4. What is electromotive force ? State its SI Unit. [Punjab 2000]
5. State the condition in which terminal voltage across a secondary cell is equal to its emf ? [CBSE D 2000]
6. Define an emf of one volt.
7. State Ohm's law. [ISCE 95]
8. Name the colours corresponding to the digits 4 and 7 in the colour code scheme for carbon resistors. [CBSE SP 15]
9. Define resistance and state its SI unit. [CBSE D 92C]
10. Define Ohm.
11. Define conductance of a material. Give its SI unit. [CBSE D 02]
12. Define electrical conductivity of a material. Give its SI unit. [CBSE D 03, 14]
13. How much is the resistance of an air-gap ?
14. How much is the resistance of a closed plug-key ?
15. Which metal has the lowest resistivity ?
16. Define resistivity of a material. State its SI unit. [ISCE 93]
17. What is the order of resistivity of an insulator ? [Punjab 97C]
18. What is the ratio of the resistivity of a typical insulator to that of a metal ?
19. What is the average velocity of free electrons in a metal at room temperature ?
20. Give the order of magnitude of the number density of free electrons in a metal.
21. Give the order of magnitude of thermal velocity and drift velocity of free electrons in a conductor carrying current at room temperature.
22. What is the order of resistivity of conductor ?
23. Define temperature coefficient of resistivity.
24. How does the random motion of free electrons in a conductor get affected when a potential difference is applied across its ends ? [CBSE D 14C]
25. Define the term 'drift velocity' of charge carriers in a conductor and write its relationship with the current flowing through it. [CBSE D 14]
26. Write the expression for the drift velocity of charge carriers in a conductor of length 'l' across which a potential difference 'V' is applied. [CBSE OD 14C]
27. How does one explain increase in resistivity of a metal with increase of temperature ? [CBSE OD 14C]
28. Define the term mobility of charge carriers in a conductor. Write its SI unit. [CBSE D 14 ; OD 15]
29. Plot a graph showing variation of resistivity of a conductor (copper) with temperature. [CBSE D 14 ; F 15]
30. Plot a graph showing variation of current versus voltage for the material GaAs. [CBSE D 14]
31. Sketch a graph showing variation of resistivity of carbon with temperature. [CBSE D 06]

    Or

    Show on a graph, the variation of resistivity with temperature for a typical semiconductor Si. [CBSE D 05, 12, 14]
32. Name two materials whose resistivity decreases with the rise of temperature.
33. How does the conductance of a semi-conducting material change with rise in temperature ?
34. Of copper and nichrome, which one has possibly larger value of temperature coefficient of resistance ? [CBSE D 95C]
35. How does resistivity of alloy manganin change with temperature ?
36. How does the resistance of an insulator change with temperature ?
37. Name two parameters which determine the resistivity of a material.
38. How is the conductivity of an electrolyte affected by the increase of temperature ? [CBSE D 95]
39. If potential difference V applied across a conductor is increased to $2V$, how will the drift velocity of the electrons change ? [CBSE OD 2000C]
40. What is a non-ohmic device ? State one example. [Punjab 02]
41. What is a linear resistor ?
42. Give an example of non-ohmic device which shows up negative resistance.

43. A cell of emf '$\mathcal{E}$' and internal resistance 'r' draws a current 'I'. Write the relation between terminal voltage 'V' in terms of $\mathcal{E}$, I and r. [CBSE OD 13]

44. Two identical cells, each of emf $\mathcal{E}$, having negligible internal resistance r, are connected in parallel with each other across an external resistance R. What is the current through this resistance ? [CBSE OD 13]

45. A 4 Ω non-insulated resistance wire is bent 180° in the middle and the two halves are twisted together. What will be its new resistance ? [CBSE D 10C]

46. Can Kirchhoff's laws be applied to both d.c. and a.c. circuits ?

47. On what conservation principle is the Kirchoff's first law based ?

48. On what conservation principle is the Kirchhoff's second law based ?

49. Name the device used for measuring the emf of a cell. [CBSE D 96]

50. Name the device used for measuring the internal resistance of a secondary cell. [CBSE D 96]

51. Define potential gradient. Give its SI unit.

52. Name the principle on which a metre bridge works.

53. What is a Wheatstone bridge ? [CBSE D 03]

54. The given graph shows the variation of resistance of mercury in the temperature range $0 < T < 4$ K. Name the phenomenon shown by the graph. [CBSE OD 03]

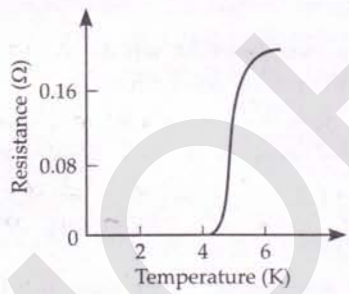


Fig. 3.325

55. If the resistances in the three successive arms of a balanced Wheatstone bridge are 1, 2 and 36 Ω respectively, what will be the resistance of the galvanometer placed in the fourth arm ?

56. Current I flows through a potential drop V across a conductor. What is the rate of production of heat ? [CBSE D 93C]

57. The rate of production of heat is given by $P = VI$. Is this relation valid for a non-ohmic conductor ?

58. How is electric energy related to electric power ?

59. Of which physical quantity is the unit kilowatt hour ?

60. What do you mean by 1 unit of electric energy in domestic use ?

61. How many joules of energy are equivalent to 1 kWh ?

62. How many kilowatt hours (kWh) are there in one joule ? [CBSE OD 99C]

63. The applied p.d. across a given resistance is altered so that heat produced per second increases by a factor of 9. By what factor does the applied p.d. change ? [CBSE OD 99C]

64. Two electric bulbs are rated at 220 V – 100 W and 220 V – 60 W. Which one of these has greater resistance and why ? [CBSE Sample Paper 03]

65. The maximum power dissipated in a 10,000 Ω resistor is 1 W. What is the maximum current ? [ISCE 93]

66. What is the safest voltage you can safely put across a 98 Ω, 0.5 W resistor ? [ISCE 97]

67. How much charge flows through a 250 V, 1,000 W heater in one minute ? [ISCE 96]

68. A heating element is marked 210 V, 630 W. What is the value of the current drawn by the element when connected to a 210 V dc source ? [CBSE D 13]

69. A heating element is marked 210 V, 630 W. Find the resistance of the element when connected to a 210 V dc source. [CBSE D 13]

70. Two resistors of 2 Ω and 4 Ω are connected in parallel to a constant d.c. voltage. In which case more heat is produced ? [CBSE D 98C]

71. Two bulbs whose resistances are in the ratio 1 : 2 are connected in parallel to a source of constant voltage. What will be the ratio of power dissipation in these bulbs ? [CBSE D 2000C]

72. Distinguish between kilowatt and kilowatt hour.

73. Which has a greater resistance–kW electric heater or a 100 W filament bulb both marked for 220 V ? [CBSE D 01C]

74. The coil of a heater is cut into two equal halves and only one of them is used into heater. What is the ratio of the heat produced by this half coil to that by original coil ?

75. What do you mean by the maximum power rating of a resistor ?

76. Express power transferred per unit volume into joule heat in a resistor in terms of current density.

77. Write two special characteristics of the wire of an electric heater. [CBSE D 94]

78. What are the characteristics of a fuse wire ?

79. What is the difference between a heater wire and a fuse wire ?

80. Two identical heaters rated 220 V, 1000 W are placed in series with each other across a 220 V line. What is their combined power ?

81. Write an expression for the resistivity of a metallic conductor showing its variation over a limited range of temperature. [CBSE D 08C]

82. The plot of the variation of potential difference across a combination of three identical cells in series, versus current is as shown in Fig. 3.326. What is the emf of each cell ? [CBSE D 08]

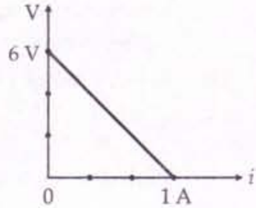


Fig. 3.326

83. A (*i*) series (*ii*) parallel combination of two given resistors is connected one-by-one, across a cell. In which case will the terminal potential difference, across the cell, have a higher value ? [CBSE OD 08C]

84. The *I-V* characteristics of a resistor are observed to deviate from a straight line for higher values of current as shown in Fig. 3.327. Why ? [CBSE SP 08]

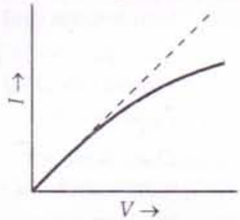


Fig. 3.327

85. Two identical slabs of given metal are joined together, in two different ways, as shown in Figs. 3.328(*i*) and (*ii*). What is the ratio of the resistances of these two combinations ? [CBSE D 10C]

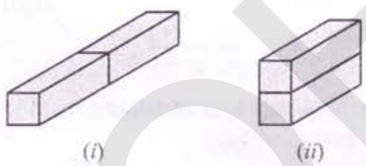


Fig. 3.328

86. A resistance R is connected across a cell, of emf $\mathcal{E}$ and internal resistance r. A potentiometer now measures the p.d., between the terminals of the cell, as V. State the expression for 'r' in terms of $\mathcal{E}$, V and R. [CBSE D 11]

87. A parallel combination of two cells of emf's $\mathcal{E}_1$ and $\mathcal{E}_2$, and internal resistances, r_1 and r_2, is used to supply current to a load of resistance R. Write the expression for the current through the load in terms of $\mathcal{E}_1, \mathcal{E}_2, r_1$ and r_2. [CBSE Sample Paper 2011]

88. Under what condition can we draw maximum current from a secondary cell ? [CBSE F 10]

89. Write any two factors on which the internal resistance of a cell depends. [CBSE OD 10]

90. Write two factors on which the sensitivity of a potentiometer depends. [CBSE D 13C]

91. Graph showing the variation of current versus voltage for a material GaAs is shown in the figure. Identify the region of

    (*i*) negative resistance

    (*ii*) where Ohm's law is obeyed. [CBSE D 15]

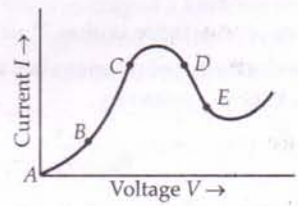


Fig. 3.329

Answers

1. The electric current is defined as the rate of flow of electric charge through any section of a conductor.

$$\text{Electric current} = \frac{\text{Total charge flowing}}{\text{Time taken}}$$

or $$I = \frac{q}{t}$$

The SI unit of electric current is *ampere* (A).

2. $1 \text{ ampere} = \dfrac{1\,\text{coulomb}}{1\,\text{second}}$.

3. The direction of conventional current in an electric circuit tells the direction of flow of positive charges in that circuit.

4. The work done per unit charge by a source in taking the charge once round the complete circuit is called electromotive force or emf of the source. SI unit of emf is volt.

5. When no current is drawn from the cell, its terminal voltage is equal to its emf.

6. If an electric cell supplies an energy of 1 joule for the flow of 1 coulomb of charge through the whole circuit (including the cell), then its emf is said to be 1 volt.

7. Ohm's law states that the electric current I passing through a conductor is proportional to the potential

difference V applied across its ends, provided the temperature and other physical conditions remain unchanged, *i.e.*, $V \propto I$ or $V = RI$

where R is called resistance of the conductor.

8. Yellow and violet respectively.
9. Resistance of a conductor is the property by virtue of which it opposes the flow of current through it. It is equal to the ratio of the potential difference applied across the conductor to the current flowing through it. SI unit of resistance is *ohm* (Ω).
$$R = \frac{V}{I}$$
10. The resistance of a conductor is said to be 1 ohm if 1 ampere of current flows through it on applying a potential difference of 1 volt across its ends
$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}} \quad \text{or} \quad 1\Omega = \frac{1\text{ V}}{1\text{ A}}.$$
11. The ease with which a conductor allows a current to flow through it is called its conductance. It is equal to the reciprocal of resistance.
$$\text{Conductance } (G) = \frac{1}{\text{Resistance } (R)}$$
SI unit of conductance is ohm^{-1} or mho.
12. The conductivity of a material is equal to the reciprocal of its resistivity.
$$\text{Conductivity } (\sigma) = \frac{1}{\text{Resistivity } (\rho)}$$
SI unit of conductivity is $\text{ohm}^{-1}\text{m}^{-1}$ or mho m^{-1}.
13. Infinity.
14. Negligibly small.
15. Silver.
16. The resistivity of a material is the resistance offered by a unit cube of that material. Its SI unit is Ω m.
17. The resistivity of an insulator like glass or rubber is of the order of $10^8 - 10^{15}$ Ωm.
18. The ratio of 10^{22}.
19. Zero.
20. Number density of free electrons in a metal
$$= 10^{29} \text{ m}^{-3}.$$
21. Thermal velocity of free electrons = 10^5 ms^{-1}.
Drift velocity of free electrons = 1 mm s^{-1}.
22. Conductors have resistivities less than 10^{-6}Ωm.
23. The temperature coefficient of resistivity is defined as the change in resistivity per unit resistivity per degree rise in temperature. Mathematically,
$$\alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)} = \frac{1}{\rho_0} \cdot \frac{d\rho}{dT}.$$
The unit of α is $\text{C}^{\circ -1}$ or K^{-1}.
24. Random motion gets partially directed towards the higher potential side.
25. Refer to point 17 of Glimpses on page 3.168.
26. $v_d = \dfrac{eV\tau}{ml}$
27. With the increase in temperature, the relaxation time τ decreases and hence resistivity $\left(\rho = \dfrac{m}{ne^2\tau}\right)$ increases.
28. Refer to point 21 of Glimpses on page 3.169.
29. See Fig. 3.20(*a*).
30. See Fig. 3.27.
31. See Fig. 3.21 on page 3.25.
32. Germanium and silicon.
33. With the rise in temperature, the conductance of a semi-conducting material increases exponentially.
34. Copper.
35. The resistivity of alloy manganin is nearly independent of temperature.
36. The resistance of an insulator decreases with the increase of temperature.
37. The resistivity of a material depends on (*i*) its number density of free electrons, (*ii*) the relaxation time.
38. The conductivity of an electrolyte increases with the increase in its temperature.
39. Drift velocity,
$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$$
Clearly, when V is increased to $2V$, drift velocity also gets doubled.
40. A device which does not obey Ohm's law is called a non-ohmic device. Semiconductor diodes, thermistors, etc. are non-ohmic devices.
41. A linear resistor is one which obeys Ohm's law or for which voltage-current graph is a straight line passing through origin.
42. Thyristor.
43. $V = \mathcal{E} - Ir$
44. Effective emf of the parallel combination
$$= \text{emf of any one cell} = \mathcal{E}$$
$$\therefore \quad I = \frac{\text{Total emf}}{\text{Total resistance}} = \frac{\mathcal{E}}{R}.$$
45. The length of the wire becomes half of the original length while area of cross-section is doubled.
$$\therefore \quad \frac{R'}{R} = \frac{l'/A'}{l/A} = \frac{l'}{l} \cdot \frac{A}{A'} = \frac{l/2}{l} \cdot \frac{A}{2A} = \frac{1}{4}$$
$$\text{or} \quad R' = \frac{1}{4}R = \frac{1}{4} \times 4 = 1\,\Omega.$$

46. Yes, Kirchhoff's laws can be applied to both d.c. and a.c. circuits.
47. Kirchhoff's first law is based on the law of conservation of charge.
48. Kirchhoff's second law is based on the law of conservation of energy.
49. Potentiometer.
50. Potentiometer.
51. The potential drop per unit length of the potentiometer wire is known as its potential gradient. Its SI unit is volt per metre (Vm^{-1}).
52. The working of a metre-bridge is based on the principle of Wheatstone bridge.
53. A Wheatstone bridge is an arrangement of four resistances used to determine quickly and accurately one of these resistances in terms of other three resistances.
54. Superconductivity.
55. Here $P = 1\Omega, Q = 2\Omega, R = 36\Omega, S = ?$

For a balanced Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

$$\therefore \quad S = \frac{R \times Q}{P} = \frac{36 \times 2}{1} = \mathbf{72\,\Omega.}$$

56. Rate of production of heat is $P = VI$
57. Yes, it is valid.
58. Electric energy = Electric power × time.
59. Kilowatt hour is the unit of electrical energy.
60. 1 unit of electric energy = 1 kWh. This means that when an appliance of power 1000 watt is operated on mains for 1 hour, it consumes 1 unit of electric energy.
61. 1 kWh = 1 killowatt × 1 hour

$$= 1000 \text{ watt} \times 3600 \text{ s} = \mathbf{3.6 \times 10^6 \text{ J}}.$$

62. $1\text{J} = \frac{1}{3.6 \times 10^6} \text{ kWh} = \mathbf{2.778 \times 10^{-7} \text{ kWh}}.$

63. Heat produced per second, $P = \frac{V^2}{R}$

$$\therefore \quad V \propto \sqrt{PR}$$

or $\quad V \propto \sqrt{P}$

As the heat produced per second increases 9 times, so the applied p.d. must increase 3 times the original p.d.

64. $R = \frac{V^2}{P}$. For a given voltage, $R \propto \frac{1}{P}$.

So 60 W bulb has greater resistance than 100 W bulb.

65. $I_{max} = \sqrt{\frac{P_{max}}{R}} = \sqrt{\frac{1}{10,000}}$

$$= \frac{1}{100} = \mathbf{0.01 \text{ A}.}$$

66. $V_{max} = \sqrt{P_{max} R} = \sqrt{0.5 \times 98} = \mathbf{7 \text{ V}.}$

67. Current, $I = \frac{P}{V} = \frac{1000 \text{ W}}{250 \text{ V}} = 4 \text{ A}$

Charge that flows in 1 minute,

$$q = It = 4 \times 60 = \mathbf{240 \text{ C}.}$$

68. $I = \frac{P}{V} = \frac{630}{210} = \mathbf{3 \text{ A}.}$

69. $R = \frac{V^2}{P} = \frac{210 \times 210}{630} = \mathbf{70\,\Omega}.$

70. Heat produced, $H = \frac{V^2 t}{R}$ *i.e.*, $H \propto \frac{1}{R}$.

Thus heat produced in $2\,\Omega$ resistor is more than that in $4\,\Omega$ resistor.

71. $\frac{P_1}{P_2} = \frac{V^2/R_1}{V^2/R_2} = \frac{R_2}{R_1} = \frac{2}{1} = \mathbf{2:1.}$

72. Kilowatt is the unit of electric power while kilowatt hour is the unit of electric energy.

1 kilowatt = 1000 W = 1000 Js^{-1}

1 kilowatt hour = 3.6×10^6 J.

73. Resistance of heater $= \frac{V^2}{P_1} = \frac{220 \times 220}{1000} = 48.4\,\Omega$

Resistance of bulb $= \frac{V^2}{P_2} = \frac{220 \times 220}{100} = 484\,\Omega$

Thus the 100 W bulb has a greater resistance.

74. Let original heat produced,

$$H_1 = \frac{V^2 t}{R}$$

Resistance of half coil = $R/2$

Heat produced in half coil,

$$H_2 = \frac{V^2 t}{R/2} = \frac{2V^2 t}{R} \quad \therefore \quad \frac{H_2}{H_1} = \mathbf{2:1}$$

75. The maximum power rating of a resistor is the maximum power that it can dissipate in the form of heat without undergoing melting.

76. $P = I^2 R = I^2 \cdot \frac{\rho l}{A}$;

Volume, $V = Al$

∴ Power transferred per unit volume

$$= \frac{P}{V} = \frac{I^2 \rho l / A}{Al} = \left(\frac{I}{A}\right)^2 \rho = J^2 \sigma$$

where J is the current density.

77. A heater wire should have
(*i*) high melting point, and
(*ii*) high resistivity.

78. A fuse wire must have high resistivity and low melting point.

79. The melting point of a heater wire is very high while that of a fuse wire is very low.

80. Total power P dissipated by the series combination is given by

$$\frac{1}{P}=\frac{1}{P_1}+\frac{1}{P_2}=\frac{1}{1000}+\frac{1}{1000}=\frac{1}{500}$$

or $P = \mathbf{500\ W}$.

81. The resistivity ρ at any temperature T is given by

$$\rho=\rho_0\,[1+\alpha(T-T_0)]$$

where ρ_0 is the resistivity at a lower reference temperature T_0 and α is the temperature coefficient of resistivity.

82. Total emf of three cells in series

= P.D. corresponding to zero current = 6 V

$\therefore$ The emf of each cell $=\frac{6}{3}=\mathbf{2\ V}$.

83. In the case of series combination of the two given resistors, the the terminal p.d. will have a higher value.

84. At higher values of current, the resistor gets heated up and its resistance increases. The resistor becomes non-ohmic and hence I-V graph deviates from the straight line.

85. For each slab, $R=\rho\frac{l}{A}$

$$R_1=\rho\frac{2l}{A}=2R\ ;\ R_2=\rho\frac{l}{2A}=\frac{R}{2}$$

$$\therefore\quad \frac{R_1}{R_2}=\frac{2R}{R/2}=\mathbf{4:1}.$$

86. Internal resistance, $r=\left(\frac{\mathcal{E}-V}{V}\right)R$.

87. $$I=\frac{\mathcal{E}_{eq}}{R+r_{eq}}=\frac{\dfrac{\mathcal{E}_1r_2+\mathcal{E}_2r_1}{r_1+r_2}}{R+\dfrac{r_1r_2}{r_1+r_2}}=\frac{\mathcal{E}_1r_2+\mathcal{E}_2r_1}{R(r_1+r_2)+r_1r_2}$$

88. When the external resistance in the circuit is zero, the current drawn from the secondary cell is max.

$$I_{max}=\frac{\mathcal{E}}{r}.$$

89. The internal resistance of a cell depends on
(*i*) the nature of the electrolyte and
(*ii*) concentration of the electrolyte.

90. The sensitivity of a potentiometer depends on the potential gradient along its wires. This, in turn, depends on (*i*) length of the potentiometer wire and (*ii*) the value of resistance put in series with the driver cell.

91. (*i*) In the region DE, I decreases with increasing V.

$$\therefore\quad \frac{+\text{ve}\,\Delta V}{-\text{ve}\,\Delta I}=-\text{ve}\,R.$$

(*ii*) AB / BC is the region where Ohm's law is obeyed.

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. Distinguish between electromotive force and terminal potential difference of a cell. What are their units ? [CBSE OD 14C]

2. Explain how the average velocity of free electrons in a metal at constant temperature, in an electric field, remains constant even though the electrons are being constantly accelerated by this electric field ?

3. Define the terms resistivity and conductivity and state their SI units. Draw a graph showing the variation of resistivity with temperature for a typical semiconductor. [CBSE D 05]

4. Define the electrical resistivity of a material. How it is related to the electrical conductivity ? Of the factors, length, area of cross-section, nature of material and temperature – which ones control the resistivity value of conductor ? [CBSE F 98]

5. Explain the term 'drift velocity' of electrons in a conductor. Hence obtain the expression for the current through a conductor in terms of 'drift velocity'. [CBSE OD 13, 13C, 15C]

6. Prove that the current density of a metallic conductor is directly proportional to the drift speed of electrons. [CBSE D 08]

7. What is meant by drift velocity of free electrons ? Derive Ohm's law on the basis of the theory of electron drift. [CBSE D 03 ; Haryana 94]

8. Are the paths of electrons straight lines between successive collisions (with positive ions of the metal) in the (*i*) absence of electric field (*ii*) presence of electric field ? Establish a relation between drift velocity 'v_d' of an electron in a conductor of cross-section 'A', carrying current 'I' and concentration

'n' of free electrons per unit volume of conductor. Hence obtain the relation between current density and drift velocity. [CBSE OD 03]

9. Define relaxation time of electrons in a conductor. Explain how it varies with increase in temperature of a conductor. State the relation between resistivity and relaxation time. [CBSE D 2000]

10. A conductor of length 'l' is connected to a d.c. source of potential 'V'. If the length of the conductor is tripled, by stretching it, keeping 'V' constant, explain how do the following factors vary in the conductor :

    (*i*) Drift speed of electrons, (*ii*) Resistance and (*iii*) Resistivity. [CBSE D 2000]

11. Write the mathematical relation between mobility and drift velocity of charge carriers in a conductor. Name the mobile charge carriers responsible for conduction of electric current in

    (*i*) an electrolyte (*ii*) an ionised gas. [CBSE D 06]

12. Define the term current density of a metallic conductor. Deduce the relation connecting current density (J) and the conductivity (σ) of the conductor, when an electric field E, is applied to it. [CBSE D 06]

13. Define ionic mobility. Write its relationship with relaxation time. Give its SI unit. How does one understand the temperature dependence of resistivity of a semiconductor. [CBSE F 10 ; OD 13C]

14. Define the terms (*i*) drift velocity, (*ii*) relaxation time.

    A conductor of length L is connected to a *dc* source of emf $\mathcal{E}$. If this conductor is replaced by another conductor of same material and same area of cross-section but of length $3L$, how will the drift velocity change ? [CBSE D 11]

15. Define relaxation time of the free electrons drifting in a conductor. How is it related to the drift velocity of free electrons ? Use this relation to deduce the expression for the electrical resistivity of the material. [CBSE OD 12]

16. Define the term resistivity of a conductor. Give its SI unit. Show that the resistance of a conductor is given by

$$R = \frac{ml}{ne^2 \tau A},$$

    where the symbols have their usual meanings. [CBSE OD 02]

17. Define resistivity of a conductor. Plot a graph showing the variation of resistivity with temperature for a metallic conductor. How does one explain such a behaviour, using the mathematical expression of the resistivity of a material. [CBSE D 01, 08]

18. Draw a plot showing the variation of resistivity of a (*i*) conductor and (*ii*) semiconductor, with the increase in temperature.

    How does one explain this behaviour in terms of number density of charge carriers and the relaxation time ? [CBSE D 14C]

19. Define conductivity of a conductor and state its SI unit. State and explain the variation of conductivity of (*a*) good conductor (*b*) ionic conductor with temperature. [CBSE D 01, 08]

20. Establish the relation between drift velocity of electrons and the electric field applied to the conductor. [Punjab 02]

    Or

    Derive an expression for drift velocity of free electrons in a conductor in terms of relaxation time. [CBSE D 09, OD 15]

21. Establish a relation between current and drift velocity. [Himachal 03 ; CBSE OD 15C]

22. Define the term resistance. Give physical explanation of the opposition offered by a conductor to the flow of current through it. [Haryana 94]

23. Explain the colour code for carbon resistors with illustrations. [Haryana 95, 98]

24. Three resistances R_1, R_2 and R_3 are connected in series. Find their equivalent resistance. [CBSE D 92]

25. Three resistances R_1, R_2 and R_3 are connected in parallel. Find the equivalent resistance of the parallel combination.

    [CBSE D 92 ; Himachal 98C ; Punjab 03]

26. What is superconductivity ? Explain. State two applications of superconductors. [Punjab 03]

27. What are superconductors ? Give two applications of the phenomenon of superconductivity.

    [Haryana 94 ; Punjab 95 ; CBSE F 03]

28. What is Meissner effect ? What does it indicate about the magnetic nature of superconductors ?

29. What are ohmic and non-ohmic resistors ? Give one example of each. [Haryana 02]

30. State the conditions under which Ohm's law is not obeyed in a conductor. [CBSE D 92]

31. What is internal resistance of a cell ? On what factors does it depend ?

32. Define emf of a cell. Show that the voltage drop across a resistor connected in parallel with a cell is different from the emf of the cell. [CBSE OD 94C]

33. When a battery of emf $\mathcal{E}$ and internal resistance r is connected to a resistance R, a current I flows through it. Derive the relation between $\mathcal{E}$, I, r and R. [CBSE D 92]

34. Figure 3.330 shows a cell of emf $\mathcal{E}$ and internal resistance r, connected to a voltmeter V and a variable resistance R. Deduce the relationship among V, $\mathcal{E}$, R and r. How will V vary when R is reduced. [ISCE 98]

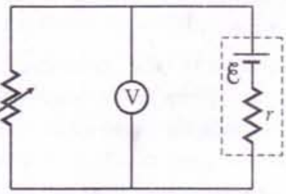


Fig. 3.330

35. Define internal resistance of a cell. Prove that

$$r = \left(\frac{\mathcal{E}}{V} - 1\right) R,$$

where R is the external resistance used. [Himachal 99]

36. A cell of emf '$\mathcal{E}$' and internal resistance 'r' is connected across a variable load resistor R. Draw the plots of the terminal voltage V versus (*i*) R and (*ii*) the current I. [CBSE D 15]

37. Distinguish between emf ($\mathcal{E}$) and terminal voltage (V) of a cell having internal resistance 'r'. Draw a plot showing the variation of terminal voltage (V) *vs.* the current (I) drawn from the cell. Using this plot, how does one determine the emf and the internal resistance of the cell ? [CBSE OD 14, 14C]

38. A cell of emf $\mathcal{E}$ and internal resistance r is connected across a variable resistance R. Plot graphs showing the variation of (*i*) $\mathcal{E}$ and R, (*ii*) terminal p.d. V with R. Predict from the second graph under which V becomes equal to $\mathcal{E}$. [CBSE D 09]

39. Two identical cells, each of emf $\mathcal{E}$ and internal resistance r are connected in parallel to an external resistance R. Find the expression for the total current flowing in the circuit. [CBSE F 96]

40. Derive the formula for the equivalent EMF and internal resistance for the parallel combination of two cells with EMFs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 respectively. What is the corresponding formula for the series combination ? [CBSE Sample Paper 08]

41. Name any one material having a small value of temperature coefficient of resistance. Write one use of this material. [CBSE D 97]

42. Define the terms electric energy and electric power. Give their units. [Haryana 92 ; Punjab 93]

43. Obtain the formula for the 'power loss' in a conductor of resistance R, carrying a current I. [CBSE D 09C]

44. Two heating elements of resistances R_1 and R_2 when operated at a constant supply of voltage, V, consume powers P_1 and P_2 respectively. Deduce the expressions for the power of their combination when they are, in turn, connected in (*i*) series and (*ii*) parallel across the same voltage supply. [CBSE OD 11]

45. Give four reasons why nichrome element is commonly used in household heating appliances.

46. What is a safety fuse ? Explain its function. [Punjab 99]

47. State the two Kirchhoff's rules used in electric networks. How are these rules justified ? [CBSE D 14, OD 15]

48. State the working principle of a potentiometer. Explain, with the help of a circuit diagram, how the emfs of two primary cells are compared by using a potentiometer. How can the sensitivity of a potentiometer be increased ? [CBSE D 05, 06C ; OD 15 C]

49. State the principle of a potentiometer. With the help of a circuit diagram, describe a method to find the internal resistance of a primary cell. [CBSE D 03 ; OD 13]

50. You are required to find the internal resistance of a primary cell in the laboratory. Draw a circuit diagram of the apparatus you will use to determine it. Explain the principle of the experiment. Give the formula used. [CBSE D 08C]

51. Why is the use of a potentiometer preferred over that of a voltmeter for the measurement of emf of a cell ? [Himachal 01]

52. Use Kirchhoff's rules to obtain conditions for the balance condition in Wheatstone bridge. [CBSE D 15]

53. For the circuit diagram of a Wheatstone bridge shown in the figure, use Kirchhoff's laws to obtain its balance condition. [CBSE D 09C]

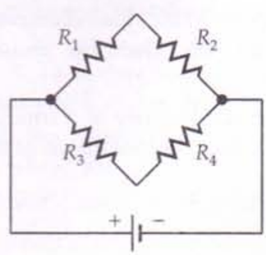


Fig. 3.331

54. State, with the help of a suitable diagram, the principle on which the working of a metre bridge is based. Under what condition is the error in determining the unknown resistance minimized ? [CBSE F 10, 13, OD 15C]

55. Draw a circuit diagram which can be used to determine the resistance of a given wire. Explain the principle of the experiment and give the formula used. [CBSE D 03C]

56. Draw a circuit diagram using a metre bridge and write the necessary mathematical relation used to determine the value of an unknown resistance. Why cannot such an arrangement be used for measuring very low resistances ? [CBSE D 06]

57. Draw a circuit diagram of a metre bridge to compare two resistances. Write the formula used. Why is this method suitable only for two resistances of the same order of magnitude ? [CBSE F 99]

58. Derive an expression for the heat produced in a resistor R when voltage drop across it is V. [CBSE F 93]

Answers

1.

	EMF	*Terminal Voltage*
(*i*)	It is the potential difference between two terminals of the cells when no current is drawn from it.	It is the potential difference between two terminals when a current passes through it.
(*ii*)	It is a cause.	It is an effect.
(*iii*)	The SI unit is volt.	The SI unit is volt.

2. Refer to the solution of Problem 1(*b*) on page 3.122.

3. Refer to points 10 and 13 of glimpses on page 168.

4. Resistivity of a material is the resistance of a conductor of that material having unit length and unit area of cross-section.

$$\text{Conductivity} = \frac{1}{\text{Resistivity}}$$

Resistivity of conductor depends on the nature of its material and its temperature.

5. Refer answer to Q. 19 on page 3.16

6. Refer answer to Q. 19 on page 3.16.

7. Refer answer to Q. 19 on page 3.16.

8. Refer answer to Problem 1(*e*) on page 3.122 and Q. 19 on page 3.16.

9. The average time that elapses between two successive collisions of an electron in a conductor is called relaxation time (τ). It is related to resistivity ρ as

$$\rho = \frac{m}{ne^2\tau}.$$

With the increase in temperature, the electrons collide more frequently with positive metal ions. So their relaxation time decreases.

10. Drift speed, $v_d = \frac{eV}{ml}\tau$; Resistance, $R = \rho\frac{l}{A}$

When l is tripled

(*i*) drift-speed becomes 1/3 times the original v_d

(*ii*) resistance becomes 3 times the original resistance

(*iii*) resistivity is not affected.

11. $\text{Mobility} = \frac{\text{Drift velocity}}{\text{Electric field}}$ or $\mu = \frac{v_d}{E}$

(*i*) The charge carriers in an electrolyte are positive and negative ions.

(*ii*) The charge carriers in an ionised gas are electrons and positively charged.

12. Refer answer to Q. 12 on page 3.7

13. Refer to point 21 of Glimpses.

Resistivity, $\rho = \frac{m}{ne^2\tau}$

As temperature increases, average speed of electrons increases. This increases collision frequency and decreases relaxation time τ. But n increases more rapidly with temperature. The increase in n more than compensates the decrease in τ. So ρ of semiconductors decreases with temperature.

14. Refer to point 17 of Glimpses.

$$v_d = \frac{e\mathcal{E}}{mL}\tau$$

When length is increased to $3L$, drift velocity becomes 1/3 times the original v_d.

15. Refer answer to Q. 19 on page 3.16.

16. Refer answer to Q. 19 on page 3.16.

17. Resistivity of a material is the resistance of a conductor of that material having unit length and unit area of cross-section. The SI unit of resistivity is ohm metre (Ωm).

$$\text{Resistivity of a conductor} = \frac{m}{ne^2\tau}$$

With the increase in temperature, the amplitude of vibration of positive ions increases. The electrons suffer collisions more frequently. The relaxation time τ decreases. Hence the resistivity of a conductor increases with the increase in temperature.

18. Refer answer to Q. 25 on page 3.24.

19. The conductivity of a conductor is the reciprocal of its resistivity. Its SI unit is $\Omega^{-1}m^{-1}$.

The conductivity of an ionic conductor increases with the increase of temperature. As the temperature increases, the electrostatic attraction between cations and anions decreases, the ions are more free to move and so the conductivity increases.

20. Refer answer to Q. 18 on page 3.15.
21. Refer answer to Q. 19 on page 3.16.
22. Refer answer to Q. 21 on page 3.17.
23. Refer answer to Q. 15 on page 3.9.
24. Refer answer to Q. 32 on page 3.30.
25. Refer answer to Q. 33 on page 3.31.
26. Refer answer to Q. 28 on page 3.29.
27. Refer answer to Q. 30 on page 3.30.
28. Refer answer to Q. 29 on page 3.29.
29. Refer answer to Q. 27 on page 3.28.
30. Refer answer to Q. 27 on page 3.28.
31. Refer answer to Q. 34 on page 3.45.
32. Refer answer to Q. 35 on page 3.46.
33. Refer answer to Q. 35 on page 3.46.
34. Refer answer to Q. 35 on page 3.46.

We get $V = \dfrac{\varepsilon R}{R+r} = \dfrac{\varepsilon}{1+(r/R)}$

Clearly V decreases when R is reduced.

35. Refer answer to Q. 35 on page 3.46.
36. (*i*) See Fig. 3.91(*b*) (*ii*) See Fig. 3.91(*c*) on page 3.46.
37. Refer answer to Q. 35 on page 3.46.
38. Refer answer to Q. 35 on page 3.46.
39. $I = \dfrac{\varepsilon}{R+\dfrac{r\times r}{r+r}} = \dfrac{\varepsilon}{R+\dfrac{r}{2}} = \dfrac{2R\varepsilon}{2R+r}$
40. Refer to the answers of Q. 37, 38 on pages 3.51 & 3.52.
41. Alloy like manganin has a small value of temperature coefficient of resistivity. It is used for making standard resistances.
42. Refer answer to Q.44 and Q. 45. on pages 3.60 & 3.61.
43. Refer answer to Q. 44 on page 3.60.
44. Refer to answers of Q. 47, 48 on page 3.61 & 3.62.
45. Refer answer to Q. 54 (Application 1) on page 3.64.
46. Refer answer to Q. 54 (Application 3) on page 3.64.
47. Refer to point 47 of Glimpses on page 3.172.
48. Refer answers to Q. 57 and Q. 58 on page 3.96.
49. Refer answer to Q. 59 on page 3.97.
50. Refer answer to Q. 59 on page 3.97.
51. Refer answer to Q. 60 on page 3.97.
52. Refer answer to Q. 62 on page 3.104.
53. Refer answer to Q. 62 on page 3.104.
54. See Fig. 3.213. The working of a metre bridge is based on the principle of Wheatstone bridge. When the bridge is balanced *i.e.*, no current flows through the galvanometer arm, $\dfrac{P}{Q} = \dfrac{R}{S}$

Error in determination of resistance can be minimised by adjusting the balance point near the middle of the metre bridge wire.

55. Refer answer to Q. 65 on page 3.105.
56. Refer answer to Q. 65 on page 3.105.

Metre bridge becomes insensitive for very low resistance. Moreover, the end resistances become comparable to the unknown low resistance and cannot be neglected.

57. Refer answer to Q. 65 on page 3.105 and Problem 71 on page 3.120.
58. Refer answer to Q. 43 on page 3.60.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. Define the term resistivity and write its SI unit. Derive the expression for the resistivity of a conductor in terms of number density of free electrons and relaxation time. [CBSE D 05]
2. What do you understand by the resistivity of a conductor ? Discuss its temperature dependence for a (*i*) conductor (*ii*) semiconductor, and (*iii*) electrolyte. [CBSE D 92C]
3. A battery of n cells, each of emf ε and internal resistance r, is connected across an external resistance R. Find the current in the circuit. Discuss the special cases when (*i*) $R >> nr$ and (*ii*) $R << nr$.
4. n cells, each of emf ε and internal resistance r are connected in parallel across an external resistance R. Determine the condition for maximum current in the circuit.
5. A mixed grouping of cells has m rows of cells connected in parallel across an external resistance R. Each row contains n cells in series. Each cell has emf ε and internal resistance r.

Show that the current in the circuit will be maximum when $R = nr/m$.

6. State Kirchhoff's laws for an electrical network.

Using Kirchhoff's laws, find the relation between the resistances of four arms of a Wheatstone bridge when the bridge is balanced.

Draw a circuit diagram to determine the unknown resistance of a metallic conductor using a metre bridge. [CBSE OD 03C ; D 13]

7. Define the term potential gradient. Using this concept, explain the method for comparison of

emfs of two primary cells using a potentiometer. Establish the relation used. Write two possible causes of potentiometer giving only one-sided deflection. [CBSE D13]

8. (*a*) State the working principle of a potentiometer. Draw a circuit diagram to compare the emfs of two primary cells. Derive the formula used. (*b*) Which material is used for potentiometer wire and why ? (*c*) How can the sensitivity of a potentiometer be increased ? [CBSE D 11 C]

9. Deduce the condition for balance in a Wheatstone bridge. Using the principle of Wheatstone bridge, describe the method to determine the specific resistance of a wire in the laboratory. Draw the circuit diagram and write the formula used. Write any two precautions you would observe while performing the experiment. [CBSE D 04]

10. (*a*) State, with the help of a circuit diagram, the working principle of a metre bridge. Obtain the expression used for determining the unknown resistance.
    (*b*) What happens if the galvanometer and cell are interchanged at the balance point of the bridge ?
    (*c*) Why it is considered important to obtain the balance point near the midpoint of the wire ? [CBSE D 11 C]

Answers

1. Refer answer to Q. 19 on page 3.16.
2. Refer answer to Q. 25 on page 3.24.
3. Refer answer to Q. 39 on page 3.53.
4. Refer answer to Q. 40 on page 3.53.
5. Refer answer to Q. 41 on page 3.54.
6. Refer answer to Q. 62 on page 3.104 and see Fig. 3.214 on page 3.106.
7. Refer answer to Q. 58 on page 3.96 and Problem 42 on page 3.142.
8. (*a*) Refer answer to Q. 57 and Q. 58 on page 3.96.
   (*b*) Refer to the solution of Problem 49 on page 3.119.
   (*c*) Refer to the solution of Problem 56 on page 3.119.
9. Refer answers to Q. 62 on page 3.104 and Q. 65 on page 3.105.
10. (*a*) Refer answer to Q. 65 on page 3.105.
    (*b*) There is no change in the position of the balance of the bridge.
    (*c*) Refer to the solution of Problem 65 on page 3.119.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Mrs. Sharma parked her car and forgot to switch off the car headlights. When she returned, she could not start the car. Rohit, a passerby, came to her for help. After knowing about her problem, he went to a nearby garage and called mechanic Ramu. Ramu noticed that the car battery has been discharged as the headlights were left on for a long time. He brought another battery from his garage and connected its terminals to the terminals of the car battery. He succeeded in starting the engine and then disconnected his battery. This is called 'jump starting', Mrs. Sharma felt happy and thanked both Rohit and Ramu. Answer the following questions based on the above information :
   (*a*) What values were displayed by Rohit ?
   (*b*) A storage battery of emf 12 V and internal resistance 0.5 Ω is to be charged by a battery charger which supplies 110 V dc. How much resistance must be connected in series with the battery to limit the charging current to 5 A. What will be the p.d. across the terminals of the battery during charging ? What is the purpose of having a series resistor in the charging circuit ?

2. Manish and Rajnish lived in an unauthorised colony. They found that most people of that colony stole power from transmission lines using hooks. They had read in the newspapers about different fire accidents caused due to electric short circuits. Alongwith some of their friends and some responsible representatives of that area, they visited house to house of that colony and made people aware of the risks involved in short circuiting. They also explained the people the importance of paying electricity bills. They succeeded in changing the mindset of the people. Answer the following questions based on the above information :
   (*a*) What according to you, are the values of displayed by Manish and Rajnish ?
   (*b*) A household circuit has a fuse of 5 A rating. Find the maximum number of bulbs of rating 60 W – 220 V each which can be connected in this circuit.

3. Abhishek went to meet his grandfather who lived in a village. Both were resting and gossiping under a fan to get relief from the scorching heat of summer. The lights suddenly went off. On seeing that, that all their neighbourers had electricity,

grandfather told Abhishek the fuse might have blown up. Abhishek immediately changed the fuse. Grandfather blessed Abhishek and told him that if had he not come to the village, he would have to sleep the whole night without fan. Abhishek realised his grandfather's problem and decided to replace the fuse with a circuit breaker which uses a solenoid with a core. When the current exceeds a safety limit, the breaker is activated and thus breaks the circuit. The circuit can be closed by a manual switch.

Answer the following questions based on the above information :

(*a*) What were the values displayed by Abhishek ?

(*b*) A low voltage supply from which one needs high current must have very high internal resistance. Why ?

4. Ameen had been getting huge electricity bill for the past few months. He was upset about this. One day his friend Rohit, an electrical engineer by profession, visited his house. When he pointed out his anxiety about this to Rohit, his friend found that Ameen was using traditional incandescent lamps and using old fashioned air conditioner. In addition there was no proper earthing in the house. Rohit advised him to use CFL bulbs of 28 W instead of 1000 W – 220 V and also advised him to get proper earthing in the house. He made some useful suggestion and asked him to spread this message to his friends also. [CBSE D 15C]

(*a*) What qualities/values, in your opinion did Rohit possess ?

(*b*) Why CFLs and LEDs are better than traditional incandescent lamps ?

(*c*) In what way earthing reduces electricity bill ?

Answers

1. (*a*) Helpful, aware of his limits, ability to take quick decisions.

(*b*) Net emf, $\varepsilon = 110 - 12 = 98\text{V}$

If R is the series resistor, then the charging current will be

$$I = \frac{\varepsilon}{R+r} = \frac{98}{R+0.5}\text{A} = 5\text{ A (given)} \quad \therefore\ R = \mathbf{19.1\,\Omega}$$

Terminal p.d. of the battery during charging,

$$V = \varepsilon + Ir = 12 + 5\times 0.5 = \mathbf{14.5\ V}$$

If the series resistor R were not included in charging circuit, the charging current would be $\frac{98}{0.5} = 196\text{A}$, which is dangerously high.

2. (*a*) Critical thinking, decision making, team spirit and assertive communication.

(*b*) Current drawn by one bulb $= \frac{P}{V} = \frac{60}{220} = \frac{3}{11}\text{A}$

Number of bulbs that can be safely used with 5 A fuse $= \frac{5}{3/11} = \frac{55}{3} = 18.33$

Hence 18 bulbs can be safely used with 5 A fuse.

3. (*a*) Empathy, dutifulness, determination, responsibility and compassion.

(*b*) The maximum current that can be drawn from a voltage supply is given by

$$I_{max} = \frac{\varepsilon}{r}$$

Clearly, I_{max} will be large if r is small.

4. (*a*) Helpfulness, co-operative attitude and scientific temperament.

(*b*) CFLs and LEDs have following advantages :

(*i*) Low operational voltage and less power consumption.

(*ii*) Fast action and require no warm up time.

(*c*) In the absence of proper earthing, the consumer can get extra charges in his bill for the electrical energy not actually consumed by his/her devices.

COMPETITION SECTION

Current Electricity

GLIMPSES

1. Current electricity. The study of electric charges in motion is called current electricity.

2. Electric current. The flow of electric charges through a conductor constitutes electric current. Quantitatively, electric current across an area held perpendicular to the direction of flow of charge is defined as the amount of charge flowing across that area per unit time.

   For a steady flow of charge, $I = \frac{Q}{t}$

   If the rate of flow of charge varies with time, then

   $$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}.$$

   SI unit of current is *ampere* (A).

   $$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}} \quad \text{or} \quad 1\text{ A} = 1\text{ Cs}^{-1}.$$

3. Conventional and electronic currents. The direction of motion of positive charges is taken as the direction of conventional current. Electrons being negatively charged, so the direction of electronic current is opposite to that of the conventional current.

4. Electric current is a scalar quantity. Although we represent current with an arrow, yet it is a scalar quantity. Electric currents do not obey the laws of vector addition.

5. Electromotive force (emf). The emf of a source may be defined as the work done by the source in taking a unit positive charge from its lower potential terminal to the higher potential terminal. Or, it is the energy supplied by the source in taking a unit positive charge once round the complete circuit. It is equal to the terminal p.d. measured in open circuit.

   $$\text{EMF} = \frac{\text{Work done}}{\text{Charge}} \quad \text{or} \quad \mathcal{E} = \frac{W}{q}.$$

6. SI unit of emf is volt. If an electrochemical cell supplies energy of 1 joule for the flow of 1 coulomb of charge through the whole circuit (including the cell), then its emf is said to be one volt.

7. Ohm's law. The current flowing through a conductor is directly proportional to the potential difference across its ends, provided the temperature and other physical conditions remain unchanged.

   $$V \propto I \quad \text{or} \quad V = RI \quad \text{or} \quad \frac{V}{I} = R$$

   Here R is called the resistance of the conductor.

8. Resistance. It is the property by virtue of which a conductor opposes the flow of charges through it. It is equal to the ratio of the potential difference applied across the conductor to the current flowing through it. It depends on the length l and area of cross-section A of the conductor through the relation :

   $$R = \rho \frac{l}{A}, \; \rho = \text{resistivity of the material.}$$

9. SI unit of resistance is ohm (Ω). The resistance of a conductor is 1 ohm if a current of 1 ampere flows through it on applying a potential difference of 1 volt across its ends.

   $$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}} \quad \text{or} \quad 1\,\Omega = 1\text{ VA}^{-1}$$

10. Resistivity or specific resistance. It is the resistance offered by a unit cube of the material of a conductor.

$$\rho = \frac{RA}{l}$$

SI unit of $\rho = \Omega$ m

It depends on the nature of the material of the conductor and the physical conditions like temperature, pressure, etc.

11. Current density. It is the amount of charge flowing per second per unit area normal to the flow of charge. It is a vector quantity having the same direction as that of the motion of the positive charge.

For normal flow of charge,

$$j = \frac{q/t}{A} = \frac{I}{A}$$

In general, $I = jA \cos\theta = \vec{j} \cdot \vec{A}$

SI unit of current density = Am^{-2}.

12. Conductance. It is the reciprocal of resistance.

$$\text{Conductance} = \frac{1}{\text{Resistance}}$$

or $$G = \frac{1}{R}$$

SI unit of conductance = ohm^{-1} = mho
= siemen (S).

13. Conductivity or specific conductance. It is the reciprocal of resistivity.

$$\text{Conductivity} = \frac{1}{\text{Resistivity}}$$

or $$\sigma = \frac{1}{\rho}$$

SI unit of conductivity

$$= \text{ohm}^{-1}\ \text{m}^{-1} = \text{mho m}^{-1} = \text{S m}^{-1}$$

14. Resistivities of different substances. *Metals* have low resistivities in the range of $10^{-8}\ \Omega$ m to $10^{-6}\ \Omega$ m. *Insulators* have resistivities more than $10^{4}\ \Omega$ m. *Semiconductors* have intermediate resistivities lying between $10^{-6}\ \Omega$ m to $10^{4}\ \Omega$ m.

15. Colour code for carbon resistors.

Colour	Number	Multiplier
Black	0	10^0
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Grey	8	10^8
White	9	10^9

How to remember colour code :

B	B	R	O	Y	of	Great	Britain	had	Very	Good	Wife
↓	↓	↓	↓	↓		↓	↓		↓	↓	↓
0	1	2	3	4		5	6		7	8	9

Tolerence :

Gold	Silver	No colour
± 5%	± 10%	± 20%

A set of coloured co-axial bands is printed on the resistor which reveals the following facts :

(1) The first band indicates the *first significant figure.*

(2) The second band indicates the *second significant figure.*

(3) The third band indicates the *power of ten* with which the above two significant figures must be multiplied to get the resistance value in ohms.

(4) The last band indicates the tolerence in per cent of the indicated value.

16. Carriers of current. In metals, free electrons are the charge carriers. In ionised gases, electrons and positively charged ions are the charge carriers. In an electrolyte, both positive and negative ions are the charge carriers. In semiconductors like Ge and Si, conduction is due to electrons and holes. A hole is a vacant state from which an electron has been removed and acts as a positive charge carrier.

17. Drift velocity and relaxation time. The average velocity acquired by the free electrons of a conductor in the opposite direction of the externally applied electric field is called drift velocity (v_d). The average time that elapses between the two successive collisions of an electron is called relaxation time (τ).

$$v_d = \frac{eE\tau}{m} \;;\quad R = \frac{ml}{ne^2 \tau A} \;;\quad \rho = \frac{m}{ne^2 \tau};$$

$$I = en\,A v_d \;;\; j = en\,v_d$$

Here n = no. of free electrons per unit volume or free electron density and m = mass of an electron.

18. Other forms of Ohm's law. In terms of vector quantities like current density $\vec{j}$ and electric field $\vec{E}$, Ohm's law may be expressed as

$$\vec{j} = \sigma \vec{E} \quad \text{or} \quad \vec{E} = \rho \vec{j}$$

The equation $\vec{E} = \rho \vec{j}$ leads to another statement of Ohm's law *i.e.*, a conducting material obeys Ohm's law when the resistivity of the material does not depend on the magnitude and direction of the applied electric field.

19. Temperature coefficient of resistance (α). It is defined as the change in resistance per unit original resistance per degree rise in temperature. It is given by

$$\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$$

If $t_1 = 0°C$ and $t_2 = t°C$, then

$$\alpha = \frac{R_t - R_0}{R_0 \times t} \quad \text{or} \quad R_t = R_0 (1 + \alpha t)$$

The unit of α is $°C^{-1}$ or K^{-1}.

20. Effect of temperature on resistance. *For metals* α is positive *i.e.*, resistance of metals increases with the increase in temperature.

For semiconductors and insulators, α is negative *i.e.*, their resistance decreases with the increase in temperature.

For alloys like constantan and manganin, the temperature coefficient of resistance α is very small. So they are used for making standard resistors.

21. Mobility of a charge carrier. The mobility of a charge carrier is the drift velocity acquired by it in a unit electric field. It is given by

$$\mu = \frac{v_d}{E} = \frac{q\tau}{m}$$

For an electron, $\mu_e = \dfrac{e\tau_e}{m_e}$

For a hole, $\mu_h = \dfrac{e\tau_h}{m_h}$

SI unit of mobility = $m^2\,V^{-1}\,s^{-1}$

Practical unit of mobility = $cm^2\,V^{-1}\,s^{-1}$.

22. Relation between electric current and mobility. For a conductor,

$$I = en\,A\mu_e\,E$$

For a semiconductor,

$$I = e\,AE(n\mu_e + p\mu_h)$$

and $\quad \sigma = e(n\mu_e + p\mu_h)$

where n and p are the electron and hole densities.

23. Ohmic conductors. The conductors which obey Ohm's law are called Ohmic conductors. For these conductors, V-I graph is a straight line passing through the origin. For example, a metallic conductor for small currents is an Ohmic conductor.

24. Non-ohmic conductors. The conductors which do not obey Ohm's law are called non-ohmic conductors. The non-ohmic situations may be of the following types :

(*i*) The straight line V-I graph does not pass through the origin.

(*ii*) V-I relationship is non-linear.

(*iii*) V-I relationship depends on the sign of V for the same absolute value of V.

(*iv*) V-I relationship is non-unique.

Examples of non-ohmic conductors are water voltameter, thyristor, a p-n junction, etc.

25. Superconductivity. The phenomenon of complete loss of resistivity by certain metals and alloys when they are cooled below a certain temperature is called superconductivity. The temperature at which a substance undergoes a transition from normal conductor to superconductor in a zero magnetic field is called transition or critical temperature (T_c).

26. Meissner effect. The expulsion of the magnetic flux from a superconducting material when it is cooled to a temperature below the critical temperature in a magnetic field is called Meissner effect.

27.. Resistances in series. When a number of resistances are connected in series, their equivalent resistance (R_s) is equal to the sum of the individual resistances.

$$R_s = R_1 + R_2 + R_3 + ...$$

28. Resistances in parallel. When a number of resistances are connected in parallel, the reciprocal of their equivalent resistance (R_p) is equal to the sum of the reciprocals of the individual resistances.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

For two resistances in parallel,

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

29. Division of current in resistors joined in parallel. The current is divided in resistors, connected in parallel, in the inverse ratio of their resistances.

$$I_1 = \frac{R_2}{R_1 + R_2} . I$$

$$I_2 = \frac{R_1}{R_1 + R_2} . I$$

30. Internal resistance (r). The resistance offered by the electrolyte of a cell to the flow of current between its electrodes is called internal resistance of the cell. It depends on (*i*) nature of the electrolyte, (*ii*) concentration of the electrolyte, (*iii*) distance between the electrodes, (*iv*) common area of the electrodes dipped in the electrolyte and (*v*) temperature of the electrolyte.

31. Relations between emf ($\mathcal{E}$), terminal potential difference (V) and internal resistance (r). The potential drop across the terminals of a cell when a current is being drawn from it is called its terminal potential difference. It is less than the emf of the cell in a closed circuit.

$$\mathcal{E} = V + Ir; \quad V = \mathcal{E} - Ir; \quad V = IR = \frac{\mathcal{E} R}{R + r}$$

$$r = \frac{\mathcal{E} - V}{I} = R\left[\frac{\mathcal{E} - V}{V}\right]; \; I = \frac{\mathcal{E}}{R + r}; \; I_{max} = \frac{\mathcal{E}}{r}$$

Terminal p.d. of a cell when it is being charged is

$$V = \mathcal{E} + Ir, \quad V > \mathcal{E}.$$

32. Cells in series. If n cells of emf $\mathcal{E}$ and internal resistance r each are connected in series, then current drawn through external resistance R is

$$I = \frac{n\mathcal{E}}{R + nr}$$

33. Cells in parallel. If m cells are connected in parallel, then current drawn through external resistance R is

$$I = \frac{m\mathcal{E}}{mR + r}$$

34. Cells in mixed grouping. If n cells are connected in series in each row and m such rows are connected in parallel, then current drawn through an external resistance R is

$$I = \frac{mn\mathcal{E}}{mR + nr}$$

For maximum current, the external resistance must be equal to the total internal resistance, *i.e.*,

$$R = \frac{nr}{m}$$

or $\quad mR = nr.$

35. Heating effect of current. The phenomenon of the production of heat in a resistor by the flow of an electric current through it is called heating effect of current or Joule heating. It is an irreversible process.

36. Joule's law of heating. It states that the amount of heat H produced in a resistor is

(*i*) directly proportional to the square of current for a given R,

(*ii*) directly proportional to the resistance R for a given I,

(*iii*) inversely proportional to the resistance R for a given V, and

(*iv*) directly proportional to the time t for which the current flows through the resistor.

Mathematically, this law can be expressed as

$$H = VIt \text{ joule}$$

$$= I^2Rt \text{ joule} = \frac{V^2 t}{R} \text{ joule}$$

or

$$H = \frac{VIt}{4.18} \text{ cal}$$

$$= \frac{I^2Rt}{4.18} \text{ cal} = \frac{V^2 t}{4.18\, R} \text{ cal}.$$

37. Electric power. It is the rate at which an electric appliance converts electric energy into other forms of energy. Or, it is the rate at which work is done by a source of emf in maintaining an electric current through a circuit.

Electric power,

$$P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$$

38. SI unit of power is watt. The power of an appliance is one watt if one ampere of current flows through it on applying a potential difference of 1 volt across it.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}}$$

$$= 1 \text{ volt} \times 1 \text{ ampere}$$

or $$1 \text{ W} = 1 \text{ Js}^{-1} = 1 \text{ VA}$$

1 kilowatt (kW) = 1000 W.

39. Electric energy. It is the total work done in maintaining an electric current in an electric circuit for a given time.

Electric energy = Electric power × time

$$W = Pt$$

$$= VI\,t \text{ joule} = I^2Rt \text{ joule}$$

40. Units of electric energy. The commercial unit of electric energy is kilowatt-hour (kWh) or Board of Trade (B.O.T.) unit. It is the electric energy consumed by an appliance of power 1000 watt in one hour.

$$1 \text{ kWh} = 1000 \text{ Wh}$$

$$= 1000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J}$$

Another unit of electric energy is watt hour.

$$1 \text{ watt hour} = 1 \text{ watt} \times 1 \text{ hour}$$

$$= 3.6 \times 10^3 \text{ J.}$$

41. Power rating. The power rating of an electrical appliance is the electrical energy consumed per second by the appliance when connected across the marked voltage of the mains.

$$P = \frac{V^2}{R} = I^2R = VI.$$

42. Power consumed by a series combination of appliances. The reciprocal of the effective power is equal to the sum of the reciprocals of the individual powers of the appliances which have been manufactured for working on the same voltage.

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \ldots\ldots$$

43. Power consumed by a parallel combination of appliances. The effective power is equal to the sum of the powers of the individual appliances.

$$P = P_1 + P_2 + P_3 + \ldots\ldots$$

44. Efficiency of a source of emf. It is the ratio of the output power to the input power. If a source of emf $\mathcal{E}$ and internal resistance r is connected to an external resistance, then its efficiency will be

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$= \frac{VI}{\mathcal{E}I} = \frac{V}{\mathcal{E}} = \frac{R}{R+r}.$$

45. Maximum Power Theorem. It states that the output power of a source of emf is maximum when the external resistance in the circuit is equal to internal resistance of the circuit *i.e.,* when $R = r$.

$$P_{\max} = \frac{\mathcal{E}^2}{4r}.$$

The efficiency of a source of emf is 50% when it delivers maximum power.

46. Efficiency of an electric device. It is the ratio of the output power to the input power.

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

(*i*) For an electric motor,

$$\eta = \frac{\text{Output mechanical power}}{\text{Input electric power}}$$

(*ii*) Input electric power

= Output mechanical power

+ Power lost as heat

The power output of an electric motor is maximum when its back emf is one-half the source emf, provided the resistance of the windings of the motor is negligible.

47. Kirchhoff's laws. These laws enable us to determine the currents and voltages in different parts of the electrical circuits.

First law or junction rule. In an electric circuit, the algebraic sum of currents at any junction is zero. Or, the sum of currents entering a junction is equal to the sum of the currents leaving that junction.

Mathematically,

$$\Sigma I = 0$$

Justification. This law is based on the law of conservation of charge. When the currents in a circuit are steady, charges cannot accumulate or originate at any point of the circuit.

Second law or loop rule. Around any loop of a network, the sum of changes in potential must be zero. Or, the algebraic of the emfs in any loop of a circuit is equal to the sum of the products of currents and resistances in it.

$$\Sigma \Delta V = 0$$

or $$\Sigma \mathcal{E} = \Sigma IR$$

Justification. This law is based on the law of conservation of energy. As the electrostatic force is a conservative force, the total work done by it along any closed path must be zero.

48. Galvanometer. It is a sensitive device to detect current in a circuit. It produces a deflection proportional to the electric current flowing through it.

49. Potentiometer. It is a device used to compare emfs of two sources. Its working is based on the principle that when a constant current flows through a wire of uniform cross-sectional area and composition, the p.d. across any length of the wire is directly proportional to that length.

$$V \propto l$$

or $$V = kl$$

where k is the potential drop per unit length which is called potential gradient. Potentiometer has two main uses.

(*i*) ***To compare the emfs of two cells.*** If l_1 and l_2 are the balancing lengths of the potentiometer wire for the cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ respectively, then

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1}.$$

(*ii*) ***To find the internal resistance r of a cell.*** If l_1 is the balancing length of the potentiometer wire without shunt and l_2 the balancing length with shunt R across the cell, then internal resistance of the cell will be

$$r = \frac{\mathcal{E} - V}{V} \times R = \frac{l_1 - l_2}{l_2} \times R$$

50. Wheatstone bridge. It is an arrangement of four resistances P, Q, R and S joined to form a quadrilateral $ABCD$ with a battery between A and C and a sensitive galvanometer between B and D. The resistances are so adjusted that no current flows through the galvanometer. The bridge is then said to be balanced. In the balanced condition,

$$\frac{P}{Q} = \frac{R}{S}$$

Knowing any three resistances, the fourth resistance can be computed. A wheatstone bridge is most sensitive when the resistances in its four arms are of the same order.

51. Slide wire bridge or metre bridge. It is an application of wheatstone bridge in which R is fixed and a balance point is obtained by varying P and Q *i.e.*, by adjusting the position of a jockey on a 100 cm long resistance wire stretched between two terminals. If the balance point is obtained at length l, then

$$\frac{P}{Q} = \frac{R}{S} = \frac{l}{100 - l}$$

or $$S = \left(\frac{100 - l}{l}\right) R$$

Resistivity, $$\rho = \frac{SA}{l} = \frac{S \times \pi r^2}{l}$$